Electroweak Physics at the LHC
— Lecture 3 —
Electroweak Di-boson Production

Stefan Dittmaier
Albert-Ludwigs-Universität Freiburg
Contents

Electroweak di-boson production – brief overview

$W\gamma / Z\gamma$ production

$WW / WZ / ZZ$ production

Gauge-invariance issues in EW multi-boson production

Outlook to electroweak tri-boson production
Electroweak di-boson production — brief overview
**EW di-boson production**

\[
\begin{align*}
V & \quad V' \\
V' & \quad V \\
V & \quad V', \quad V, V' = \gamma, Z, W^\pm
\end{align*}
\]

**Physics issues:**

- **triple-gauge-boson couplings**, especially at high momentum transfer
  - EW corrections significant
  - anomalous TGC: “formfactor approach” to switch off unitarity violation
    \[\rightarrow\] element of arbitrariness, avoid when possible

- **important background processes**
  - to Higgs production, \( H \rightarrow WW^*/ZZ^* \rightarrow 4f \)
    \[\rightarrow\] invariant masses below \( VV \) thresholds,
    proper description of off-shell \( V^*V^* \rightarrow 4f \) production required!
  - to searches at high invariant masses
    \[\rightarrow\] EW corrections
State-of-the-art predictions

\(W\gamma/Z\gamma\) (with leptonic decays)

- **NNLO QCD** Grazzini, Kallweit, Rathlev ’14,’15
- **NLO EW** Denner, S.D., Hecht, Pasold ’14 (\(Z\gamma\) in preparation)

\(WW, WZ, ZZ\)

- **NNLO QCD**
  - \(ZZ\) (on-shell and off-shell) Cascioli et al. ’14; Grazzini, Kallweit, Rathlev ’15
  - \(WW\) (on-shell) Gehrmann et al. ’14
  - \(gg \rightarrow VV \rightarrow 4\) leptons Binoth et al. ’05,’06

- **NLO EW**
  - stable \(W/Z\) bosons Bierweiler, Kasprzik, Kühn ’12/’13
  - \(pp \rightarrow WW \rightarrow 4\) leptons in DPA Baglio, Le, Weber ’13
  - approximative inclusion in \textsc{Herwig++} Gieseke, Kasprzik, Kühn ’14
  - full off-shell calculation in progress Denner et al.
$W\gamma / Z\gamma$ production
Example of $W\gamma$ production

Issues / physics goals:

- **clean photon–jet separation**
  - quark-to-photon fragmentation function
    - Glover, Morgan '94
  - or Frixione isolation
    - Frixione '98

- **stronger bounds on anomalous $WW\gamma$ coupling:**

$$
\begin{align*}
W_{\mu}^+ (q) \to & \gamma (p) \\
W_{\nu}^- (\bar{q}) \to & \gamma (\bar{p})
\end{align*}
$$

$$
\begin{align*}
&= e \left\{ \frac{q^\mu g^{\nu \rho}}{q^2} \left( \Delta \kappa^\gamma + \lambda^\gamma \frac{q^2}{M_W^2} \right) - q^\nu g^{\mu \rho} \left( \Delta \kappa^\gamma + \lambda^\gamma \frac{q^2}{M_W^2} \right) \\
&\quad + (\bar{q}^\rho - q^\rho) \frac{\lambda^\gamma}{M_W^2} \left( p^\mu p^\nu - \frac{1}{2} g^{\mu \nu} p^2 \right) \right\} \times \left( 1 + \frac{M_W^2 \gamma}{\Lambda^2} \right)^2
\end{align*}
$$

**ATLAS limits ’12:** $\Delta \kappa^\gamma = 0.41, \quad \lambda^\gamma = 0.074$ for $\Lambda = 2$ TeV
Photon–jet separation via photon fragmentation function $D_{q\rightarrow \gamma}$  

Glover, Morgan ’94

Why?

- QCD radiation cannot be suppressed by cuts  
  $\leftrightarrow$ treat at least soft/collinear jets inclusively

- separation of collinear quarks and photons  
  leads to IR-unstable corrections $\propto \ln(m_q^2/Q^2)$  
  $\leftrightarrow$ recombine collinear quarks and photons

- quark and gluon jets cannot be distinguished event by event  
  $\leftrightarrow$ common recombination required for quarks/gluons with photons

$\Rightarrow$ ($g_{\text{hard}} + \gamma_{\text{soft}}$) and ($g_{\text{soft}} + \gamma_{\text{hard}}$) both appear as 1 jet  

EW corr. to $X+$jet  

QCD corr. to $X+\gamma$

Problem: signatures of $X+$jet and $X+\gamma$ overlap!
Photon–jet separation via photon fragmentation function $D_{q \rightarrow \gamma}$  

Solution:

- **idea:** declare photon/jet systems as photon or jet according to energy share

- determine photon energy fraction $z_{\gamma} = \frac{E_{\gamma}}{E_{\text{jet}} + E_{\gamma}}$ of photon/jet system

  $\leftrightarrow$ event selection:
  
  $z_{\gamma} > z_0$: photon
  
  $z_{\gamma} < z_0$: jet  
  (typical value $z_0 = 0.7$)

- **but:** cut on $z_{\gamma}$ destroys inclusiveness needed for KLN theorem

  $\leftrightarrow$ collinear singularity $\propto \alpha \ln m_q$ remains (but are universal!)

- absorb universal collinear singularity in “fragmentation function” $D_{q \rightarrow \gamma}(z_{\gamma})$

  $\leftrightarrow$ subtract convolution of LO cross section with

\[
D_{q \rightarrow \gamma}^{\text{MS}}(z_{\gamma}, \mu_{\text{fact}}) \Big|_{\text{mass.reg.}} = \frac{\alpha Q_q^2}{2\pi} P_{q \rightarrow \gamma}(z_{\gamma}) \left[ \ln \frac{m_q^2}{\mu_{\text{fact}}^2} + 2 \ln z_{\gamma} + 1 \right] \leftarrow \text{cancels coll. singularities}
\]

\[
+ D_{q \rightarrow \gamma}^{\text{ALEPH}}(z_{\gamma}, \mu_{\text{fact}}) \leftarrow \text{non-perturbative part fitted to ALEPH data}
\]

where $P_{q \rightarrow \gamma}(z_{\gamma}) = \frac{1 + (1 - z_{\gamma})^2}{z_{\gamma}}$ = quark-to-photon splitting function
Alternative: photon–jet separation via Frixione isolation

Frixione '98

Idea: suppress jets inside collinear cone around photons:

\[ p_{T,\text{jet}} < \varepsilon p_{T,\gamma} \left( \frac{1 - \cos R_{\gamma\text{jet}}}{1 - \cos R_0} \right) \quad (R_0 = \text{fixed cone size}) \]

- photon and jet collinear \((R_{\gamma\text{jet}} \to 0)\) \(\rightarrow\) event discarded
- photon soft or collinear to beams \((p_{T,\gamma} \to 0)\) \(\rightarrow\) event discarded
- jet soft or collinear beams \((p_{T,\text{jet}} \to 0)\) \(\rightarrow\) event kept \(\Rightarrow\) IR safety

Comments:

- Frixione isolation simple to implement theoretically, but problematic experimentally
- cleaner isolation of non-perturbative effects by fragmentation function
- approximate relation between the two methods:

\[ z_\gamma \sim \frac{p_{T,\gamma}}{p_{T,\gamma} + p_{T,\text{jet}}} > \frac{1}{1 + \varepsilon \frac{1 - \cos R_{\gamma\text{jet}}}{1 - \cos R_0}} \sim \frac{1}{1 + \varepsilon} \quad \text{for} \quad R_{\gamma\text{jet}} \sim R_0 \]

\(\leftrightarrow\) methods yield quite similar results for \(z_0 \sim \frac{1}{1 + \varepsilon}\)
• good agreement of experimental results with NNLO QCD (no EW corrections included)
• QCD uncertainties:  (for small/moderate $p_T, \gamma$)  
  scale: $4-5\%$, PDF: $1-2\%$  (increasing with $p_T, \gamma$)  
• LHC run 2:  higher energy reach & higher statistics  
  $\leftrightarrow$  EW corrections important
NLO EW corrections calculated with full W off-shell/decay effects
(complex-mass scheme)

more + more complicated diagrams than in QCD

particular focus on:

- high energies (e.g. large $p_T$):
  - large EW corrections ↔ sensitivity to anomalous couplings
  - missing corrections could fake anomalous couplings
- photon-induced contributions
Rapidity distributions in $W\gamma$ production

$pp \rightarrow l^+ \nu_l / \gamma$ (jet)

- huge QCD corrections ($\sim 100\%$), only mildly reduced by jet veto $p_{T,\text{jet}} < 100$ GeV
- EW corrections and $q\gamma$ channels (few %) small and flat
  $\leftrightarrow$ resemble corrections to integrated cross section

$\sqrt{s} = 14$ TeV

\[ \delta_{\text{QCD}} \approx \delta_{\text{QCD,LO}} + \delta_{\text{QCD,NLO}} \]

\[ \delta_{\text{EW,LO}} \approx \delta_{\text{EW,LO},q} + \delta_{\text{EW,LO},\gamma} \]

\[ \delta_{\text{EW,NLO}} \approx \delta_{\text{EW,NLO},q} + \delta_{\text{EW,NLO},\gamma} \]

\[ \delta_{\text{EW,LO},q} \approx \delta_{\text{EW,LO},q} + \delta_{\text{EW,LO},\gamma} \]

\[ \delta_{\text{EW,NLO},q} \approx \delta_{\text{EW,NLO},q} + \delta_{\text{EW,NLO},\gamma} \]

\[ \delta_{\text{EW,LO},\gamma} \approx \delta_{\text{EW,LO},\gamma} + \delta_{\text{EW,LO},\gamma} \]

\[ \delta_{\text{EW,NLO},\gamma} \approx \delta_{\text{EW,NLO},\gamma} + \delta_{\text{EW,NLO},\gamma} \]

\[ \delta_{\text{EW,LO},q} \approx \delta_{\text{EW,LO},q} + \delta_{\text{EW,LO},\gamma} \]

\[ \delta_{\text{EW,NLO},q} \approx \delta_{\text{EW,NLO},q} + \delta_{\text{EW,NLO},\gamma} \]

\[ \delta_{\text{EW,LO},\gamma} \approx \delta_{\text{EW,LO},\gamma} + \delta_{\text{EW,LO},\gamma} \]

\[ \delta_{\text{EW,NLO},\gamma} \approx \delta_{\text{EW,NLO},\gamma} + \delta_{\text{EW,NLO},\gamma} \]
• EW corrections $\sim -30\%$ in TeV range

• $\gamma$-induced corrections non-negligible in TeV range (even with jet veto)\n  $\rightarrow$ reduction of $\gamma$ PDF uncertainties mandatory!
**Wγ production – anomalous couplings**

- Denner, S.D., Hecht, Pasold ‘14

- $\sigma_{\text{NLO QCD}}$, $\Lambda = 1\text{ TeV}$
- $\sigma_{\text{AC}}^{\text{NLO QCD}}$, $\Lambda = 2\text{ TeV}$
- $\sigma_{\text{AC}}^{\text{NLO QCD}}$, $\Lambda \to \infty$

- $\Delta \kappa^\gamma = 0.41$, $\lambda^\gamma = 0.074$
- $\rightarrow$ much tighter limits expected at LHC run 2

- $\sqrt{s} = 14\text{ TeV}$
- $p_T, l \gamma, \gamma(jet)$

- $\sigma_{\text{AC}}, \Lambda =$ 1 TeV
- $\sigma_{\text{AC}}, \Lambda =$ 2 TeV
- $\sigma_{\text{AC}}, \Lambda \to \infty$

- Results shown without and with jet veto on $p_{T,\text{jet}} > 100\text{ GeV}$

- ATLAS values of 2012 used: $\Delta \kappa^\gamma = 0.41$, $\lambda^\gamma = 0.074$
WW / WZ / ZZ production
Complementarity in WW / WZ / ZZ production

WW production:

WZ production:

ZZ production:
Complementarity in WW / WZ / ZZ production

WW production:

WZ production:

ZZ production:

Sensitivity to different PDF combinations:

• $q\bar{q}$ in WW/ZZ
• $u\bar{d}/d\bar{u}$ in $W^+Z/W^-Z$
• $\gamma\gamma$ in WW
Complementarity in WW / WZ / ZZ production

WW production:

WZ production:

ZZ production:

Sensitivity to different anomalous TGCs:

- overlay of $\gamma_{WW}/ZWW$ in WW
- only $ZWW$ in WZ
- $\gamma_{ZZ}/ZZZ$ in ZZ
Complementarity in WW / WZ / ZZ production

WW production:

WZ production:

ZZ production:

Background to Higgs production in channel $H \rightarrow WW^*/ZZ^* \rightarrow 4f$

$\rightarrow$ off-shell calculation particularly important for WW/ZZ!
QCD corrections to $WW$, $WZ$, $ZZ$, $W\gamma$, $Z\gamma$ production

NLO QCD calculated (including leptonic $W/Z$ decays)

Baur, Han, Ohnemus '93-'98
Dixon, Kunszt, Signer '99
Campbell, R.K. Ellis '99
DeFlorian, Signer '00

Large positive corrections due to jet radiation, i.e. $VV + \text{jet}$ production

- reduction of corrections and scale dependence by jet veto: $p_{T,\text{jet}} < \text{cut}$?
  $\leftrightarrow$ include QCD resummation for veto

- NNLO QCD corrections important
**WW production – NNLO QCD theory versus experiment**

\[ \sigma [\text{pb}] \]

- **CMS** & **ATLAS**

\[ \sqrt{s} \text{[TeV]} \]

- **Subtlety:**
  - Separation of single-\( t \) and \( t\bar{t} \) contributions @ NNLO QCD
  - \( \leftrightarrow \) b-jet veto, etc.

- **good agreement of experimental results with NNLO QCD**

- **NNLO QCD correction** \( \sim 7(12)\% @ 8(13) \text{ TeV} \), scale uncertainty \( \lesssim 3\% \)

- **gg contribution** \( \sim 7(8)\% @ 8(13) \text{ TeV} \)

- **LHC run 2:** higher energy & higher statistics \( \rightarrow \) EW corrections important
good agreement of experimental results with NNLO QCD

NNLO QCD correction $\sim 12(17)\%$ @ 8(13) TeV, scale uncertainty $\lesssim 3\%$

gg contribution $\sim 7(10)\%$ @ 8(13) TeV

LHC run 2: higher energy & higher statistics $\rightarrow$ EW corrections important
WZ production – NLO QCD theory versus experiment

\[
\sigma(pp \rightarrow W^+Z + W^-Z) \text{ [pb]}
\]

NLO QCD+EW, \( \mu_0 = M_W + M_Z \)

- good agreement of experimental results with NLO QCD
- NLO QCD scale uncertainty \( \sim 3\% \), \( \Delta_{PDF+\alpha_s} \sim 4\% \)
- LHC run 2: higher energy & higher statistics
  \( \leftrightarrow \) NNLO QCD and NLO EW corrections important

Baglio, Le, Weber et al. ’13
EW corrections to massive di-boson production

- small for integrated XS
- growing in distributions for larger scales

Note:
- $M_{VV}$ not accessible for $W$ final states
- on-shell approximation not applicable for $M_{VV} < M_{V1} + M_{V2}$
Survey of corrections to WW production

(stable/on-shell W bosons)

Bierweiler, Kasprzik, Kühn ‘12

Baglio, Le, Weber ‘13

Note:
large contribution by γγ channel for high invariant WW masses!

Stefan Dittmaier, Electroweak Physics at the LHC – Lecture 3
Freiburg, Oct 2015 – 23
• many observables not accessible without W decays
• sizeable influence of W decays on EW corrections
• $\gamma\gamma$ contribution significant for large energies
• $q\gamma$ contribution suppressed by jet veto (otherwise overwhelmed by QCD corrections)
Relevance of EW corrections at 8 TeV

- EW corrections reach already 10–20% at scales $\sim 500–1000$ GeV
- $\gamma\gamma$ contribution sizeable for large energies
- Special situation in $p_{T,e\mu}$:
  - large positive corrections due to WW recoiling against hard $\gamma$ radiation
EW corrections vs. anomalous TGCs in gauge-boson pair production

\[ \frac{d\sigma}{dP_T^{\text{max}}(l)} \text{[fb/GeV]} \]

\[ pp \rightarrow WZ \rightarrow e\nu_e\mu^+\mu^- \]
\[ \sqrt{s} = 14 \text{ TeV} \]

Accomando, Kaiser '05

### Scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( \Delta g_1^Z )</th>
<th>( \Delta \kappa_\gamma )</th>
<th>( \lambda_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Born</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2a/2b</td>
<td>±0.02</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3a/3b</td>
<td>0</td>
<td>±0.04</td>
<td>0</td>
</tr>
<tr>
<td>4a/4b</td>
<td>0</td>
<td>0</td>
<td>±0.02</td>
</tr>
</tbody>
</table>

\[ \lambda_Z = \lambda_\gamma, \quad \Delta \kappa_Z = \Delta g_1^Z - \tan^2 \theta_W \Delta \kappa_\gamma \]

Formfactor rescaling (\( \Lambda = 1 \text{ TeV} \)):

\[ \Delta Y \rightarrow \frac{\Delta Y}{(1 + \hat{s}/\Lambda^2)^2}, \quad \Delta Y = \Delta g_1^Z, \Delta \kappa_\gamma, \lambda_\gamma \]

Note:
- EW corrections and anomalous couplings distort distributions
- neglect of EW corrections can mimick anomalous couplings
Gauge-invariance issues in EW multi-boson production
Gauge invariance implies...

- **Slavnov–Taylor or Ward identities**
  = algebraic relations of or between Greens functions
  \[ \rightarrow \text{guarantee cancellation of unitarity-violating terms,} \]
  crucial for proof of unitarity of \( S \)-matrix

- **Nielsen identities** (compensation of gauge-fixing artefacts)
  \[ \rightarrow \text{gauge-parameter independence of} \ S \text{-matrix} \]
  although Greens function (e.g. self-energies) are gauge dependent

Both statements hold order by order in standard perturbation theory!

Implications:

- **Resonances** require Dyson summation of resonant propagators
  \[ \rightarrow \text{perturbative orders mixed} \rightarrow \text{gauge invariance jeopardized!} \]
  Gauge-invariance-violating terms \( \propto \Gamma \) are formally of higher order,
  but can be dramatically enhanced if unitarity cancellations disturbed

- **Anomalous couplings** potentially enhanced
  if effective operator not gauge invariant
Important Ward identities for processes with EW gauge bosons:

Elmg. U(1) gauge invariance implies

\[ k^\mu F_1 k = 0 \quad \text{for any on-shell fields } F_l \]

\[ \rightarrow \quad \text{Identity becomes crucial for collinear light fermions:} \]

\[ \text{for fermion momenta } p_1 \sim c p_2: \]

\[ p_2 p_1 k = p_1 - p_2 = \bar{u}_2(p_2) \gamma^\mu u_1(p_1) \propto k^\mu \]

A typical situation: quasi-real space-like photons

\[ e \quad \gamma \downarrow k \sim \frac{1}{k^2} k^\mu T^\gamma_\mu \quad \text{for } k^2 \to O(m_e^2) \ll E^2 \]

Identity \( k^\mu T^\gamma_\mu = 0 \) needed to cancel \( 1/k^2 \),
otherwise gauge-invariance-breaking terms enhanced by \( E^2/m_e^2 \) \( (\sim 10^{10} \text{ for LEP2}) \)
Electroweak SU(2) gauge invariance implies

\[ k^\mu \sim Z_\mu \quad F_1 \quad = \quad iM_Z \quad \chi \quad F_n \]

\[ k^\mu \sim W^\pm_\mu \quad F_1 \quad = \pm M_W \quad \phi^\pm \quad F_n \]

\( F_i \) = on-shell fields
\( \chi, \phi^\pm \) = would-be Goldstone fields

A typical situation: high-energetic quasi-real longitudinal vector bosons

\[ \leftrightarrow \text{fermion current attached to } V(k) \text{ again } \propto k^\mu \]

\[ \sim \frac{1}{k^2 - M_V^2} k^\mu T^V_\mu \quad \text{for } k^0 \gg M_V \]

Identity \( k^\mu T^V_\mu = c_V M_V T^S \) needed to cancel factor \( k^0 \),
otherwise gauge-invariance/unitarity-breaking terms enhanced by \( k^0 / M_V \)

For on-shell \( V \):
\[ \varepsilon^\mu_{VL}(k) = \frac{k^\mu}{M_V} + \mathcal{O}(M_V / k^0) \]
Illustration of unitarity cancellations for $WV$ production ($V = Z/\gamma$)

Leading behaviour of amplitudes with $\varepsilon_{W_L^+}^{\mu}(k) = \frac{k^{\mu}}{M_V} + \ldots$ for $k^{0} \gg M_W$:

\[
\sim \frac{-ie^2 g_{VWW}}{2\sqrt{2}s_WM_W} \left[ \bar{u}\gamma_\mu \omega - u_u \right] \left\{ g_Y^{V} \left[ \varepsilon^{*\mu}_{V} - k^{\mu} \frac{\varepsilon^{*\cdot k}_{V}}{s} \right] + \kappa_{V} \left[ \varepsilon^{*\mu}_{V} + k^{\mu} \frac{\varepsilon^{*\cdot k}_{V}}{s} \right] \right\}
\]

\[
\sim \frac{ie^2 g^{V d\bar{d}}}{\sqrt{2}s_WM_W} \left[ \bar{u}\gamma_\mu \omega - u_u \right], \quad g^{Z dd} = -\frac{s_W}{c_W} Q_d - \frac{1}{2s_Wc_W}, \quad g^{\gamma dd} = -Q_d
\]

\[
\sim \frac{-ie^2 g^{V uu}}{\sqrt{2}s_WM_W} \left[ \bar{u}\gamma_\mu \omega - u_u \right], \quad g^{Z uu} = -\frac{s_W}{c_W} Q_u + \frac{1}{2s_Wc_W}, \quad g^{\gamma uu} = -Q_u
\]

Cancellation (unitarity!) of sum demands:

\[
g^{V dd} - g^{V uu} \frac{g_{VWW}}{2}(g_Y^{V} + \kappa_{V}) \equiv 0, \quad g_Y^{V} \equiv \kappa_{V}
\]

$\leftrightarrow$ SM provides unique solution: $g^{Z}_1 = \kappa_Z = g^{\gamma}_1 = \kappa_{\gamma} = 1$

Note: no constraint on coupling $\lambda_V$, since effective operator gauge invariant!

Stefan Dittmaier, Electroweak Physics at the LHC – Lecture 3
Freiburg, Oct 2015 – 31
Width schemes for LO calculations and gauge invariance

Naive propagator substitutions in full tree-level amplitudes:

\[ \frac{1}{k^2 - m^2} \rightarrow \frac{1}{k^2 - m^2 + im\Gamma(k^2)} \]

in all propagators

- constant width \( \Gamma(k^2) = \text{const.} \) \( \rightarrow \) U(1) respected, SU(2) “mildly” violated
- running width \( \Gamma(k^2) \neq \text{const.} \) \( \rightarrow \) U(1) and SU(2) violated
  \( \Leftarrow \) results can be totally wrong!

Fudge factor approaches:

Multiply full amplitudes without widths with

factors \( \frac{p^2 - m^2}{p^2 - m^2 + im\Gamma} \) for each potentially resonant propagator

\( \Leftarrow \) gauge invariant, but spurious factors of \( O(\Gamma/m) \)

Complex-mass scheme: (see lecture 1)

Consistent use of complex masses everywhere (including couplings)

For W/Z bosons:

\[ M_V^2 \rightarrow \mu_V^2 = M_V^2 - iM_V\Gamma_V, \quad V = W, Z \]

complex weak mixing angle:

\[ c_W^2 = 1 - s_W^2 = \frac{\mu_W^2}{\mu_Z^2} \]

\( \Leftarrow \) gauge invariance fully respected
An example: \( e^- e^+ \rightarrow e^- \bar{\nu}_e u \bar{d} \) result of Kurihara, Perret-Gallix, Shimizu '95

\[ \sqrt{s} = 180 \text{ GeV} \]

solid: gauge-invariant (fudge factor) scheme

dashed: constant width only in resonant propagator
\[ \leftrightarrow \text{crude U(1) gauge-invariance violation} \]

Dominant diagrams:

nearly real photon!
Example continued:

Partial amplitude from above “photon diagrams”:

\[ M_\gamma = Q_e e \bar{u}_e(k_e) \gamma^\mu u_e(p_e) \frac{1}{k_\gamma^2} T_\gamma^\mu \]

Elmg. Ward identity:

\[ 0 = k_\gamma^\mu T_\gamma^\mu \propto (p_+^2 - p_-^2)Q_W P_w(p_+^2) P_w(p_-^2) + Q_e P_w(p_+^2) - (Q_d - Q_u) P_w(p_-^2) \]

With \( Q_W = Q_e = Q_d - Q_u \) and \( P_w(p^2) = [p^2 - M_W^2 + iM_W \Gamma_W(p^2)]^{-1} \)

one obtains:

\[ \Gamma_W(p_+^2) \equiv \Gamma_W(p_-^2) \]

\[ \rightarrow \] Elmg. gauge invariance demands

common width on \( s \)- and \( t \)-channel propagators in “naive fixed width scheme”
Examples from $e^+e^-$ physics: RACOONWW (Denner et al. ’99-’01) and LUSIFER (S.D.,Roth ’02)

- $\sigma$ [ fb] for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>189 GeV</th>
<th>500 GeV</th>
<th>2 TeV</th>
<th>10 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant width</td>
<td>703.5(3)</td>
<td>237.4(1)</td>
<td>13.99(2)</td>
<td>0.624(3)</td>
</tr>
<tr>
<td>running width</td>
<td>703.4(3)</td>
<td>238.9(1)</td>
<td>34.39(3)</td>
<td>498.8(1)</td>
</tr>
<tr>
<td>complex mass</td>
<td>703.1(3)</td>
<td>237.3(1)</td>
<td>13.98(2)</td>
<td>0.624(3)</td>
</tr>
</tbody>
</table>

- $\sigma$ [ fb] for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu + \gamma$ (separation cuts for “visible” $\gamma$: $E_\gamma, \theta_{\gamma_f} >$ cut)

<table>
<thead>
<tr>
<th>$\sqrt{s} =$</th>
<th>189 GeV</th>
<th>500 GeV</th>
<th>2 TeV</th>
<th>10 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant width</td>
<td>224.0(4)</td>
<td>83.4(3)</td>
<td>6.98(5)</td>
<td>0.457(6)</td>
</tr>
<tr>
<td>running width</td>
<td>224.6(4)</td>
<td>84.2(3)</td>
<td>19.2(1)</td>
<td>368(6)</td>
</tr>
<tr>
<td>complex mass</td>
<td>223.9(4)</td>
<td>83.3(3)</td>
<td>6.98(5)</td>
<td>0.460(6)</td>
</tr>
</tbody>
</table>

- $\sigma$ [ fb] for $e^+e^- \rightarrow \nu_e\bar{\nu}_e\mu^-\bar{\nu}_\mu u\bar{d}$ (phase-space cuts applied)

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>500 GeV</th>
<th>800 GeV</th>
<th>2 TeV</th>
<th>10 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant width</td>
<td>1.633(1)</td>
<td>4.105(4)</td>
<td>11.74(2)</td>
<td>26.38(6)</td>
</tr>
<tr>
<td>running width</td>
<td>1.640(1)</td>
<td>4.132(4)</td>
<td>12.88(1)</td>
<td>12965(12)</td>
</tr>
<tr>
<td>complex mass</td>
<td>1.633(1)</td>
<td>4.104(3)</td>
<td>11.73(1)</td>
<td>26.39(6)</td>
</tr>
</tbody>
</table>
Gauge-invariant width schemes @ NLO

Problem much more complicated than at LO! (would fill own lectures)

**Complex-Mass Scheme (CMS)**

- complex, but straightforward renormalization
- NLO everywhere in phase space
- loop integrals with complex masses

**Pole Approximation (PA)** (= leading term of pole expansion)

- corrections decomposed into two types
  - factorizable: corrections to on-shell production / decay
  - non-factorizable: soft photon/gluon exchange between production / decays
- NLO in neighbourhood of resonances
- PA involves less diagrams than CMS → higher multiplicities possible

**Effective Field Theories**

- involves pole expansions → NLO in neighbourhood of resonances
- formal elegance → e.g. combination with resummations

→ For details & examples see literature ...
Outlook to
electroweak tri-boson production
Electroweak tri-boson production – overview

Typical LO diagrams (example of WWZ production):

- similarity/complementarity to vector-boson scattering (crossed kinematics)
  - sensitivity to quartic gauge couplings
  - sensitivity to electroweak symmetry breaking
- background to WH/ZH production with $H \rightarrow VV^*$ (if accessible)

$\gamma\gamma$ channel for WWZ/$\gamma$ production:
QCD corrections to WWZ production

- inclusive cross section $\sim 200$ fb @ $\sqrt{s} = 14$ TeV
- QCD corrections $\sim 100\%$
  - other final states WWW, WZZ, etc. known to NLO QCD as well
    - Lazopoulos, et al. '07; Hankele/Zeppenfeld '07; Binoth et al. '08; Nhung et al. '13
  - analyze possible jet vetoes
  - W/Z decays partially included in NWA
• sizeable EW corrections in TeV range (as expected from di-boson case)

• EW corrections only known for on-shell (stable) WWZ production
  ↦ homework for theorists to ...
  ◊ consider the other cases WWW, WZZ, etc. as well
  ◊ include W/Z decays
  ◊ combine NLO QCD (known) and EW corrections