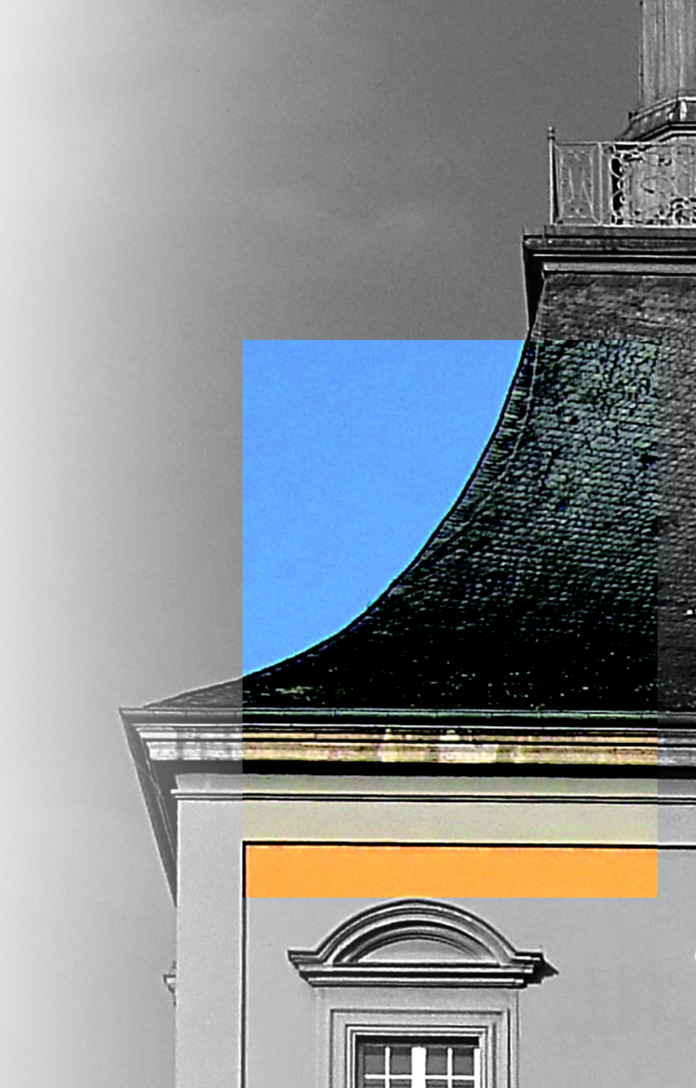
SIGNAL FORMATION AND SIGNAL PROCESSING IN DETECTORS

LECTURES AT THE UNIVERSITY OF FREIBURG

MARCH 9-14, 2020

LECTURE 1

NORBERT WERMES
UNIVERSITY OF BONN



Outline overview (signals, signal fluct., signal processing)

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1. What is a detector “signal”?

2. Charge transport in gases and solids

3. Induced signals on electrodes
   - Schottky-Ramo Theorem
   - Current, charge or voltage?
   - Applying SRT to detectors
   - SRT for structured electrodes
   - (calculation of E_W by “conformal mapping”)

4. Signal fluctuations and (electronic) noise
   - Why bother?
   - Signal fluctuations (Fano noise)
   - Electronics noise
     - shot, white, parallel, series, thermal, RTS, Johnson Nyqvist, 1/f, popcorn, flicker, resistor noise, current noise, … and all that

5. Readout of signals
   - Amplification
   - (Excursion: Laplace transform)
   - Filtering
   - Discrimination
   - Digitisation (digit. errors)
   - (Example: a readout chip)

6. Signal transmission off detector

7. (Deadtime)

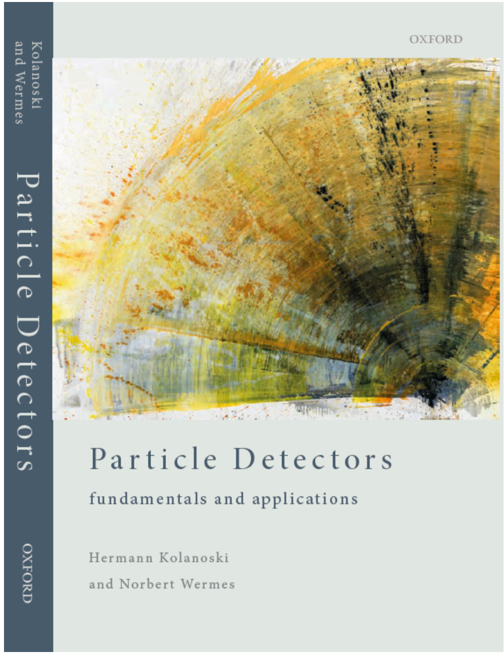
8. Noise of a readout system
   - CSA + shaper
   - Explicit calculation of noise
     - ATLAS pixel detector
     - ATLAS strip detector
     - ATLAS Liq. Argon calorimeter

Plan of Lectures

- 3 double lectures
- each 14:00 – 15:30 (with a break)
- dates

  09.03. + 10.03. + 11.03.

Content largely based on chapters of …



- Kolanoski, H. und Wermes, N.
  Teilchendetektoren – Grundlagen und Anwendungen
  (Springer/Spektrum 2016)

- Kolanoski, H. and Wermes, N. **(new edition)**
  Particle Detectors – fundamentals and applications
  (Oxford University Press 2020, in print)

What is a detector “signal”?

“Signals“ for detectors

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Any form of elementary excitation can be a “radiation signal“

• ionization	→	direct electrical signals (e^--ion, e-h)
• scintillation	→	excitation of optical states → light
• lattice vibrations	→	phonons
• break-up of Cooper pairs	→	superconducting detectors

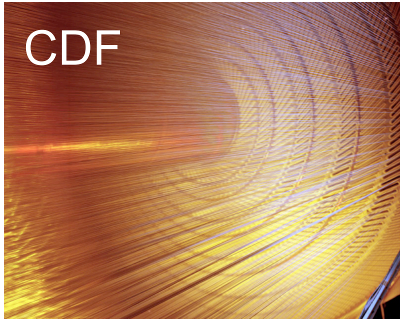
• necessary energies for the production of quanta

~ 30 eV	ionization in gases
~ 1- 5 eV	ionization in semiconductors
~ 10 eV	scintillating materials
~ meV	phonon excitation
~ meV	Cooper pair break up

Ionisation detectors

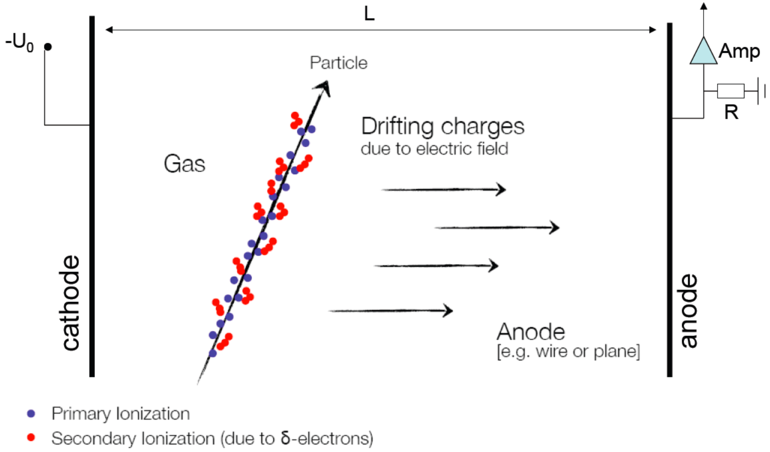
gaseous

MWPC



driftchamber

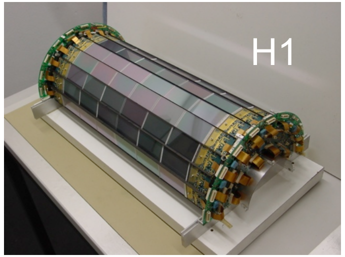
- 94 e/ion pairs per cm (in Ar)
  => (gas-)amplification needed
- "signal" due to separation and movement of charges in E-field
- => current on electrode charge -> V_{out} after amplifier



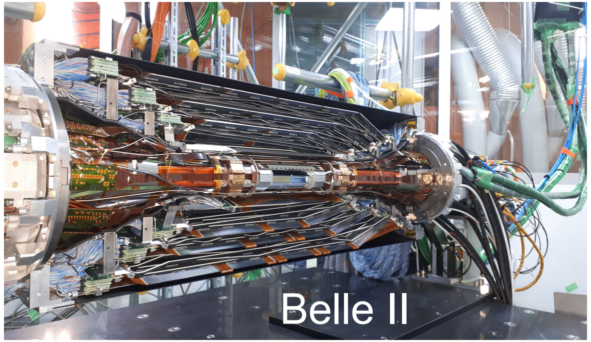
Detector types

Ionisation detectors

semiconductor

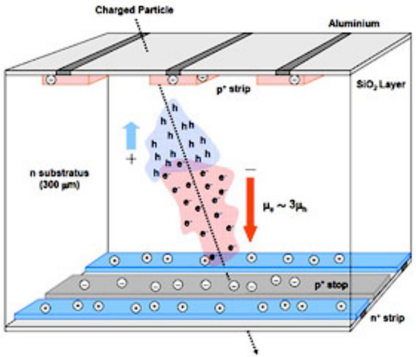


microstrips

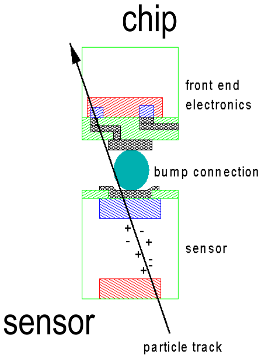


strips & pixels

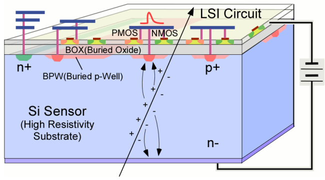
- ~10^6 e/h pairs per cm (20.000/300 µm)
- "signal" due to separation and movement of charges in E-field
- => current on electrode charge -> V_{out} after amplifier



strips



hybrid

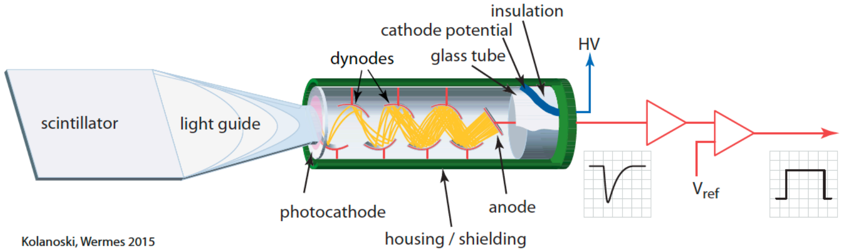


monolithic

pixels

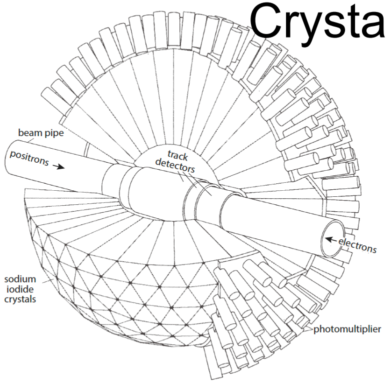
Detector types

Scintillation detectors



Plastic scintillator

Crystals

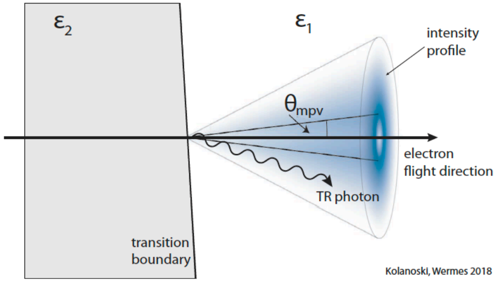


- 10.000 (plastic)/40.000 photons/MeV energy input
- "signal" due to conversion of photons (light) into charged electrons
  - intrinsic amplification => direct detection
  - detection by ionisation (PD, APD, SiPMs) then as before

Crystal Ball (NaI(Tl)) detector

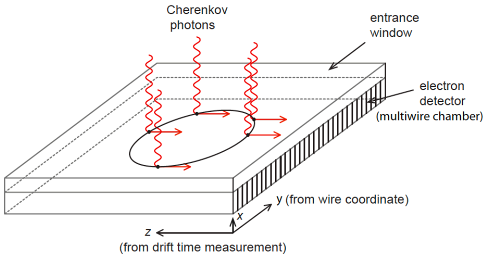
Cherenkov detectors Transition radiation detectors

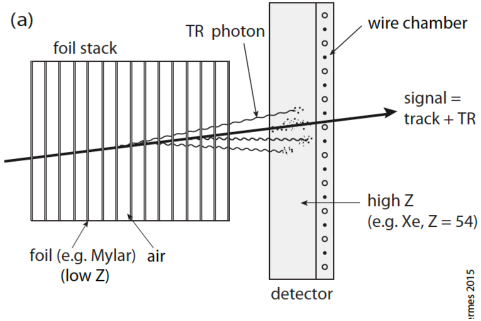
Cherenkov
UV light



Transition Radiation
X-rays

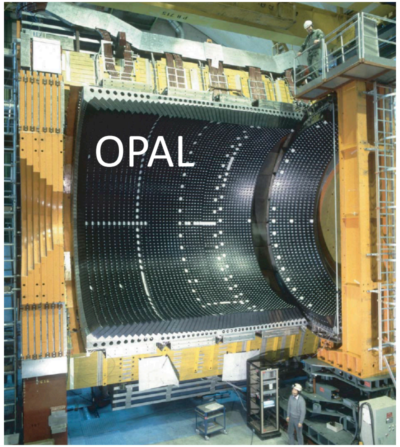
- very few to many photons emitted (UV or X-ray)
- conversion into electrons (Cherenkov) or
- direct absorption (TR) -> conversion into e/ion or e/h
- detection by ionisation detector
  then as before

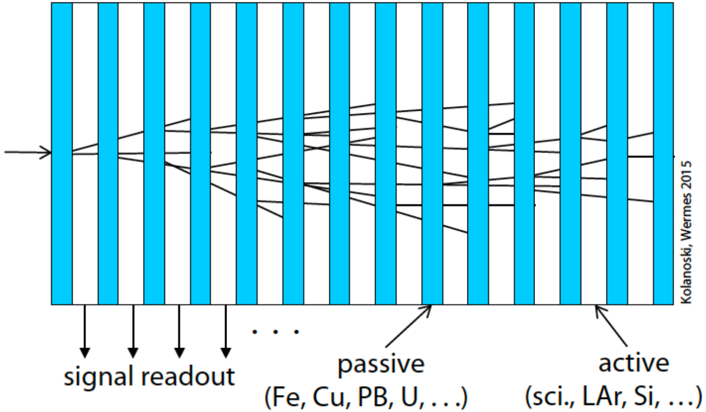




Calorimeters

crystal calo





sampling calo

- many e+/e- (ECAL), many charged particles (HCAL)
  neutral particles (n,γ,K^0) converted into charged
- "signal" due to
  - conversion of scintillation or Cherenkov radiation
    photons (light) converted into electrons (PMT) or e/h (PD; APD)
    (in crystal calorimeters)
    -> then amplification & direct detection
  - detection by ionisation (liq. Ar, Si pad, drift tubes (bad))
    then as before

(by far) the main signal producer for HEP applications

ionisation of separable charges
in a medium

=> focus in this lecture

TRANSPORT OF CHARGES IN DETECTORS

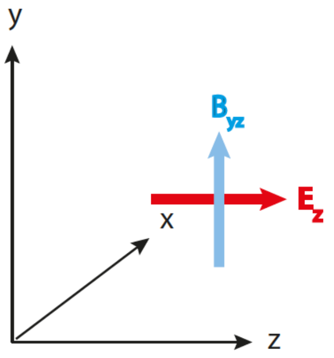
Transport of charges to the R/O electrode

Generally described by the Boltzmann Transport Equation

explicit t-dep. -> diffusion -> drift in E,B) interactions w/ medium atoms

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{d\vec{r}}{dt}\vec{\nabla}_{\vec{r}} f + \frac{d\vec{v}}{dt}\vec{\nabla}_{\vec{v}} f = \left.\frac{\partial f}{\partial t}\right|_{coll}}$$

total deriv. partial deriv. wrt. phase space variables ‘coll. integral



with f(r, v, t) describing the probability distribution of a charge cloud in phase

$$dp(\vec{r},\vec{v},t) = f(\vec{r},\vec{v},t)\, d^3\vec{r}\, d^3\vec{v}$$

Which can treat arbitrary E and B-fields … (E in z-direction, B in z-y-direction)

$$v^B_{D,1} = -\frac{4\pi}{3}\frac{qE}{m}\int_0^\infty \tau \frac{\omega_2\tau}{1+\omega^2\tau^2}\left(\frac{2\epsilon}{m}\right)^{3/2}\frac{\partial f_0}{\partial \epsilon}d\epsilon = \frac{qE}{m}\left\langle \tau \frac{-\omega_2\tau}{1+\omega^2\tau^2}\right\rangle_\epsilon$$

$$v^B_{D,2} = \frac{4\pi}{3}\frac{qE}{m}\int_0^\infty \tau \frac{\omega_2\omega_3\tau^2}{1+\omega^2\tau^2}\left(\frac{2\epsilon}{m}\right)^{3/2}\frac{\partial f_0}{\partial \epsilon}d\epsilon = \frac{qE}{m}\left\langle \tau \frac{\omega_2\omega_3\tau^2}{1+\omega^2\tau^2}\right\rangle_\epsilon$$

$$v^B_{D,3} = \frac{4\pi}{3}\frac{qE}{m}\int_0^\infty \tau \frac{1+\omega_3^2\tau^2}{1+\omega^2\tau^2}\left(\frac{2\epsilon}{m}\right)^{3/2}\frac{\partial f_0}{\partial \epsilon}d\epsilon = \frac{qE}{m}\left\langle \tau \frac{1+\omega_3^2\tau^2}{1+\omega^2\tau^2}\right\rangle_\epsilon$$

with

$\omega_i = qB_i/m$ = cyclotron frequencies

τ = mean collision time

ε = kin. energy

Usually in detectors: either $\boxed{\vec{E} \perp \vec{B}}$ or $\boxed{\vec{E} \parallel \vec{B}}$

Diffusion and drift of charge cloud on its way to the electrode

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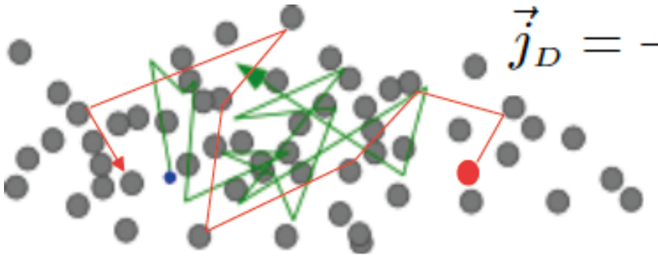
E = 0, T > 0: diffusion (needs dn/dx ≠ 0)



$$\vec{j}_D = -D\vec{\nabla}\rho$$

$$E_{th} = \frac{3}{2}kT = \frac{1}{2}m\langle v_{th}\rangle^2 \Rightarrow \langle v_{th}\rangle = \sqrt{\frac{3kT}{m}}$$

for $kT(@300K) = 25\text{meV}$

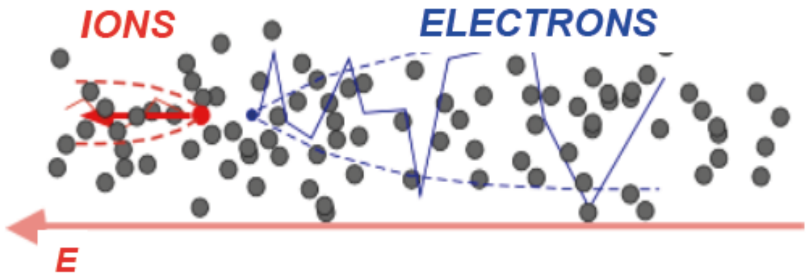
$$\Rightarrow v_{th}^e(Si) = \sqrt{\frac{75\text{ meV}}{0.5\text{MeV}}} \cdot c \approx \frac{5-10\text{cm}}{\mu s}$$

fast!!

$$D = \frac{1}{3}\langle\lambda \cdot v_{th}\rangle \approx \frac{1}{\sigma P}\frac{1}{\sqrt{m}}(kT)^{3/2}$$

$$\frac{1}{n\sigma} \approx \frac{kT}{pressure} \cdot \frac{1}{\sigma} \qquad \sqrt{\frac{3kT}{m}}$$

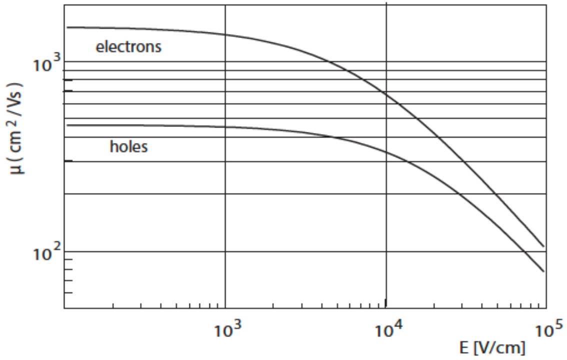
E > 0, T > 0: diffusion + drift



$$v_D = \mu(E)\,E = \frac{\mu_0 E}{\left[1 + \left(\frac{\mu_0 E}{v_{sat}}\right)^\beta\right]^{1/\beta}}$$

Drude Ansatz (emp.)
(especially semicond.)

β~1-2



MOBILITY

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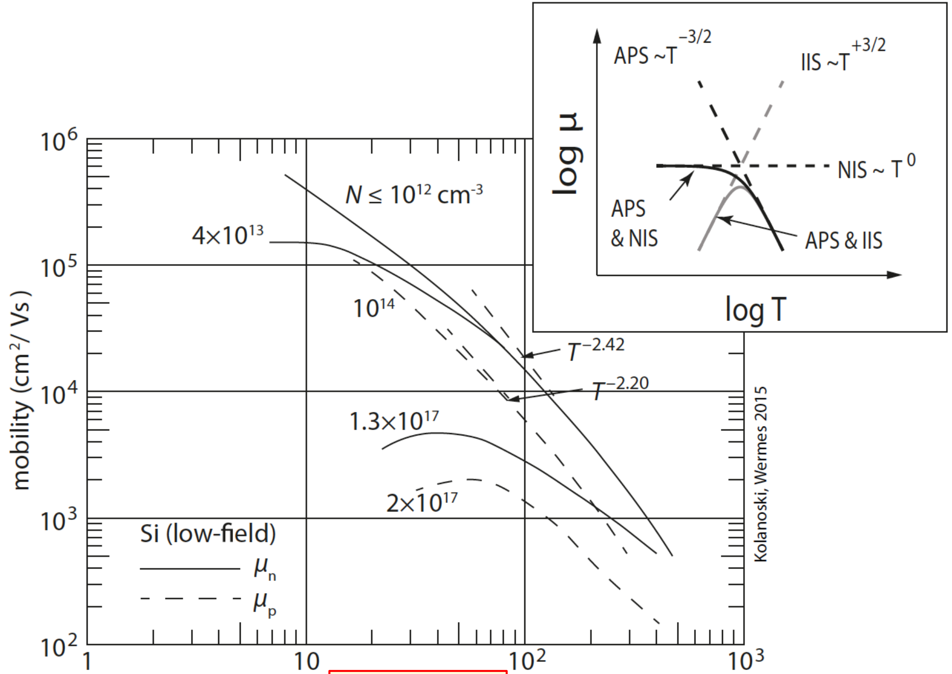
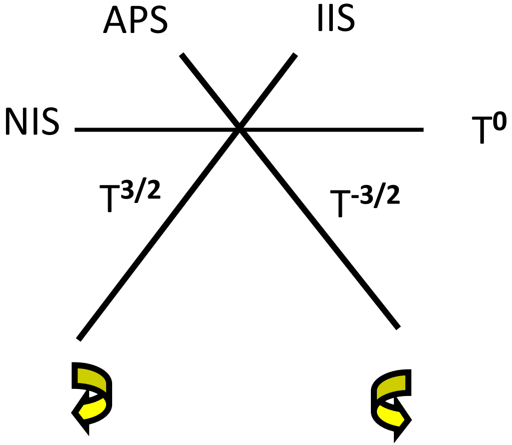
APS = acoustic phonon scattering (= lattice vibrations)
IIS = ionized impurity scattering
NIS = neutral impurity scattering



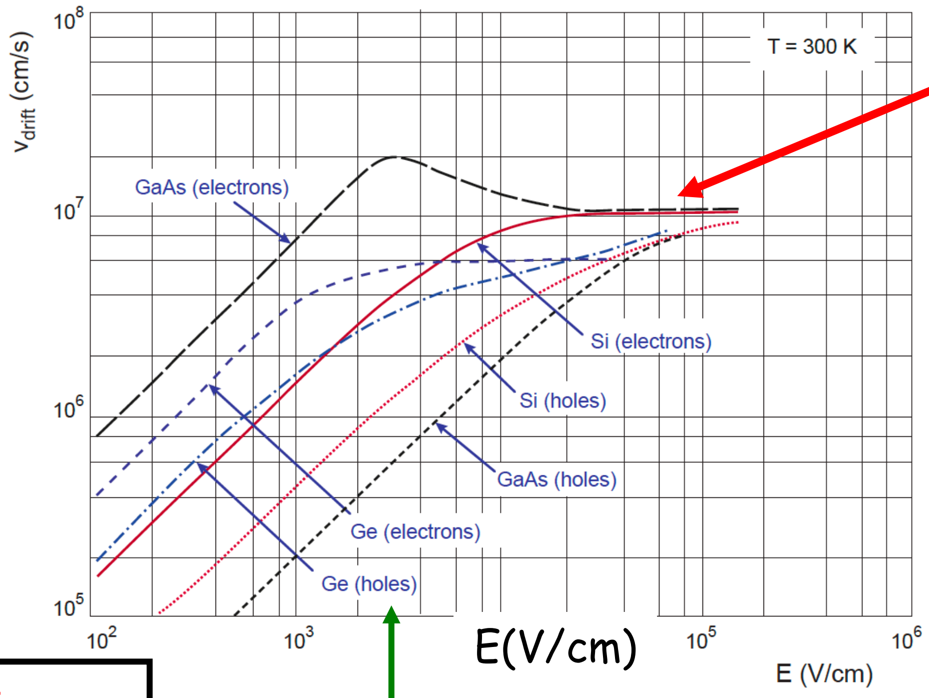


scattering on impurities
higher temp
⇒ less (~1/E²) deflection
⇒ higher mobility

scattering on the lattice
higher temp
⇒ more lattice vibrations
⇒ lower mobility

Drift of electrons and holes in semiconductors

drift velocity as a function of electric field strength

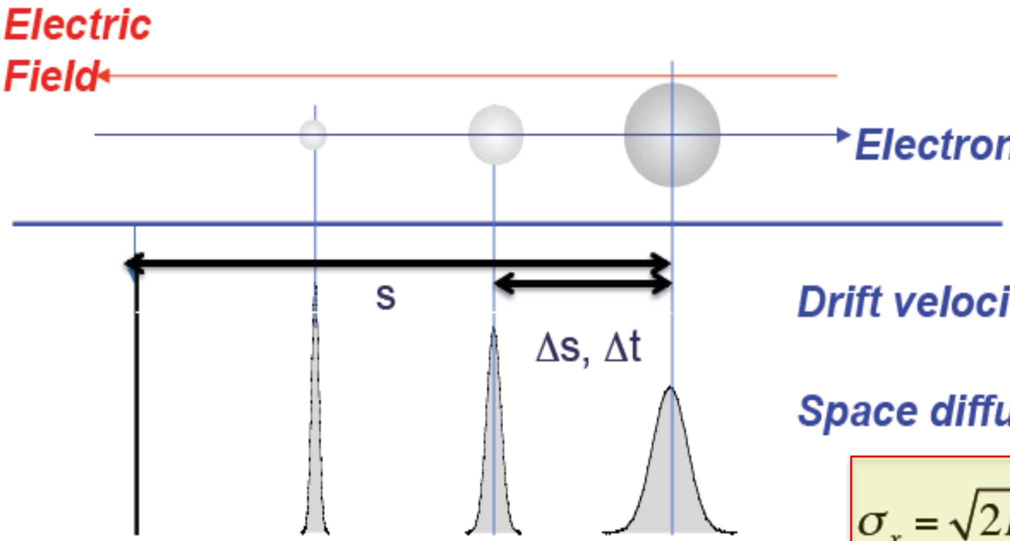


saturation
at ~10^7 cm/s

typical operation point for a Si - detector
100 V / 300 μm

remember
v = μ(E) E always !

Diffusion and drift of charge cloud on way to electrode



Drift velocity: $$v_D = \frac{\Delta s}{\Delta t}$$

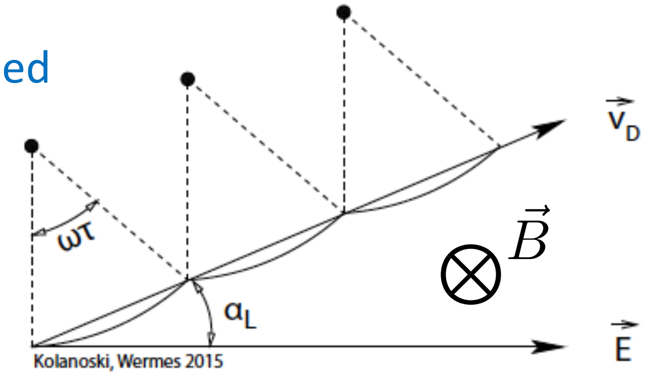
Space diffusion rms:

$$\boxed{\sigma_x = \sqrt{2Dt}} = \sqrt{2D\frac{s}{v_D}}$$

typical: $\sigma_x \approx 5\text{–}10\ \mu\text{m}$ for $d = 250\ \mu\text{m}$

Movement in the presence of a magnetic field

simplified



perp. to E

$$\boxed{\tan\,\alpha_L = \frac{v_{D,\perp}}{v_{D,||}} = \omega\tau}$$

parallel to E

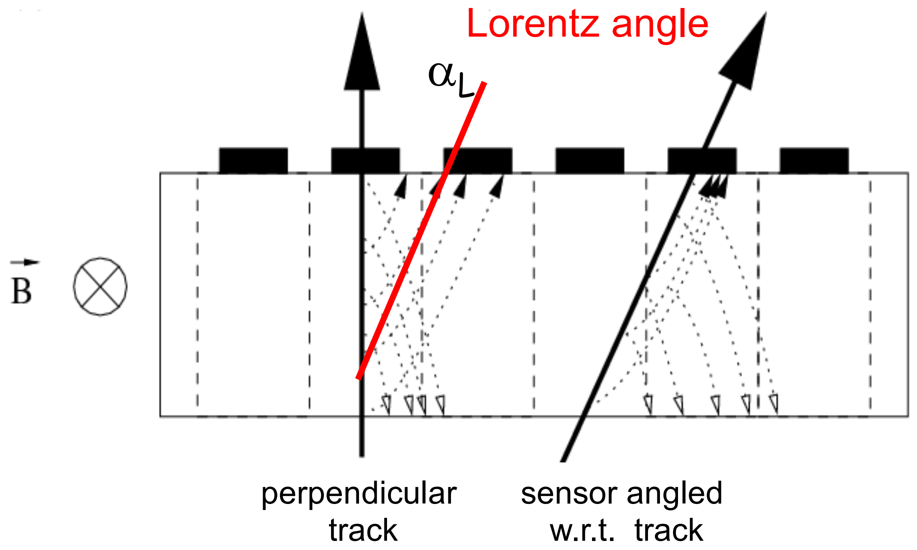
- if the electric field E is perpendicular to a magnetic field B then the charges drift on circle segments until they stop in a collision
- on average this results in a deflection of the drift path by an angle called

Lorentz angle

with

ω = qB/m = cyclotron frequency
τ = mean collision time

Charge collection in a magnetic field

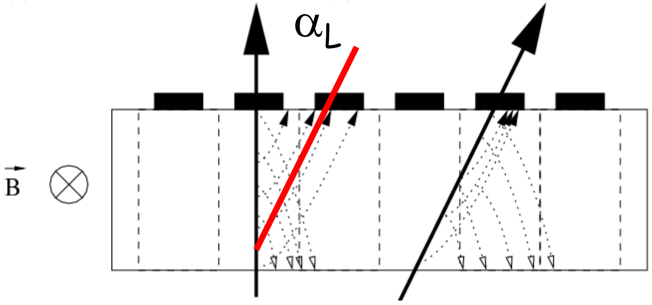


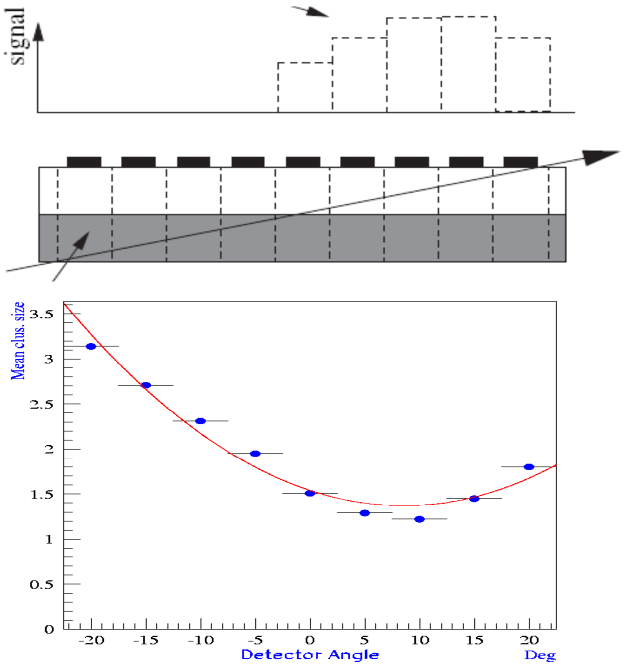
$$\boxed{\tan \alpha_L} = \omega\tau = \frac{e}{m_{\rm eff}}\tau B = \mu_H B = \boxed{\mu_0 \, r_H \, B}$$

Hall mobility ≈ μ_0 (but not equal)

Measurement of the Lorentz angle

Lorentz angle (= average deviation between collisions)





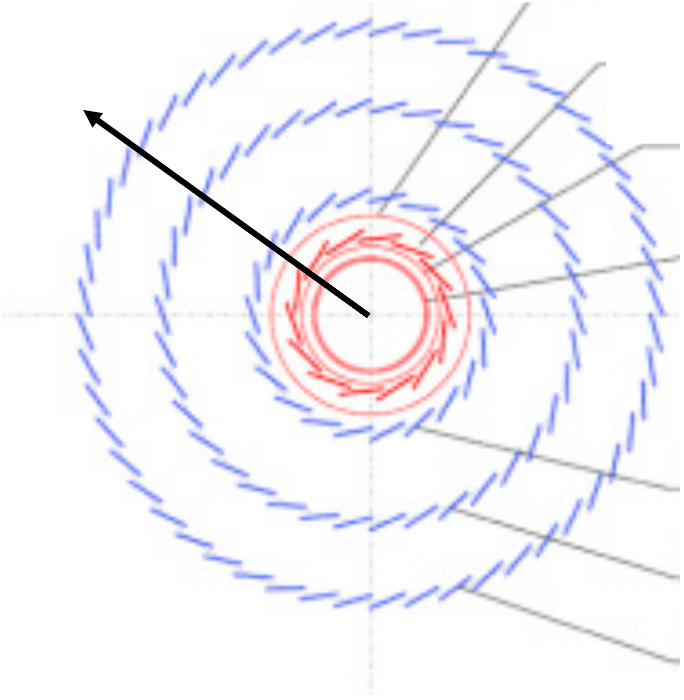
$$\tan\alpha_L = \mu_H B = \mu_0\, r_H\, B$$

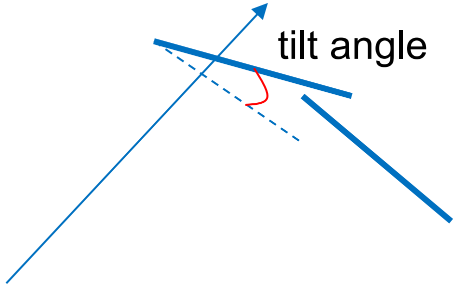
Measurement method: number of pixel hits is minimal when the particle incidence angle is equal to the Lorentz angle (ATLAS pixels)

I. Gorelov et al. NIM A481:204-221,2002

Tune Lorentz angle for optimal detector operation

- charge sharing (CMS: 100×150 μm^2 pixels) versus radiation lifetime (ATLAS: 50×250 μm^2)





- tilt angle can be "chosen"
- can compensate or increase "charge sharing" between pixels
- radiation hardness: choose little charge sharing in order to keep majority of signal on one pixel (after signal loss due to radiation)
- resolution: choose charge sharing to be able to interpolate between pixels for more precise space point reconstruction.

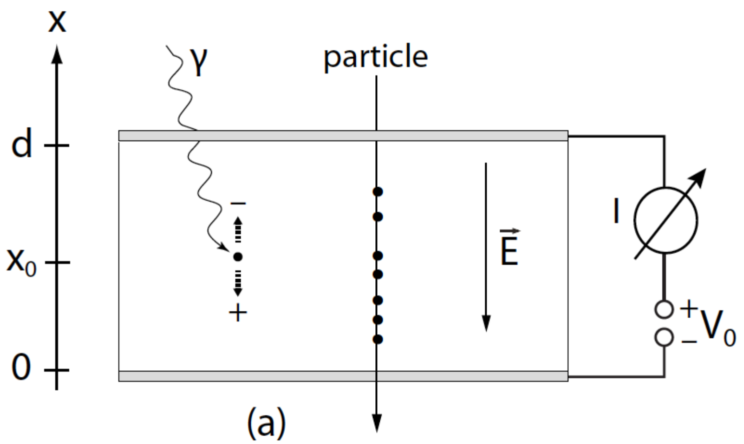
Pop-up questions

- Which effects play the dominant role in charge transport to the electrodes and what are they due to?
  - drift (in E-field) and diffusion (temperature fluctuations and concentration gradient)
- Which other (less important) effects could you imagine?
  - multiple coulomb scattering, electrostatic repulsion within charge cloud
- How wide is a charge cloud spread after having travelled a distance x?
  - $\sigma = \sqrt{2Dt} = \sqrt{2Dx/v} \propto \sqrt{x}$ (typ. 5-10 μm for 250 μm Si)
- What is the Lorentz angle and what does it depend on?
  - Angle wrt perpendicular motion due to B-field. $\tan \alpha_L = \mu_H B$.

How the signal develops

- How do particles (radiation) passing (being absorbed) in a detector generate a “signal” that one can measure?
- How electronic signals develop on electrodes.
- Use the “Shockley – Ramo Theorem” (SRT) for different detector and electrode configurations.
- What neighbouring electrodes “see” when charges move in a detector …

How to think about the problem?



Q: when does the signal current $i_S(t)$ start?

(a) when charges reach the electrode? → popular answer
(b) when charges begin to move? → correct answer

Induced charge

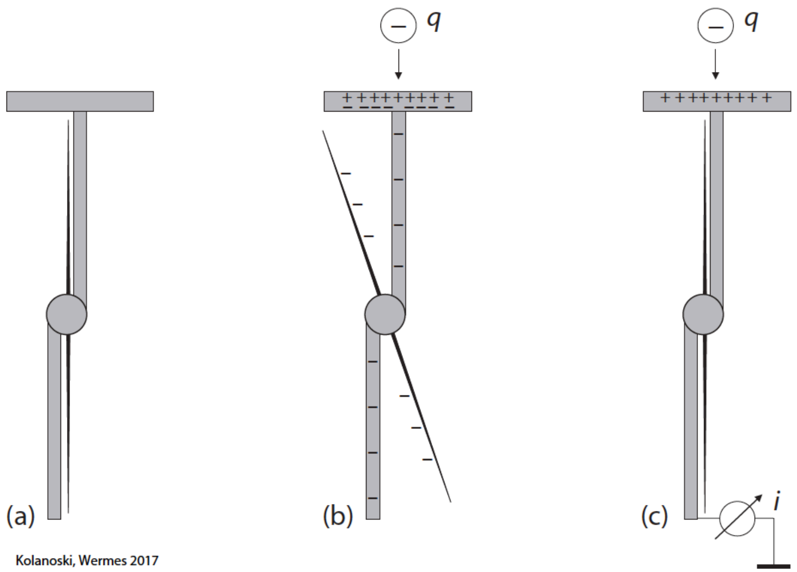
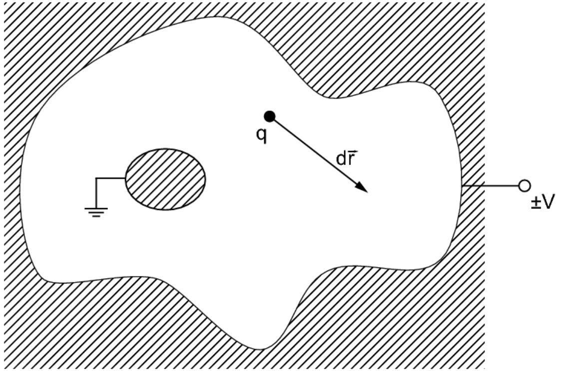


Fig. 5.1 Effect of a charge approaching an electrometer. a) An uncharged electrometer which is insulated against ground. b) A charge q, approaching from infinity, induces a counter-charge on the metal surface of the electrometer which increases as the distance becomes smaller. Within the free-floating electrometer the charge is conserved and hence can only be separated. The charge with sign opposite of that of q (here $q < 0$) accumulates on the metal surface close to q and the same sign charge accumulates on the surfaces further away. This generates a deflection of the electrometer's needle. c) Grounding the electrometer pedestal allows the (negative) charge to drain off so that a current signal can be measured on the path to ground.

Signal generation in an electrode configuration



How does a moving charge couple to an electrode ?

- respect Gauss' law and find

Shockley- Ramo theorem
(Shockley J Appl.Phys 1938, Ramo 1939)

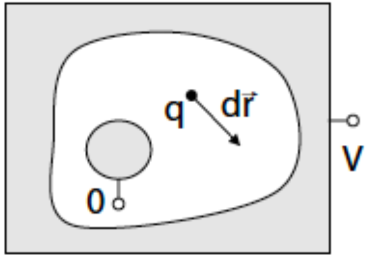
weighting field
determines how charge movement couples to a specific electrode

$$\boxed{i_S = -\frac{dQ}{dt} = q\,\vec{E}_w\,\vec{v}}$$

$$dQ = q\vec{\nabla}\Phi_W\,d\vec{r}$$

induction (weighting) potential
determines how charge movement couples to a specific electrode

Ramo Theorem (1) ... the derivation



When moving the charge q from $\mathbf{r_q}$ to $\mathbf{r_q} + \mathbf{dr_q}$ the field of the electrodes $\mathbf{E_0}$ does the work

$$dW_q = q\,\vec{E}_0\,d\vec{r}$$

This work is delivered by the power supply (W_V) and/or the field energy (W_E):

$$\boxed{dW_q + dW_V + dW_E = 0}$$

If the voltage is kept constant ($dV = 0$) the work done by the power supply is only determined by the charge dQ extracted from the power supply:

$$dW_V = dQ\,V + Q\,dV = dQ\,V$$

Here we adopt the convention that dQ is negative if negative charge is moved into the power supply (or, equivalently, if positive charge is taken from the power supply) and has the opposite sign as the charge induced on the electrode surface.

Ramo Theorem (2)

The total field energy in the volume τ bounded by the electrodes is

$$W_E = \frac{1}{2}\epsilon\epsilon_0 \int_\tau \vec{E}^2 d\tau$$

and $$\vec{E} = \vec{E}_0 + \vec{E}_q$$

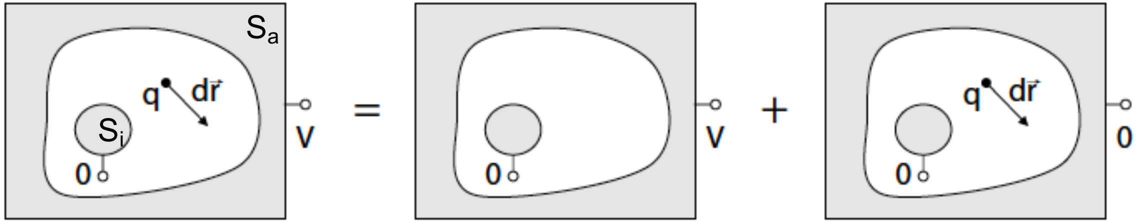
linear superposition of the two components

Accordingly, the potential $\phi(\vec{r})$ in the volume bounded by the electrodes can be represented by the sum of a potential ϕ_0 without charge q and the potential resulting from the additional charge q:

$$\phi(\vec{r}) = \phi_0(\vec{r}) + \phi_q(\vec{r})$$

with boundary conditions

$$\begin{aligned} \phi(\vec{r})|_{S_a} &= \phi_0(\vec{r})|_{S_a} = V, \\ \phi(\vec{r})|_{S_i} &= \phi_0(\vec{r})|_{S_i} = 0\,, \\ \phi_q(\vec{r})|_{S_a} &= \phi_q(\vec{r})|_{S_i} = 0\,. \end{aligned}$$



Kolanoski, Wermes 2015

Ramo Theorem (3)

One can show that also the field energy is separable into the energy originating from the static field E_0 and the point charge field E_q .

$$W_E = W_{E_0} + W_{E_q}$$

If q is moved, both field energy contributions do not change: E_0 = static, E_q does neither do work on q, nor exchange energy with the power supply (since potentials are fixed).

$$dW_E = dW_{E_0} + dW_{E_q} = 0$$

hence with $dW_q + dW_V + dW_E = 0$ and $dW_V = dQ\,V + Q\,dV = dQ\,V$

$$dW_q + dW_V = q\,\vec{E}_0\,d\vec{r} + dQ\,V = 0 \quad \Rightarrow \quad dQ\,V = -q\,\vec{E}_0\,d\vec{r}$$

Ramo Theorem (3)

This means that the work on the charge q is exclusively provided by the voltage source yielding:

$$dQ = -q\,\frac{\vec{E}_0}{V}\,d\vec{r}$$

independent of the applied voltage since $E_0 \propto V$

Therefore dQ is also independent of the applied voltage and we define the weighting field and weighting potential as

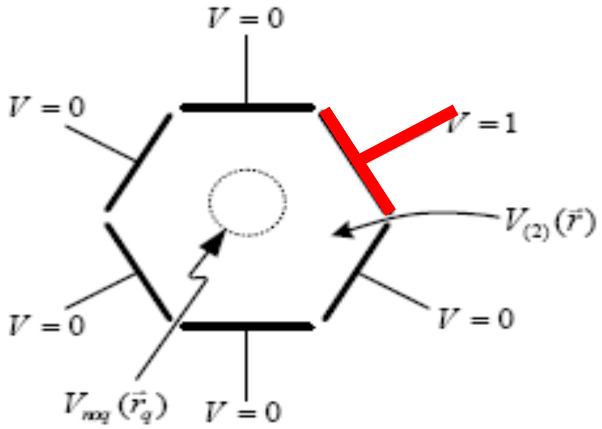
$$\phi_w = \frac{\phi_0}{V}\,, \qquad \vec{E}_w = -\vec{\nabla}\phi_w\,.$$

$$\boxed{dQ = -q\,\vec{E}_w\,d\vec{r}}$$

$$\boxed{i_S = -\frac{dQ}{dt} = q\,\vec{E}_w\,\vec{v}_D}$$

$$\vec{v}_D = \frac{d\vec{r}}{dt}$$

The result is independent of the presence of polarisation charges or space charge, as long as the polarisation dependence on the applied voltage is linear. For undepleted or partially depleted Si substrates, time dependent weighting fields are required (Riegler, doi: 10.1016/j.nima.2019.06.056.) For heavily irradiated substrates E_W is, however, almost like the one of fully depleted undamaged substrates. (Schwandt, Klanner, NIM A 942 (2019), 162418. doi: 10.1016/j.nima.2019.162418).

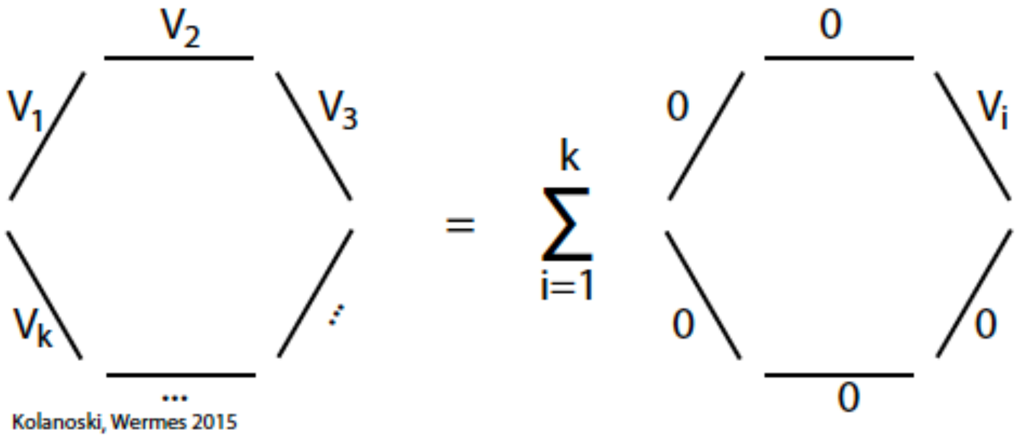


Calculate weighting potential by setting readout electrode to V = 1 and all other electrodes to V = 0.

$$dQ_i = -q\vec{\nabla}\Phi_{w,i}d\vec{r}$$

$$\boxed{i_{S,i} = q\,\vec{E}_{w,i}\,\vec{v}}$$

Ramo Theorem, weighting field



$$dQ_i = -q\,\vec{E}_{w,i}\,d\vec{r}$$

$$i_{S,i} = q\,\vec{E}_{w,i}\,\vec{v}$$

Remarks: Ramo Theorem (1)

(1) dV (the signal) does not directly depend on V (value of HV)

(2) the true electrode field ($-\nabla\Phi$) determines
- the direction of the particle
- the velocity of the generated charge movement

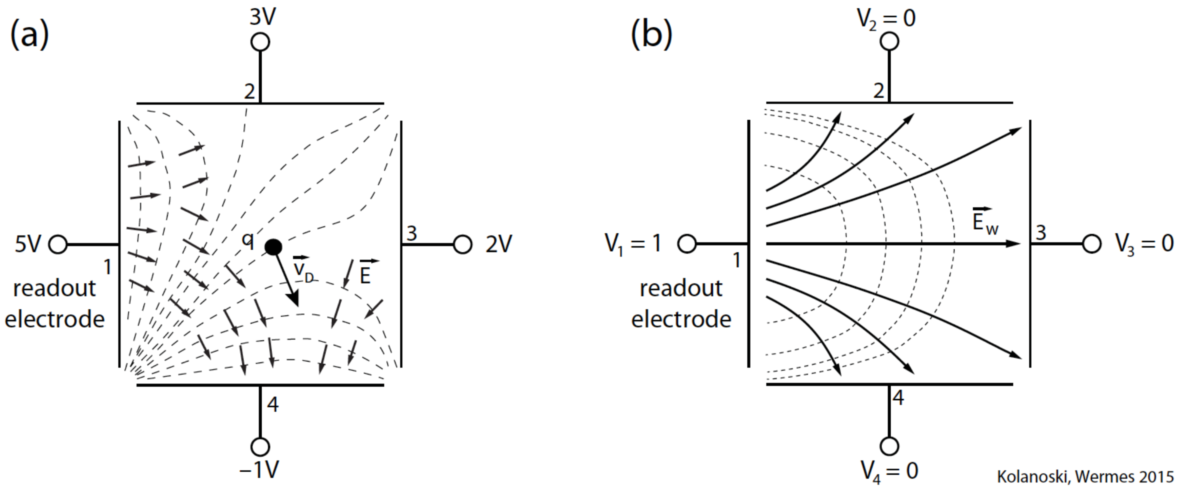
$\Rightarrow$ $\vec{v}(t)$, i.e. the time dependence of the pulse shape

(3) the „weighting field“ $E_W = -\nabla\Phi_W$ only depends on
- the geometry of the electrode configuration

and determines „How the moving charge couples to a specific electrode“

(4) in a more-electrode configuration like …

Remarks: Ramo Theorem (2)



Kolanoski, Wermes 2015

- The weighting field with respect to electrode i is obtained by setting the potential of electrode i to 1 (1V) and all other potentials to 0 (0V)
- It tells us, how the induced charge dQ (current i_S) changes when the charge moves (either by the influence of E_{true} or even moving it „by hand“)

Signal =
charge, current or voltage?

… it depends

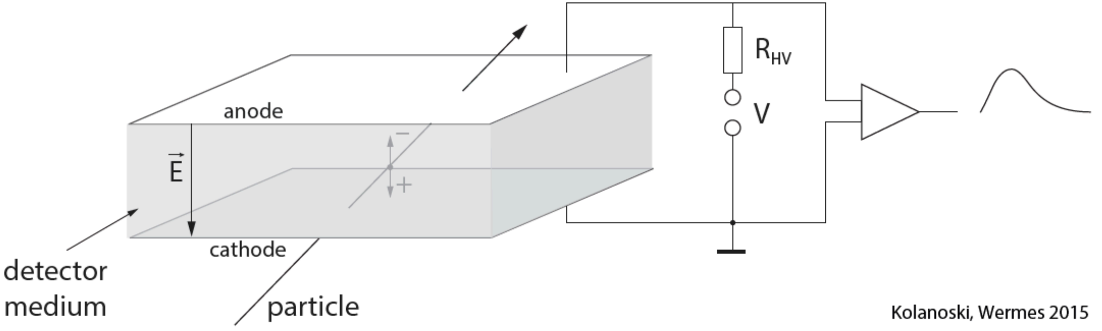
A detector is a **current source**

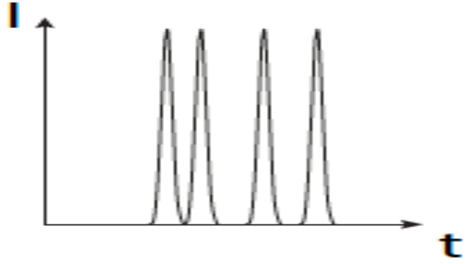
It delivers a current pulse
independent of the load

One can convert current into
charge (integral) or voltage (via R or C)

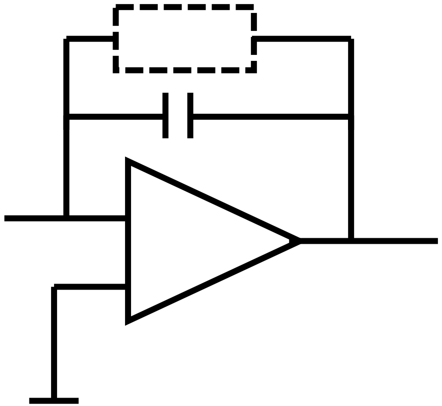
Example 1: Wire chamber or semiconductor

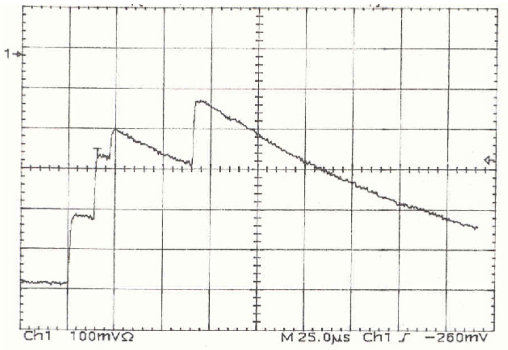
Output is a charge … or better: an integrated current





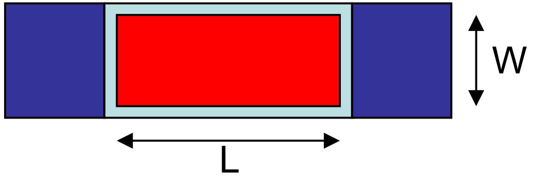
charge integrating amplification circuit (CSA)

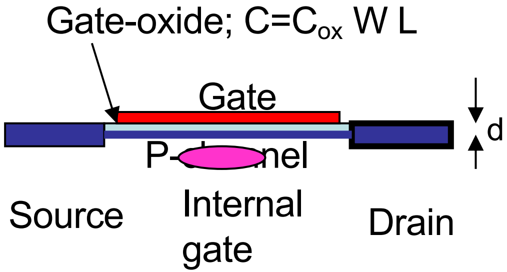


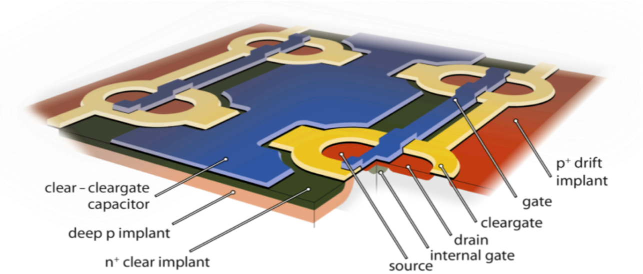


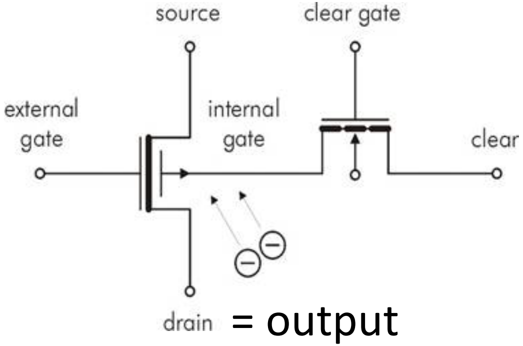
Example 2: DEPFET pixels

output is a current









= output

A charge **q** in the internal gate is – via the capacitance to the channel – a voltage which “steers” the channel current I_d together with the external gate voltage, which hence effectively changes by: $\Delta V = \alpha\, q / (C_{ox}\, W\, L)$.
$\alpha < 1$ due to stray capacitances

Kemmer, J., G. Lutz et al., Nucl. Inst. and Meth. A 288 (1990) 92

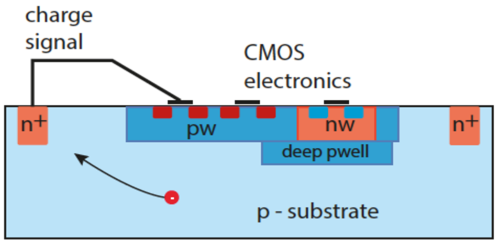
q

features:

- $g_q \sim 700$ pA/e$^-$
- small intrinsic noise
- sensitive off-state, w/o power being used

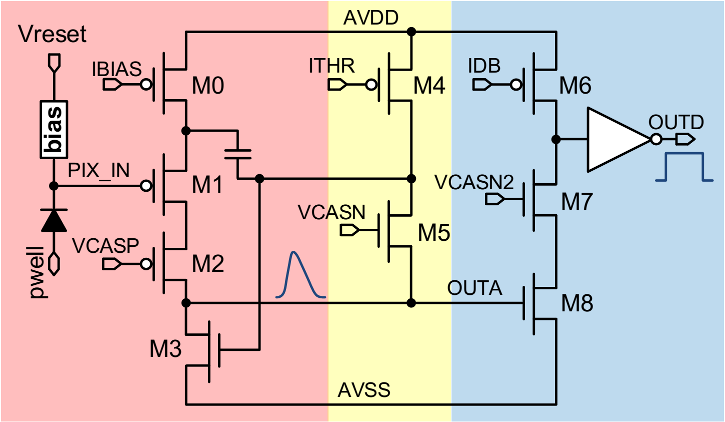
Example 3: Pixel detector with very small electrode

output is a voltage
or better: get a large voltage for
a small charge



(b) Small fill-factor

C_D tiny (~5fF)



D. Kim et al., doi 10.1088/1748-0221/11/02/C02042

$v = Q_{in}/C_D$ large
e.g. 2000 e- => 60 mV (large !)

$$\frac{S}{N} \propto \frac{Q}{C_D}\sqrt{g_m} \propto \frac{Q}{C_D}\sqrt[m]{P}\,, \quad P \propto \left(\frac{C_D}{Q}\right)^m$$

Examples of signals by moving charges

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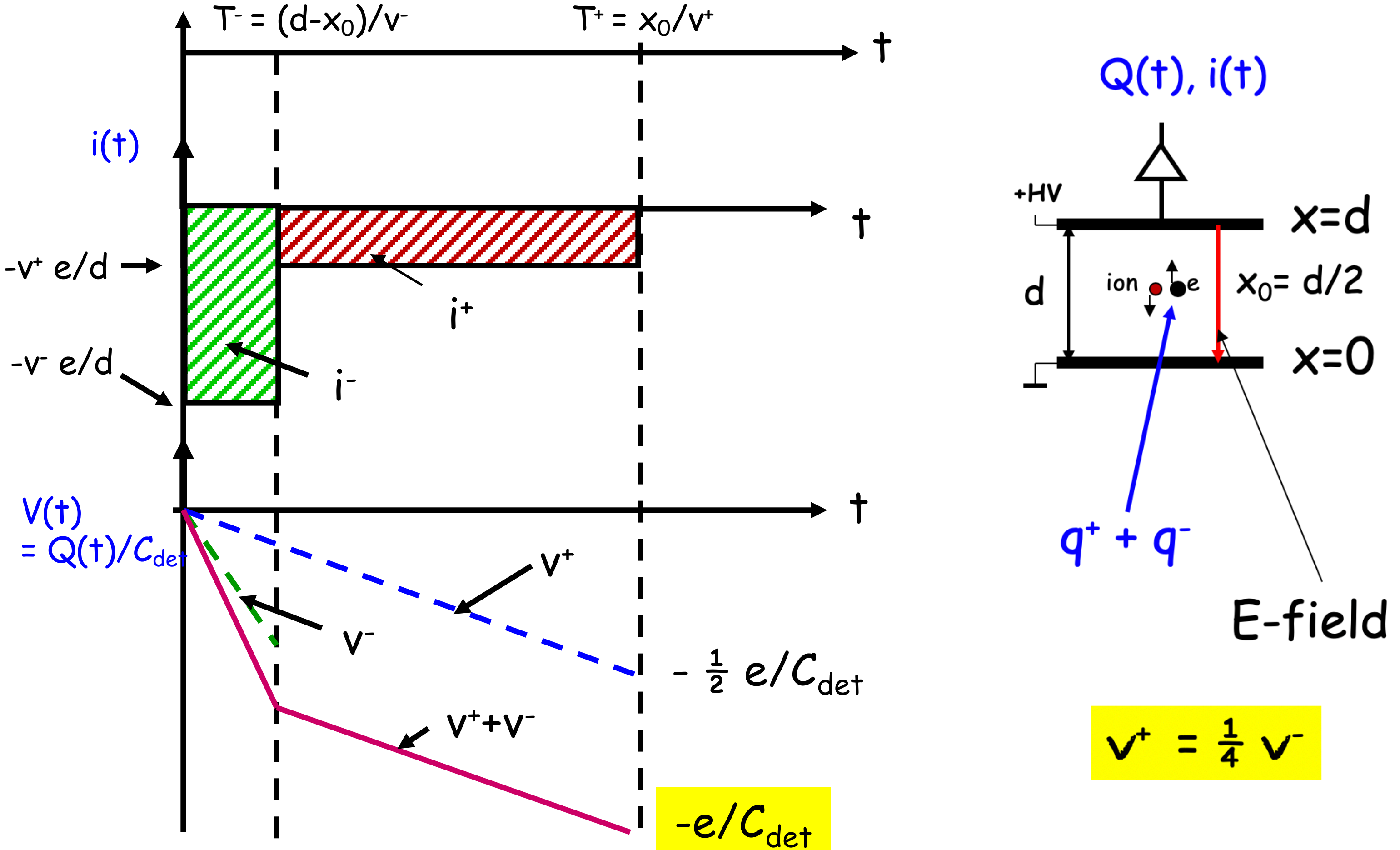
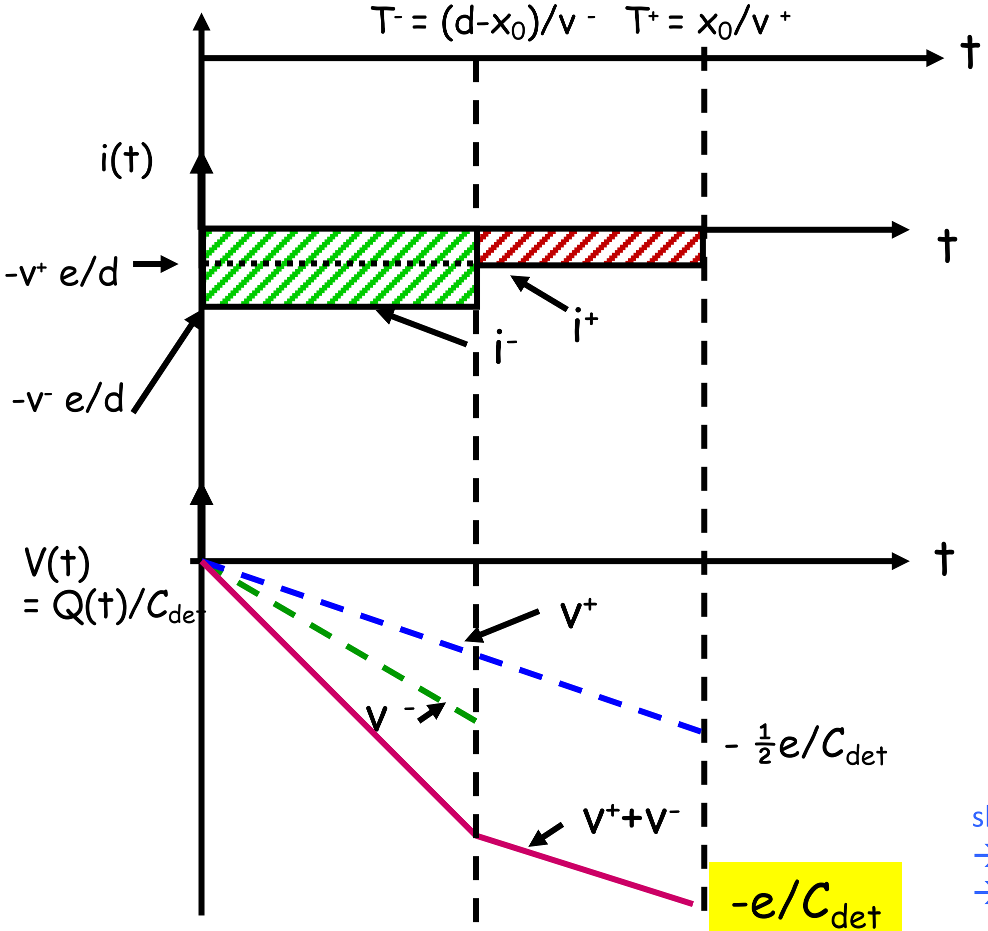
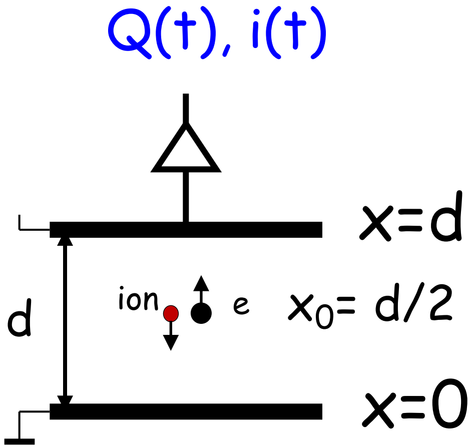
Parallel Plate Detector

Signal development in a parallel plate detector



Signal development in a parallel plate detector



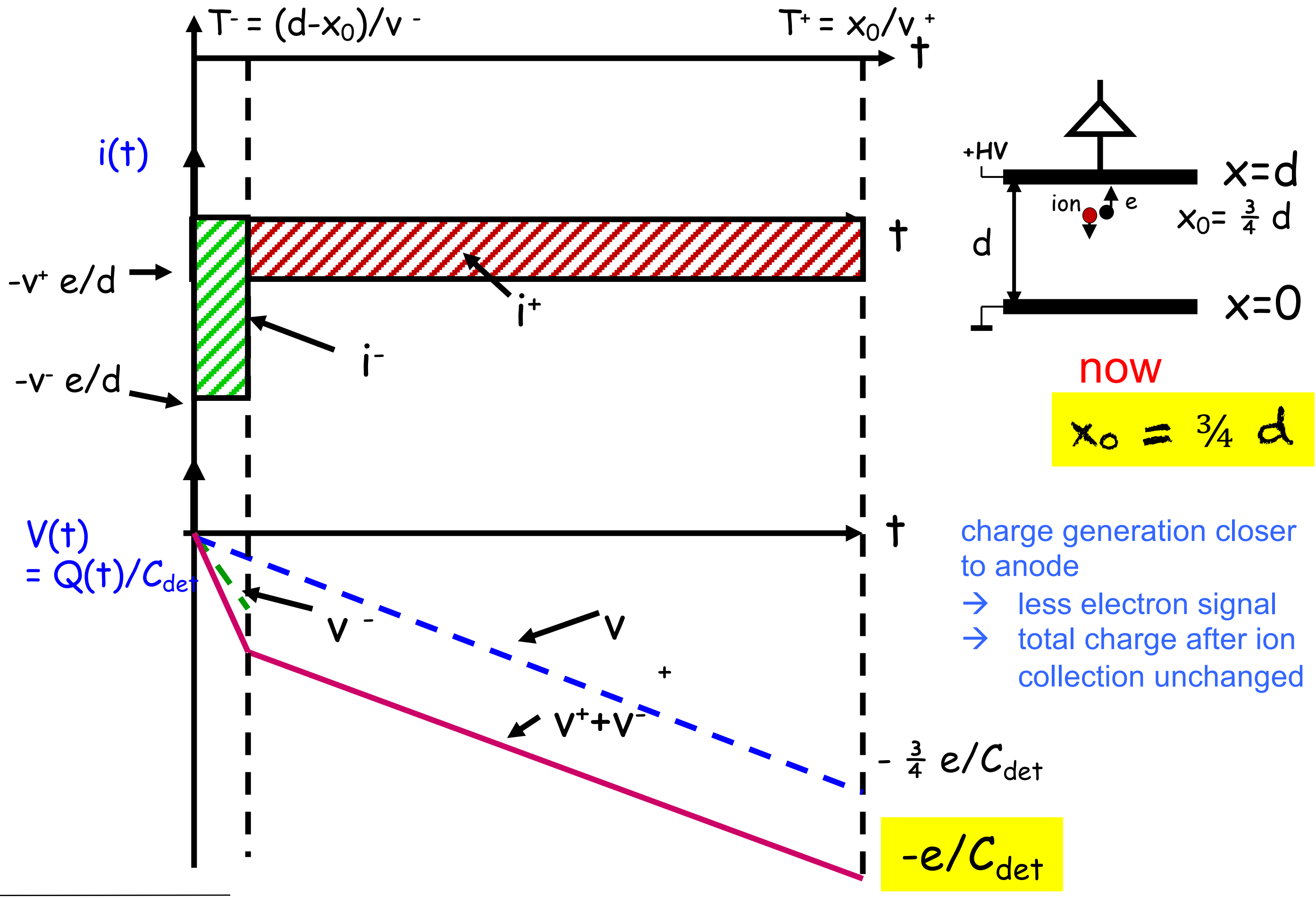


now

$v^- \rightarrow v^-/3$
$i^- \rightarrow i^-/3$
$T^- \rightarrow 3T^-$

slower electron drift
→ electron current smaller
→ electrons need longer time to arrive

Signal development in a parallel plate detector



now

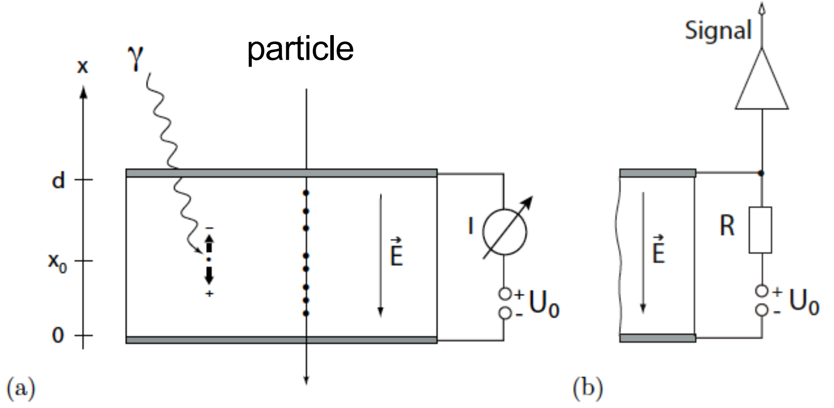
$x_0 = ¾\, d$

charge generation closer to anode

→ less electron signal

→ total charge after ion collection unchanged

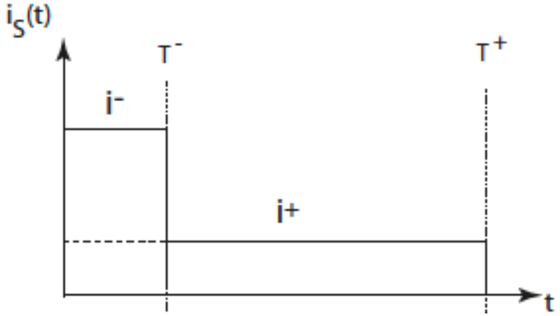
Parallel plate detector (capacitor)



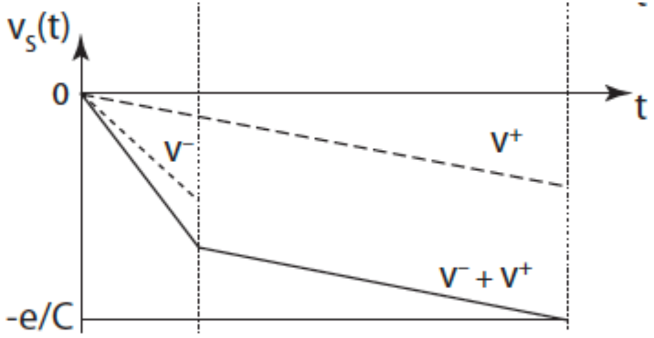
$$\vec{E} = -\frac{V_0}{d}\vec{e}_x \ , \qquad C = \frac{\epsilon\epsilon_0 A}{d}$$

- constant E-field
- almost constant velocity (v=μE)
- weighting field simple

$$dQ = -q\underbrace{\frac{\vec{E}}{V_0}}\, d\vec{r} \qquad \vec{E}_w = -\frac{1}{d}\vec{e}_x$$



$$\boxed{i_S^{\pm} = q^{\pm}\vec{E}_w\,\vec{v}^{\pm} = -\frac{q^{\pm}}{d}\,\vec{e}_x\,\vec{v}^{\pm} = \frac{e}{d}v^{\pm}}$$

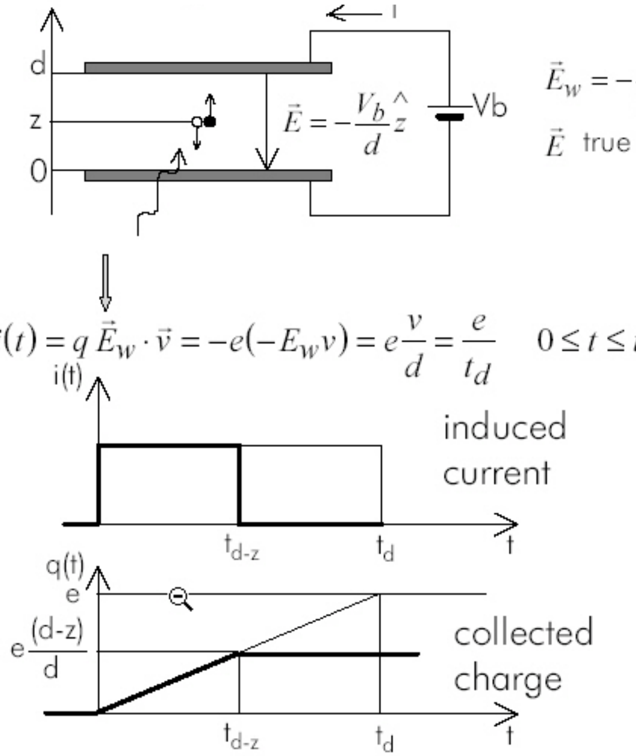


$$\boxed{Q_S^{tot} = Q_S^- + Q_S^+} = -\frac{e}{d}\left(\int_0^{T^-} v^- dt + \int_0^{T^+} v^+ dt\right)$$

$$= -\frac{e}{d}v^-\left(\frac{d-x_0}{v^-}\right) - \frac{e}{d}v^+\left(\frac{x_0}{v^+}\right) \boxed{= -e\,.}$$

Induced signal in a parallel plate detector

one e/ion pair



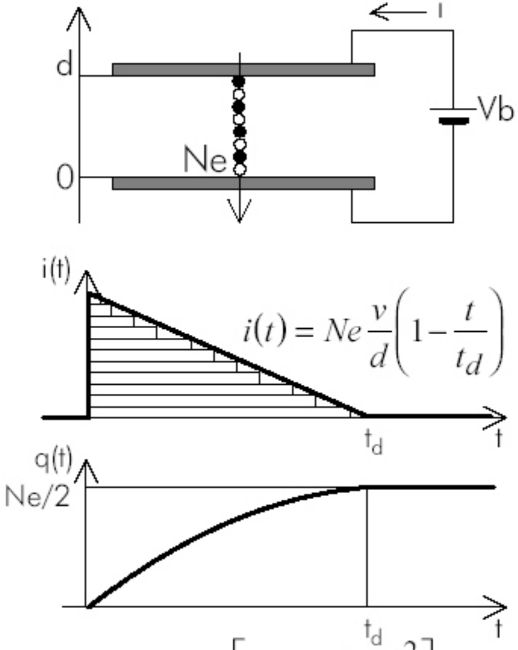
$$\vec{E}_w = -\frac{1}{d}\hat{z}$$

$$\vec{E} = -\frac{V_b}{d}\hat{z}$$

$\vec{E}$ true field

$$i(t) = q\,\vec{E}_w \cdot \vec{v} = -e(-E_w v) = e\frac{v}{d} = \frac{e}{t_d} \qquad 0 \le t \le t_d$$

continuous ionisation



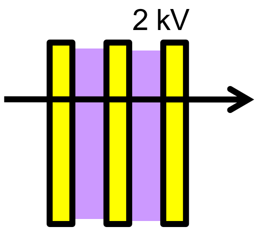
$$i(t) = Ne\frac{v}{d}\left(1 - \frac{t}{t_d}\right)$$

$$Q_s(t) = \int_0^t i(\tau)d\tau = Ne\left[\frac{t}{t_d} - \frac{1}{2}\left(\frac{t}{t_d}\right)^2\right]$$

here: $t_d = T^{\pm}$

Signal development in a parallel plate configuration

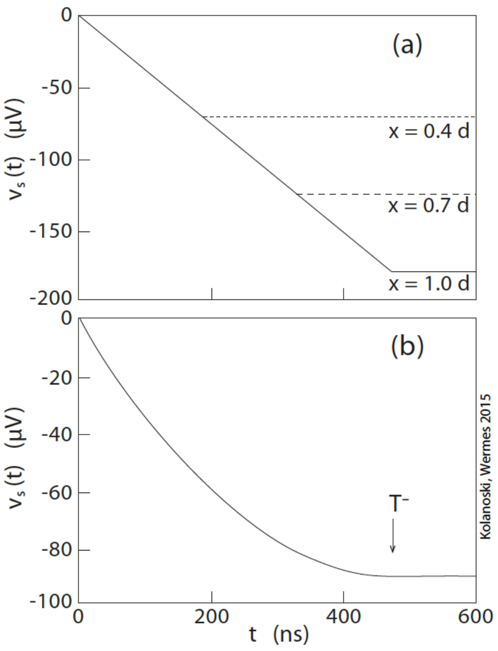
• realistic example:
Liquid Argon in between Lead Plates



d = 2.35 mm
A = 16 cm^2
ε = 1.5

$\Rightarrow$ C ≈ 9 pF

v+ = 15 cm/s
v- = 0.5 cm/µs
$\Rightarrow$ T^- = d/v- = 470 ns
$\Rightarrow$ T^+ = 16 ms



ionisation signal of a point charge

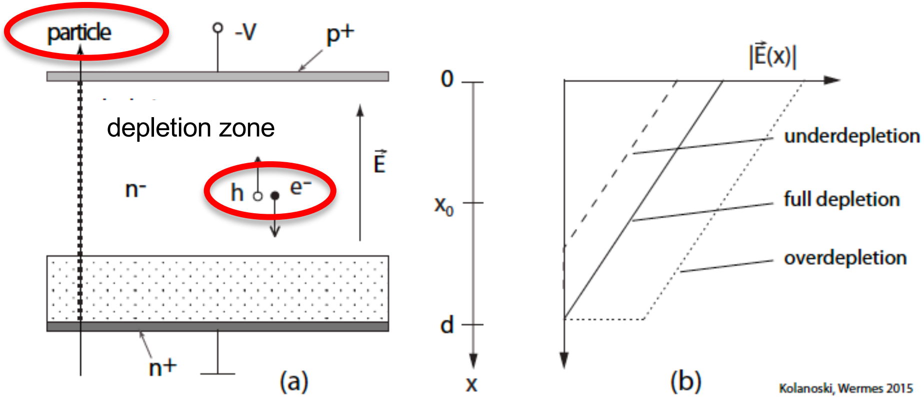
ionisation signal of a line charge

Silicon Detector
= Detector with Space Charge

note: space charge changes the electric field in the sensor

but NOT the weighting field

Signal in a Silicon detector (= parallel plate w/ space charge)



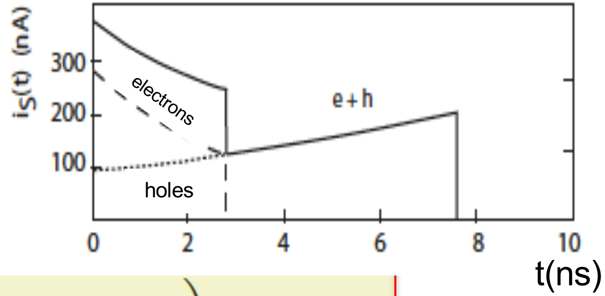
- E-field not constant
- velocity not constant
- weighting field still the same

$$\boxed{\vec{E}_w = -\frac{1}{d}\vec{e}_x}$$

$$\vec{E}(x) = -\left[\frac{2V_{dep}}{d^2}(d-x) + \frac{V - V_{dep}}{d}\right]\vec{e}_x \quad = -\left[\frac{V + V_{dep}}{d} - \frac{2V_{dep}}{d^2}x\right]\vec{e}_x = -\left(a - bx\right)\vec{e}_x$$

$$v_e = -\mu_e E(x) = +\mu_e\left(a - bx\right) = \dot{x}_e$$

$$v_h = +\mu_h E(x) = -\mu_h\left(a - bx\right) = \dot{x}_h$$

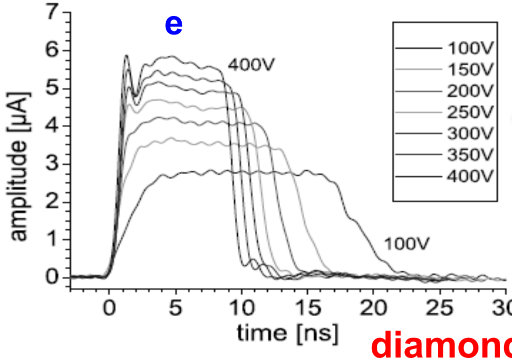


$$i_S(t) = i_S^e(t) + i_S^h(t)$$

$$= \frac{e}{d}\left(\frac{a}{b} - x_0\right)\left(\frac{1}{\tau_e}e^{-t/\tau_e}\,\Theta(T^- - t) + \frac{1}{\tau_h}e^{t/\tau_h}\,\Theta(T^+ - t)\right)$$

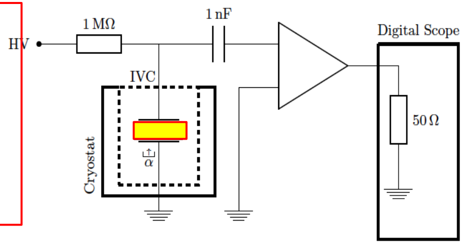
Current pulse measurements: TCT technique

transient current

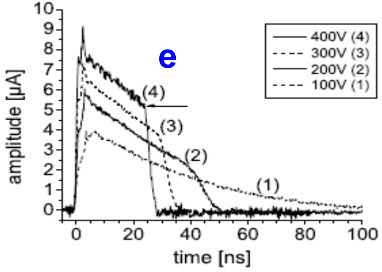


diamond

single crystal **diamond** is like a parallel plate detector filled with a dielectric w/o space charge

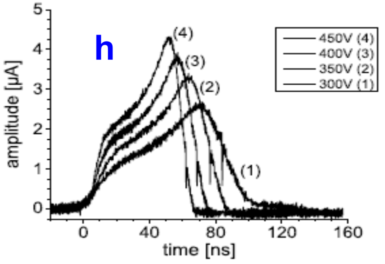


1mm pn – Diode **silicon**
- same weighting field
- different electric field



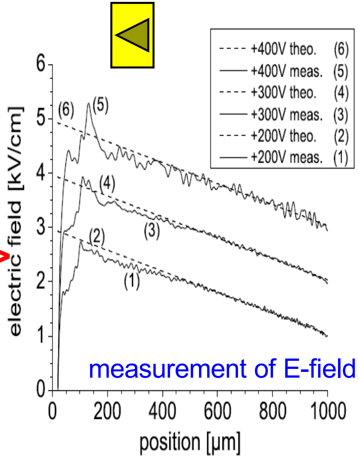
(a) Electron signals from α-particles impinging on the cathode.

Si



(b) Hole signals from α-particles impinging on the anode.

=>

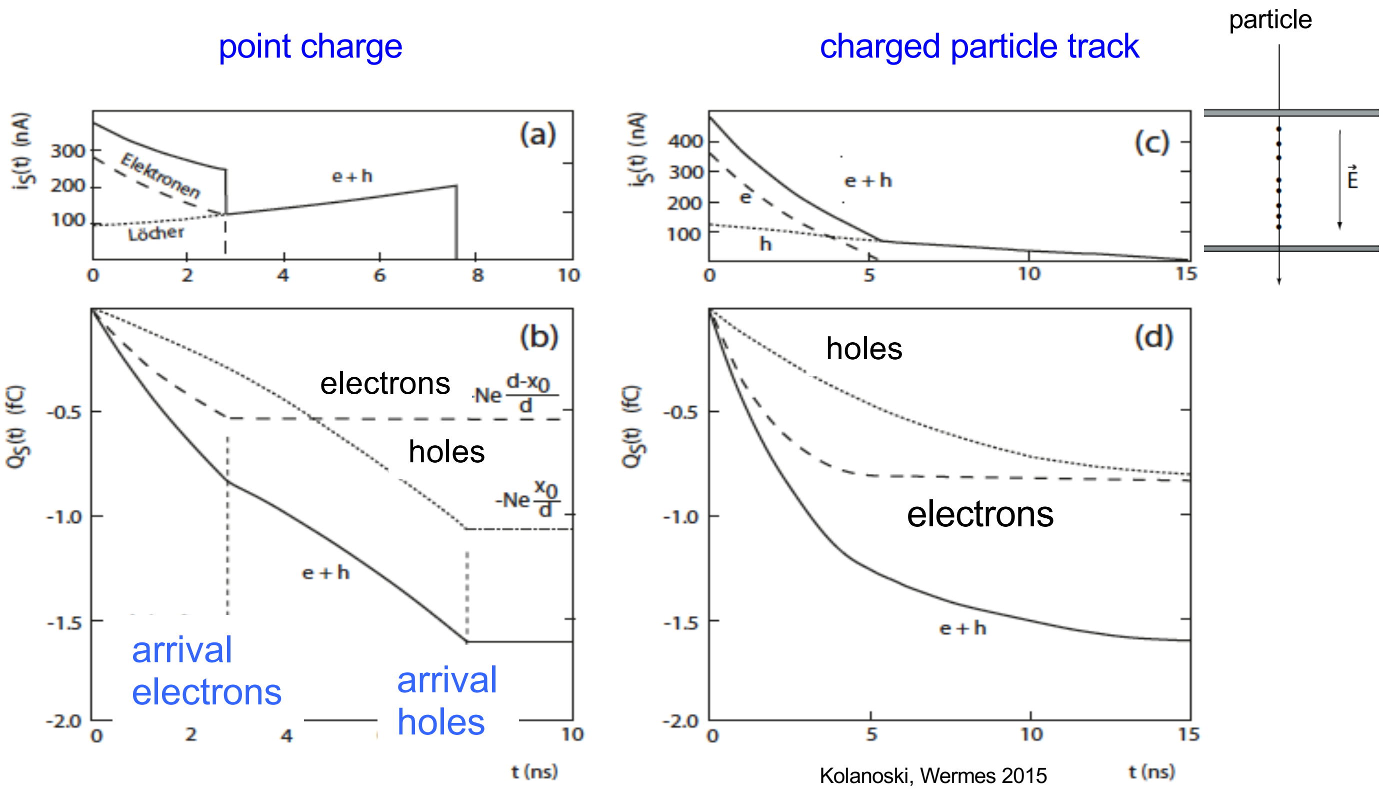


measurement of E-field

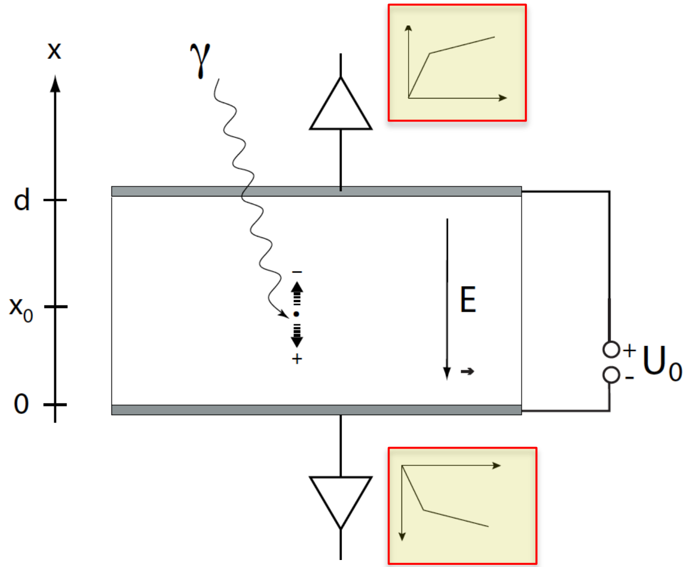
Current and charge signals

point charge

charged particle track



Note!

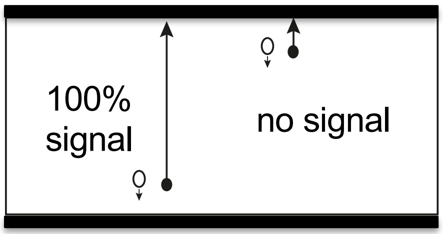


- movement of both charges create signals on both electrodes.
- on every electrode a total charge of

$$Q_S^{tot} = Q_S^- + Q_S^+ = -Ne$$

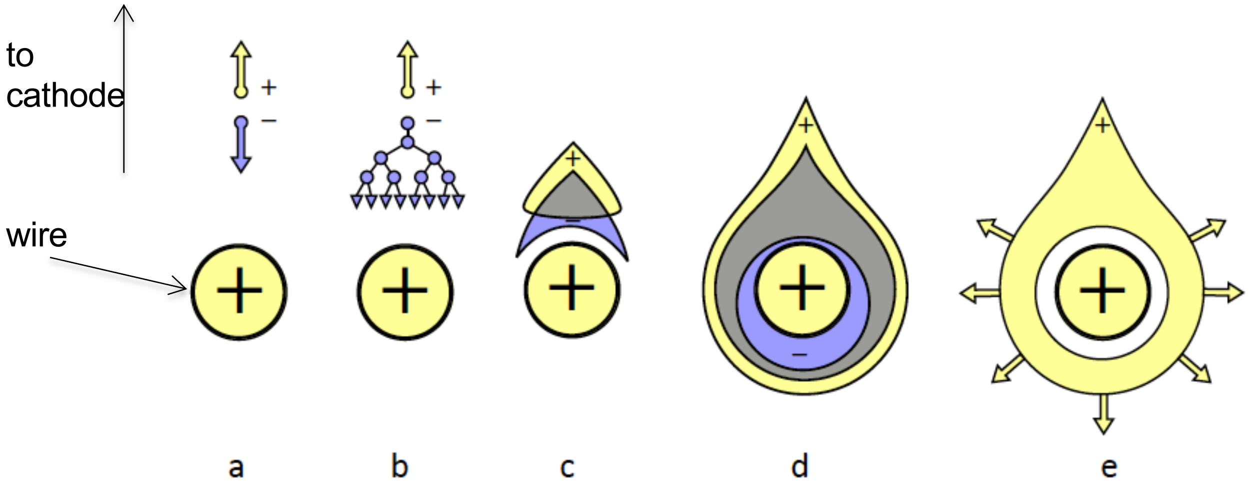
is induced.

- if a material the produced charges have very different mobilities (like CdTe) e.g. with
$\mu_h \approx 0$, then part of the signal is lost and the signal becomes dependent on where the charge was deposited.



Wire Chamber

Signal development in a wire chamber



big difference:

- ❑ electrode (wire) does not "see" (too small) the charge before gas amplification
- ❑ signal (on wire) shape is governed by the (large) ion cloud moving away from the wire to cathode

Avalanche process:

$$dN = \alpha(E)\, N\, ds$$

with

$$\alpha = \sigma_{ion}\, n = \frac{1}{\lambda_{ion}}$$

1st Townsend coefficient

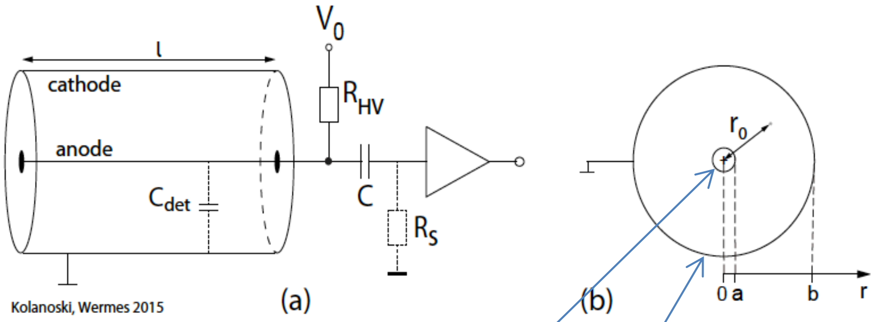
$$N(x) = N_0 e^{\alpha x}$$

gas gain

$$\boxed{\frac{N}{N_0} = G = e^{\alpha x}}$$

Signal development in a wire chamber like configuration (1)

configuration



$$\vec{E}(r) = \frac{1}{r}\frac{V_0}{\ln b/a}\frac{\vec{r}}{r}, \qquad \phi(r) = -V_0\frac{\ln r/b}{\ln b/a}, \qquad C_l = \frac{2\pi\epsilon_0}{\ln b/a}$$

- We follow our Shockley-Ramo-recipe: find the weighting field E_W or the weighting potential Φ_W by setting

$$\phi_w(a) = 1\,, \quad \phi_w(b) = 0^{(*)}$$

- We know already the shape of $\Phi_W \sim \ln r$, since $E(r) \sim 1/r$
- hence

$$\boxed{\vec{E}_w(r) = \frac{1}{r}\frac{1}{\ln b/a}\frac{\vec{r}}{r}, \qquad \phi_w(r) = -\frac{\ln r/b}{\ln b/a}}$$

which fulfils (*)

Signal development in a wire chamber like configuration (2)

- Now use Shockley-Ramc $dQ_S = -q\vec{E}_w d\vec{r}$

- we assume that N e/ion-pairs are produced at r = r_0. Note that, if there is avalanche amplification (starting only in the high field region) the vast majority of charges is produced very close to the wire (r_0 < 10 µm, see previous page)

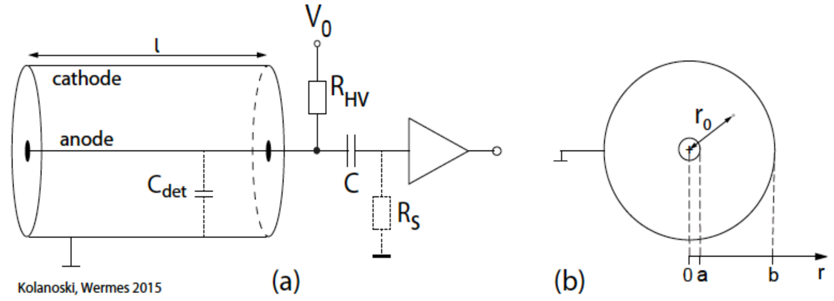
- then we get immediately

$$\boxed{\begin{aligned} Q_S^- &= -(-Ne)\frac{1}{\ln b/a}\int_{r_0}^{a}\frac{1}{r}dr = -Ne\frac{\ln\ r_0/a}{\ln\ b/a} \\ Q_S^+ &= -(+Ne)\frac{1}{\ln b/a}\int_{r_0}^{b}\frac{1}{r}dr = -Ne\frac{\ln b/r_0}{\ln b/a} \end{aligned}}$$

(**)

- and the <u>total</u> charge is $Q_S^{tot} = Q_S^- + Q_S^+ = -Ne$
  however, due to the 1/r dependence of the weighting field the situation is much different from that of a parallel plate detector: the contribution from electrons and ions is not necessarily the same but depends on r_0 (i.e where the avalanche is created, because only there N becomes large enough that the signal is “felt” by the electrode (wire).

Signal development in a wire configuration (3)



ratio depends on r_0

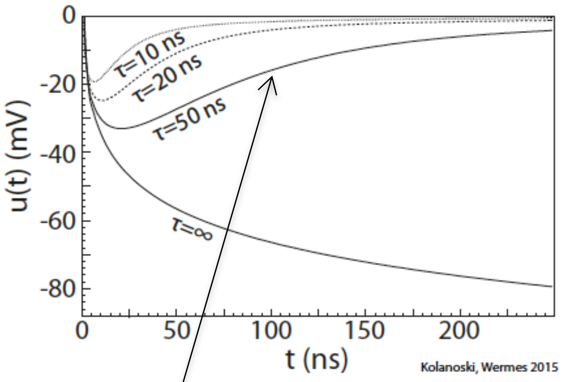
$$\left(\frac{Q_S^-}{Q_S^+}\right)_{r_0} = \frac{\ln r_0/a}{\ln b/r_0}$$

for a typical config (a=10 μm, b=10 mm)

far away from wire

near wire

$$\left(\frac{Q_S^-}{Q_S^+}\right)_{r_0=b/2} \approx 9 \qquad \left(\frac{Q_S^-}{Q_S^+}\right)_{r_0=a+\epsilon} \approx 0.01 - 0.02$$



with τ = RC filter

in wire chambers the (integrated) signal is dominated by the ion contribution. Reason: point of signal creation and specific form of the weighting field.

using Ramo and r(t) from the 1/r - E-field, we get …

$$\boxed{i_S^+(t) = \frac{Ne}{2\ln b/a}\,\frac{1}{t+t_0^+}} \qquad t_0^+ = \frac{r_0^2 \ln b/a}{2\mu^+ V_0}$$

char. time

ions only

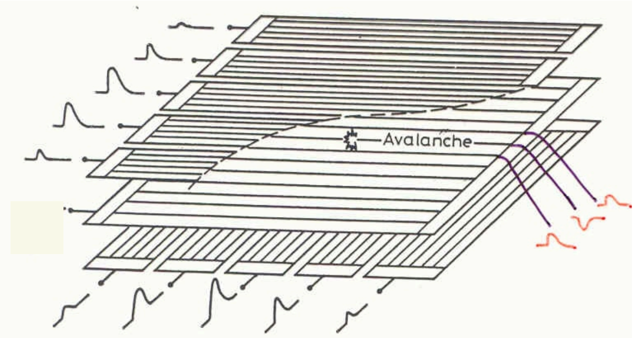
$$\boxed{v_s(t) = \frac{Q_S(t)}{C_l\, l} = -\frac{N\,e}{2\pi\epsilon_0\, l}\ln\left(1+\frac{t}{t_0^+}\right)}$$

Upshot: Signal formation characteristics in a wire chamber

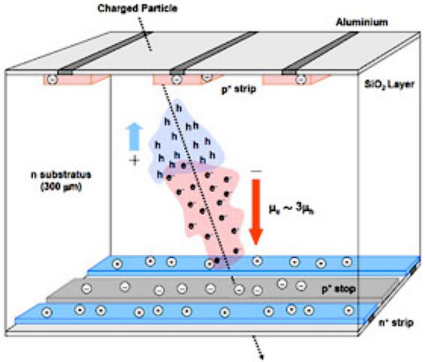
- electric field is large close to the wire @ $r \approx r_{wire}$
  => secondary ionisation has a much larger effect on signal
  than primary ionisation
  => avalanche near wire: $q \rightarrow q \times 10^{4\text{-}7}$
- from there (µm's away from wire) the electrons reach the wire fast
  => very small and fast e^- component of Q_{tot}
- ions move slowly away from wire => main component of $Q_{tot}(t)$
- signal <u>only</u> relevant after avalanche ionization $\cong$ quasi only $Q^+(t)$
- the term 'charge collection' is more justified in wire chambers than in other ionisation detectors (e.g. parallel plate detectors) since most of the signal is created only very close to the wire

Cathode readout

signals are induced on BOTH (ALL) electrodes => exploit for second coordinate readout



wire chamber
with cathode readout



double sided
silicon strip detector

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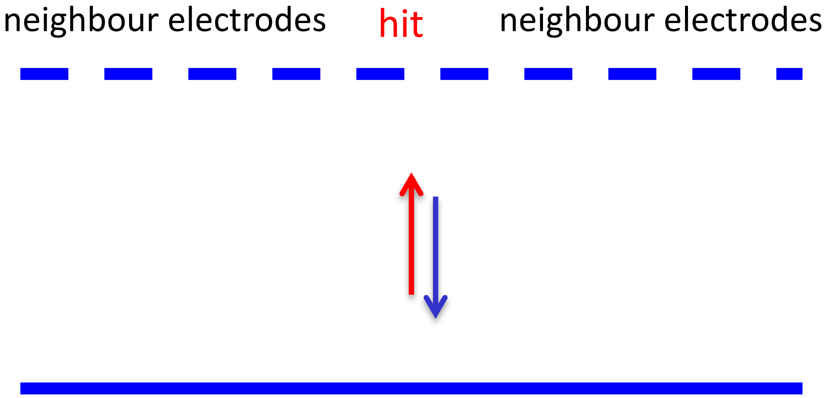
Structured Electrodes

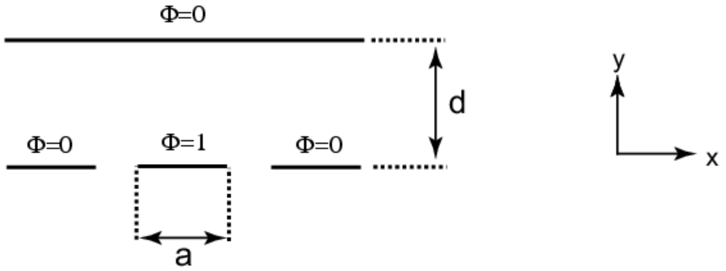
Now: Weighting field for structured electrodes



Excursion: Weighting field calculation by conformal mapping

skip?

The problem: find the weighting potential for a special geometry by solving the Poisson equation. Then apply the Shockley-Ramo theorem to find the induced pulse.



our geometry

$\boldsymbol{\Phi}_l = 1 \qquad \boldsymbol{\Phi}_{rest} = 0$

mathematical help by complex analytical functions

$$f(z) = f(x+iy) = \underbrace{u(x,y)}_{\mathrm{Re}(f)} + i\,\underbrace{v(x,y)}_{\mathrm{Im}(f)}$$

for which the Cauchy-Riemann differential equations hold:

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad ; \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$$

Weighting Potential for Strip- or Pixel Electrodes - II

example: $$f(z) = z^2 = (x+iy)^2 = x^2 - y^2 + 2ixy$$

$$Ref = x^2 - y^2 := u \qquad Imf = 2xy := v$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -2y = \frac{\partial v}{\partial x}$$

not a trivial property !

hence for any analytic function f we have that

$$\begin{aligned} \Delta u(x,y) &= \frac{\partial}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial}{\partial y}\frac{\partial v}{\partial x} = 0 \\ \Delta v(x,y) &= 0 \end{aligned}$$

i.e. the Poisson equation is fulfilled for u and v

Our problem is therefore reduced to: Find an analytic function whose real part u(x,y) or whose imaginary part v(x,y) satisfies the boundary conditions of our geometry. Since it is analytic, then the Poisson eqn. $\Delta u = 0$ is automatically solved.

Weighting Potential for Strip- or Pixel Electrodes - III

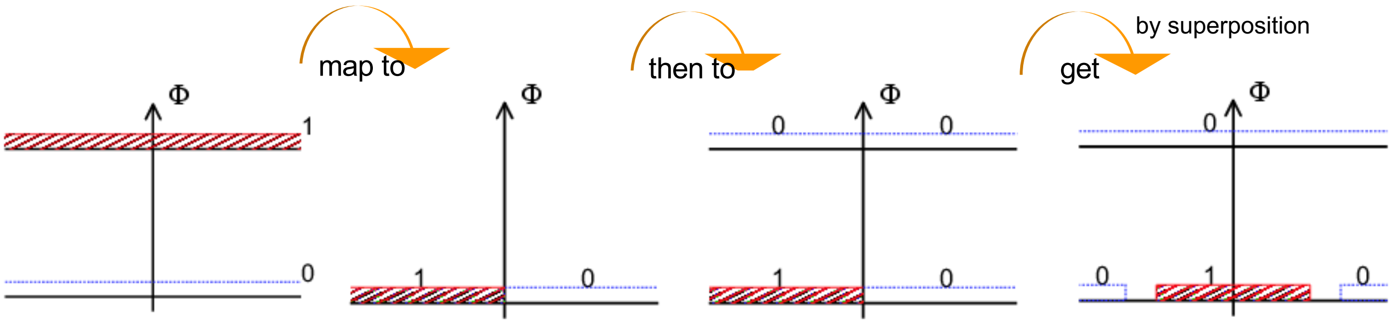
Our method: conformal (i.e. angle conserving) mapping!
Here: angles between potential and electric field lines are conserved.

We start with a simple geometry and map its boundary to the desired (more complicated) boundary.

recipe

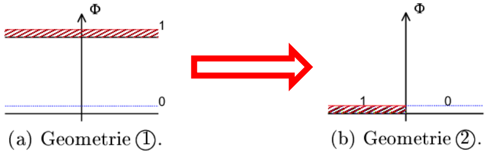
1. Assume that $z_2 = f(z_1)$ does this mapping (from simple to complicated geometry)
2. Solve the (weighting) potential for the simpler geometry
3. Invert the mapping to obtain the (weighting) potential for the new geometry.

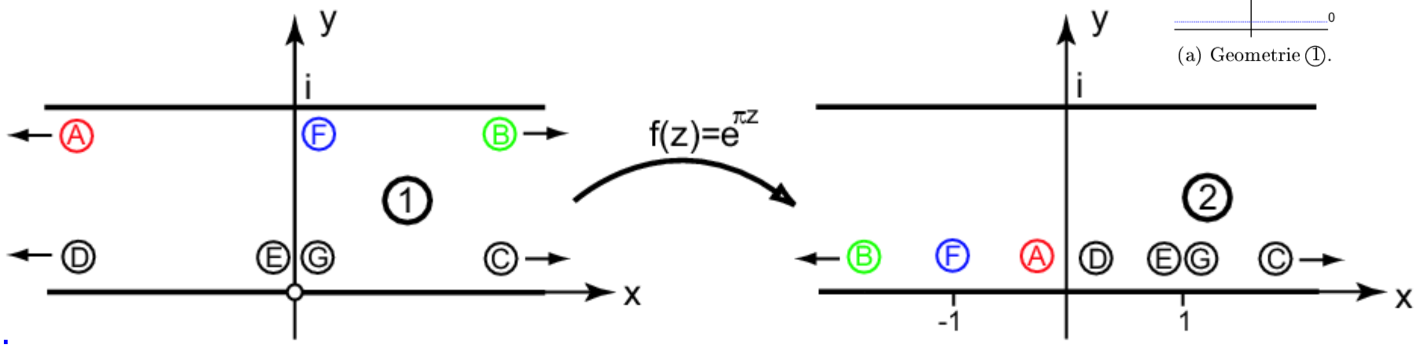
The procedure in our case: start with parallel plate configuration



(a) Geometrie ①. (b) Geometrie ②. (c) Geometrie ③. (d) Geometrie ④.

example for mapping 1: $f(z_1) = z_2 := e^{\pi \cdot z_1}$





check it using

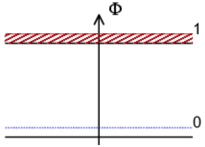
$$z_2 = x_2 + iy_2 = f(z_1) = e^{\pi(x_1+iy_1)} = e^{\pi x_1}(\cos(\pi y_1) + i\sin(\pi y_1))$$

check

- A: $f(i-\infty) = e^{-\infty} \cdot e^{i\pi} = 0^-$
- ● B: $f(i+\infty) = e^{\infty} \cdot e^{i\pi} = e^{\infty}(\cos\pi + i\sin\pi) = -\infty$
- C: $f(\infty) = e^{\pi\infty} = \infty$
- D: $f(-\infty) = e^{-\infty} = 0^+$
- ● E: $f(0^-) = e^{\pi 0^-} = 1$
- ● F: $f(i) = e^{\pi i} = -1$
- G: $f(0^+) = e^{\pi 0^+} = 1$

In geometry (1) the potential $\Phi_1(x,y)$ is given by
$$\Phi_1(z_1 = x_1 + iy_1) = \mathrm{Im}(z_1) = y_1$$
and satisfies the boundary conditions of a parallel plate geometry fulfilling (trivially) the Poisson eqn. $\Delta\Phi_1(x,y) = 0$.

Step 2: $\Phi_1 = \mathrm{Im}(z_1) = y_1$ satisfies the boundary condition $\Phi_1(z_1 = x+i1) = 1$, $\Phi_1(x,0) = 0$, and is solution to the Poisson eq. (plate capacitor).



We now search $\Phi_2(z_2) = \Phi_1(z_1(z_2)) = \mathrm{Im}\left(\frac{1}{\pi}\ln z_2\right)$

(since $e^{\pi \cdot z_1} = z_2 \quad \Leftrightarrow \quad z_1 = \frac{1}{\pi}\ln z_2$)

being the potential of the mapped geometry
with $\mathrm{Arg}(z_2) = \varphi$ (mit $0 \leq \varphi < 2\pi$)

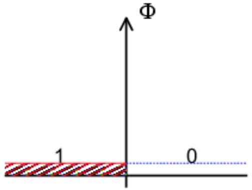
remember

$$\mathrm{z} = |\mathrm{z}|\mathrm{e}^{\mathrm{i}\phi}$$

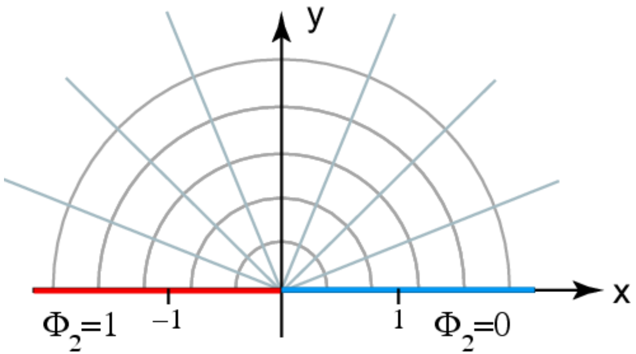
$$\ln\mathrm{z} = \ln|\mathrm{z}| + \mathrm{i}\phi$$

$$\mathrm{Im}(\ln\mathrm{z}) = \phi = \mathrm{Arg}(\mathrm{z}) = \mathrm{atan}\frac{\mathrm{Im}\,\mathrm{z}}{\mathrm{Re}\,\mathrm{z}}$$

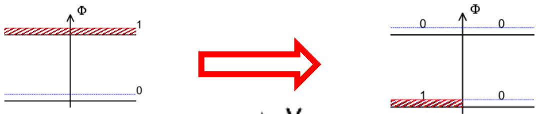
$$\Phi_2(z_2) = \frac{1}{\pi}\mathrm{atan}\left(\frac{\mathrm{Im}\,\mathrm{z}_2}{\mathrm{Re}\,\mathrm{z}_2}\right) = \frac{1}{\pi}\mathrm{atan}\left(\frac{\mathrm{y}_2}{\mathrm{x}_2}\right)$$



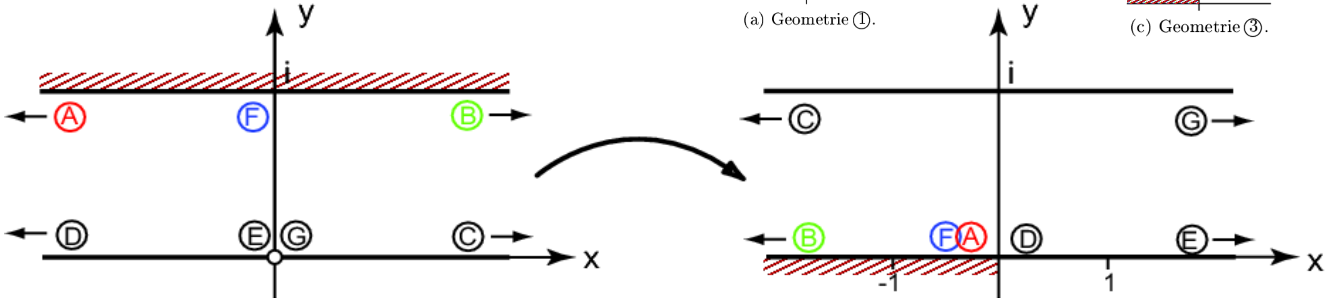
geometry (2)
… as expected



Step 3: our wanted geometry (first only part of it)



(a) Geometrie ①. (c) Geometrie ③.



Mapping Function

$$z_2 = f(z_1) = -\frac{1}{\pi}\ln(1 - e^{\pi z_1}) = -\frac{1}{\pi}\ln(|1 - e^{\pi z_1}| + in\phi)$$

$$\text{remember}: \ln(-x) = \ln(e^{\pm i\pi}\,x) = i\pi + \ln x = \ln x - i\pi$$

check

$$\left\{\begin{array}{l}
\text{G}: f(0^+) = -\frac{1}{\pi}\ln(1 - e^{\pi 0^+}) = -\frac{1}{\pi}\ln(1 - 1^+) = -\frac{1}{\pi}\ln 0^- = -\frac{1}{\pi}\ln(0^+ \cdot e^{-i\pi}) \\
\qquad = -\frac{1}{\pi}(-\infty - i\pi) = i + \infty \\
\text{E}: f(0^-) = -\frac{1}{\pi}\ln(1 - e^{\pi 0^-}) = -\frac{1}{\pi}\ln(1 - 1^-) = -\frac{1}{\pi}\ln 0^+ = -\frac{1}{\pi}(-\infty) = +\infty \\
\text{F}: f(i) = -\frac{1}{\pi}\ln(1 - e^{i\pi}) = -\frac{1}{\pi}\ln|1 - e^{i\pi}| + \arctan(0) = -\frac{\ln(2)}{\pi} \\
\text{A}: f(i - \infty) = -\frac{1}{\pi}\ln(1 - e^{i\pi}e^{-\infty}) = -\frac{1}{\pi}\ln(1 + e^{-\infty}) = -0^+ = 0^- \\
\text{D}: f(-\infty) = -\frac{1}{\pi}\ln(1 - e^{-\infty}) = \ldots = -0^- = 0^+ \\
\text{B}: f(1 + \infty) = -\frac{1}{\pi}\ln(1 + e^{\infty}) = \ldots = -\infty \\
\text{C}: f(+\infty) = -\frac{1}{\pi}\ln(1 - e^{\infty}) = -\frac{1}{\pi}(-i\pi + \infty) = i - \infty
\end{array}\right.$$

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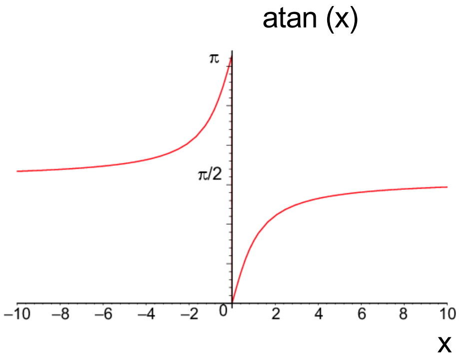
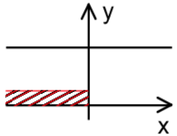
as before, the inverse mapping is $$z_1 = f^{-1}(z_2) = \frac{1}{\pi}\ln(1 - e^{-\pi z_2})$$

and hence
$$\begin{aligned}
\Phi_2(z_2) := \mathrm{Im}(z_1) &= \mathrm{Im}\left(\tfrac{1}{\pi}\ln(1 - e^{-\pi z_2})\right) \\
&= \tfrac{1}{\pi}\mathrm{Arg}(1 - e^{-\pi z_2}) \\
&= \tfrac{1}{\pi}\arctan\frac{e^{-\pi x}\sin(\pi y)}{1 - e^{-\pi x}\cos(\pi y)} \\
&= \tfrac{1}{\pi}\arctan\frac{\sin(\pi y)}{e^{\pi x} - \cos(\pi y)}
\end{aligned}$$



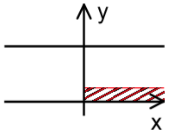
check

$x < 0, y = 0^+$:



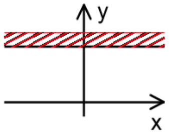
$$\Phi_2 = \frac{1}{\pi}\arctan\frac{0^+}{1^- - 1} = \frac{1}{\pi}\arctan 0^- = 1 \checkmark$$

$x > 0, y = 0^+$:



$$\Phi_2 = \frac{1}{\pi}\arctan\frac{0^+}{e^{\pi x} - 1} = 0 \checkmark$$

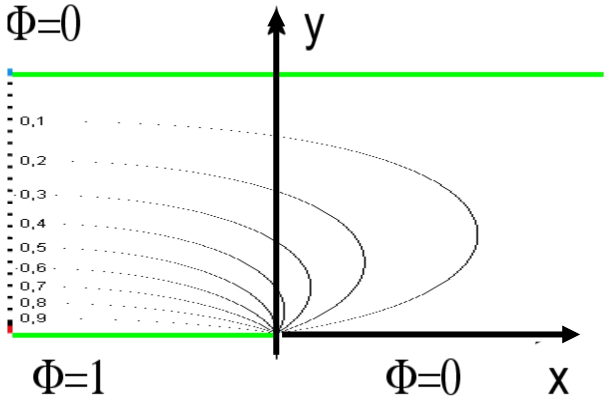
$y = 1^-$:



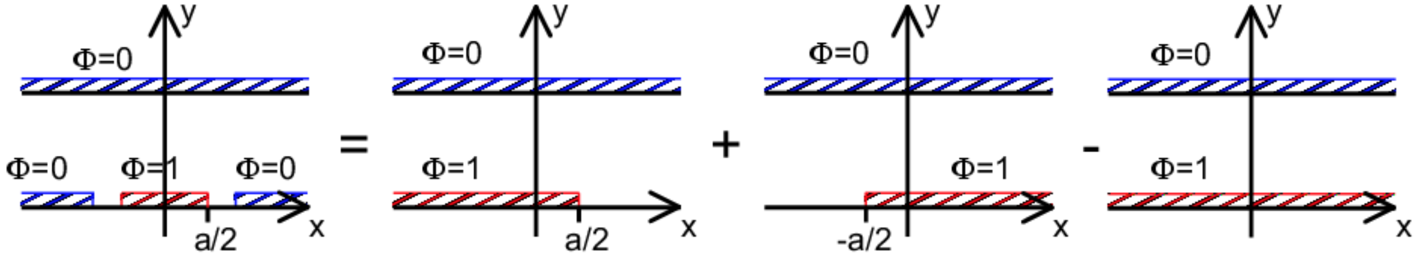
$$\Phi_2 = \frac{1}{\pi}\arctan\frac{0^+}{e^{\pi x} + 1} = 0 \checkmark$$

Weighting Potential for Strip- or Pixel Electrodes – VIII

Plotting the new potential $\Phi_2(z_2)$ then yields



to arrive at the final geometry: superimpose several geometries



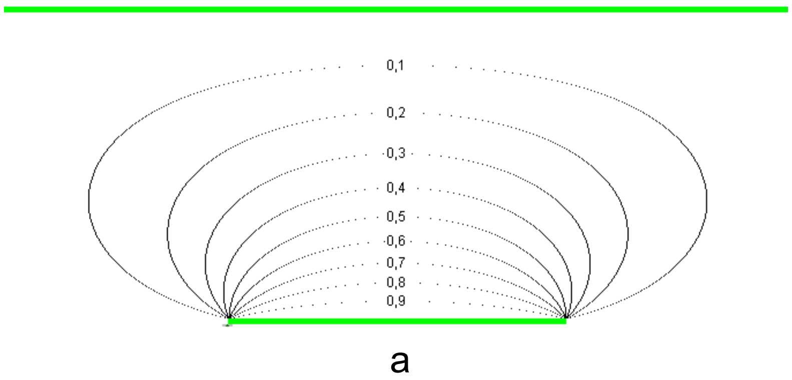
$$\boxed{\phi_{2,\text{strip}}(z_2) = \frac{1}{\pi}\arctan\frac{\sin(\pi y)}{e^{\pi(x-a/2)}-\cos(\pi y)} + \frac{1}{\pi}\arctan\frac{\sin(\pi y)}{e^{\pi(-a/2-x)}-\cos(\pi y)} - (1-y)}$$

more elegantly using the "Schwarz-Christoffel-Transformation"

$$z_2 = f(z_1) = i - \frac{1}{\pi} \ln \frac{e^{\pi z_1} - e^{\pi a}}{e^{\pi z_1} - 1}$$

leads to the simpler (but equivalent) form

$$\boxed{\Phi(x,y) = \frac{1}{\pi} \arctan \frac{\sin(\pi y) \cdot \sinh(\pi \frac{a}{2})}{\cosh(\pi x) - \cos(\pi y) \cosh(\pi \frac{a}{2})}}$$



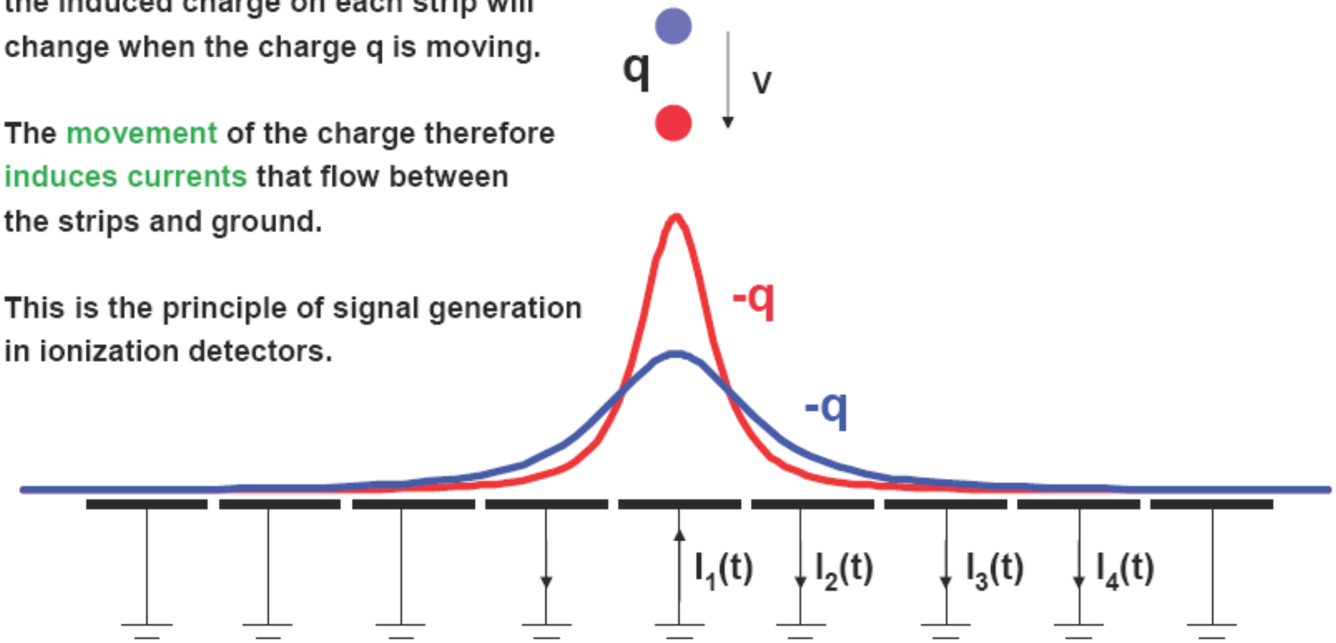
Can be calculated e.g. by "conformal mapping" technique (see e.g. Kolanoski, Wermes, Appendix B) or by using "Schwarz-Christoffel transformations" (e.g. in Morse, Feshbach: Methods of Theoretical Physics, Part I and II., McGraw-Hill)

Charge/current induction on strip electrodes

In case the strips are segmented, the induced charge on each strip will change when the charge q is moving.

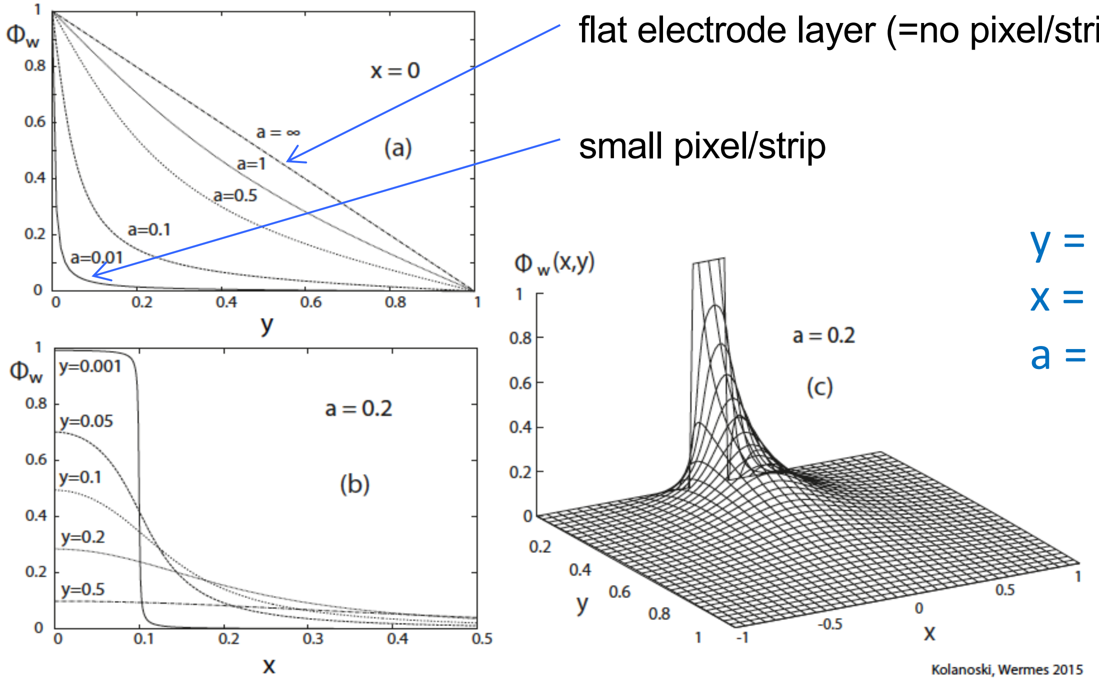
The movement of the charge therefore induces currents that flow between the strips and ground.

This is the principle of signal generation in ionization detectors.



Small pixel effect

weighting potential for different pixel sizes !



flat electrode layer (=no pixel/strip)

small pixel/strip

y = distance from strip plane
x = position on strip plane
a = strip width

Fig. 5.13 Plots of the weighting potential ϕ_w according to (5.94): (a) ϕ_w as a function of the distance y from the strip plane at $x = 0$ (strip centre) for different strip widths a; (b) ϕ_w as a function of the distance x from the strip centre for different distances y from the strip plane at fixed strip width $a = 0.2$ (c) ϕ_w in two-dimensional representation at fixed strip width $a = 0.2$. All lengths values are given in units of the detector thickness d.

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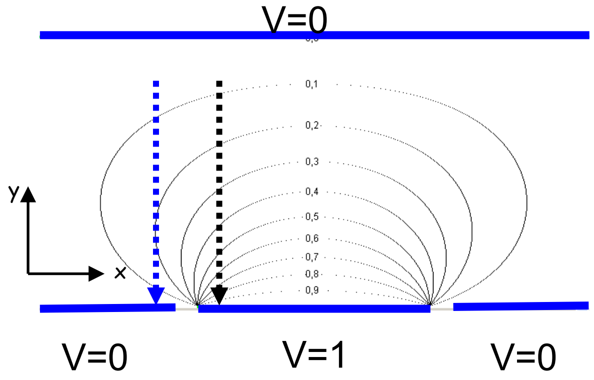
End excursion
Consequences for signal generation

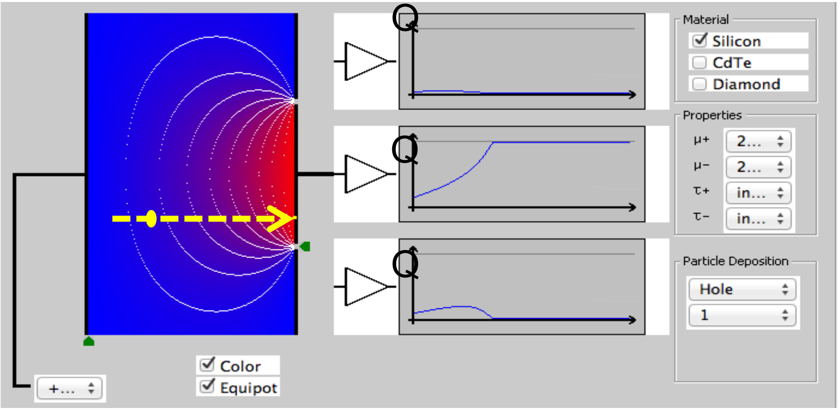
Signal generation in a segmented detector (1-dim)

Φ_W for a strip/pixel geometry

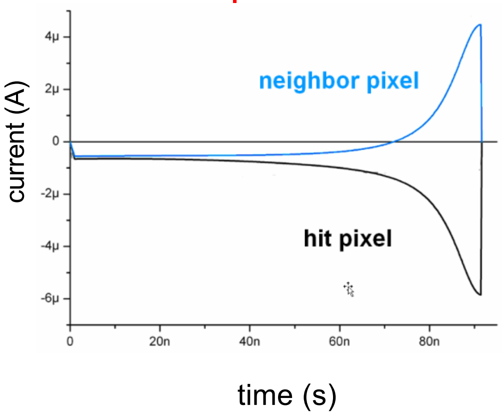
$$\Phi(x,y) = \frac{1}{\pi}\arctan\frac{\sin(\pi y)\cdot\sinh(\pi\frac{a}{2})}{\cosh(\pi x) - \cos(\pi y)\cosh(\pi\frac{a}{2})}$$

(Can be calculated e.g. by "conformal mapping" technique (see e.g. Kolanoski, Wermes, Appendix B) or by using "Schwarz-Christoffel transformations" (e.g. in Morse, Feshbach: Methods of Theoretical Physics, Part I and II., McGraw-Hill)





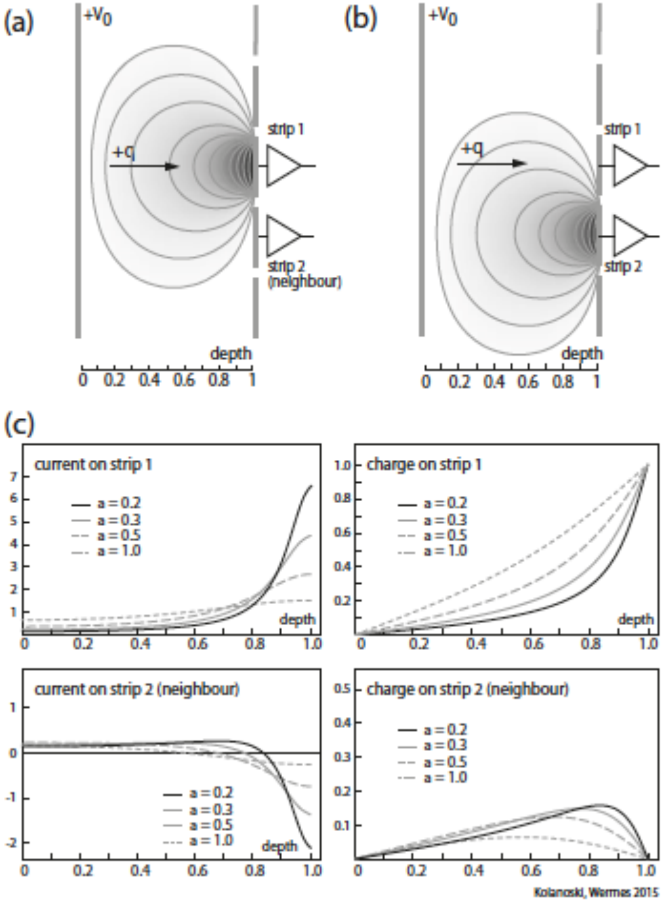
"small pixel effect" !



Weighting field in pixel (and strip) detectors

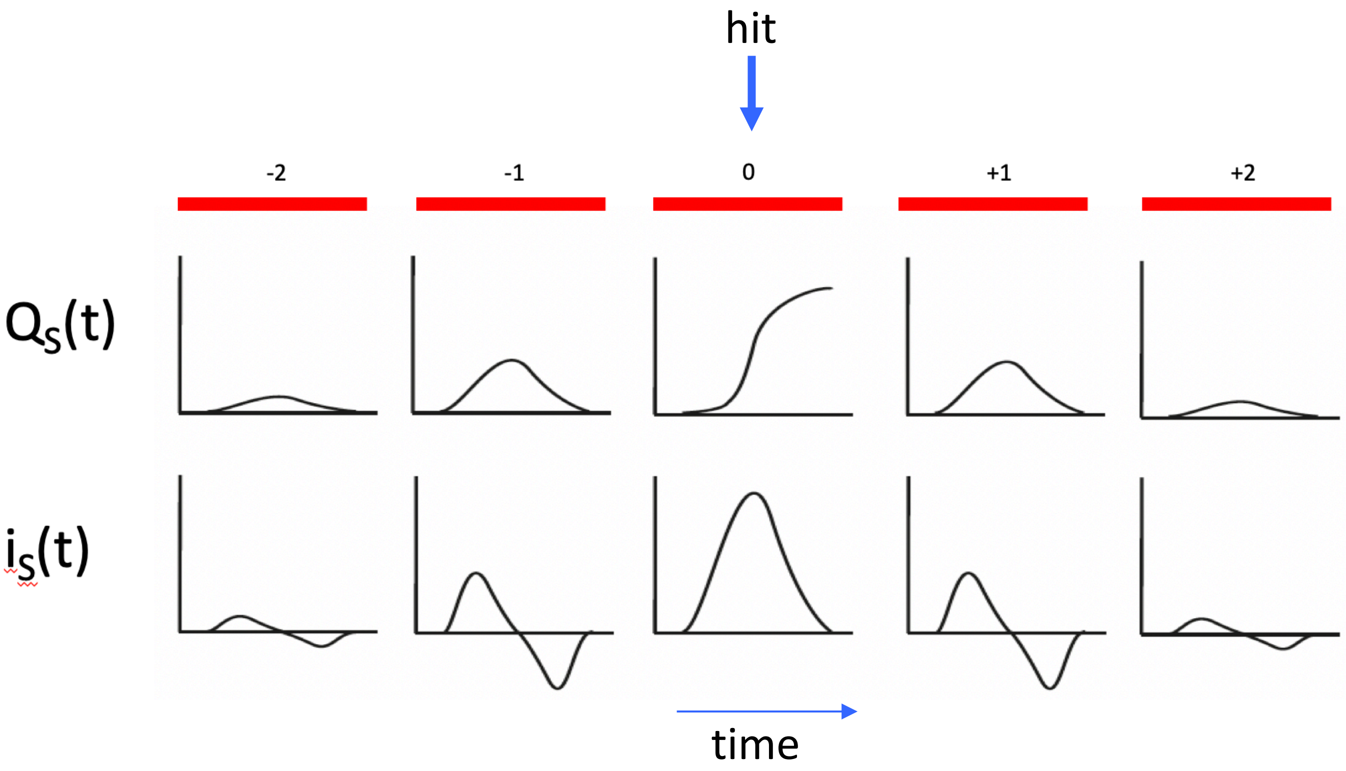
$$dQ_i = -q\,\vec{E}_{w,i}\,d\vec{r}$$

$$i_{S,i} = q\,\vec{E}_{w,i}\,\vec{v}$$



Concluding … consequences …

qualitatively the situation then roughly is like this



Pop-up questions

- What does the term “charge collection” imply and to what extend is it misleading?
  - integration of the deposited charge at the electrode until arrival of all charges misleading since the signal appears instantly, not only after arrival, term appropriate for wire chambers with intrinsic charge amplification.
- What does the “weighting field” specify?
  - the coupling of moving charges to the measurement electrode.
- Formulation of the Shockley-Ramo Theorem?
  - current: $i_{S,i} = q\vec{E}_{w,i}\vec{v}$ charge: $dQ = q\vec{\nabla}\Phi_W d\vec{r}$
- What changes regarding signal development when going from a parallel-plate gas-filled detector to a semiconductor detector with space charge?
  - Weighting field remains the same, electric field is no longer constant
- What is the reason that ion drift governs the signal development in a wire chamber?
  - Due to (a) avalanche multiplication happens only very close to the wire and (b) to the specific shape of the weighting field.
- For multi-electrode geometries … how is the signal development for the “hit” electrode compared to neighbouring electrodes?
  - (hit) current returns to zero upon arrival of last charges (unipolar); (neighbour) bipolar