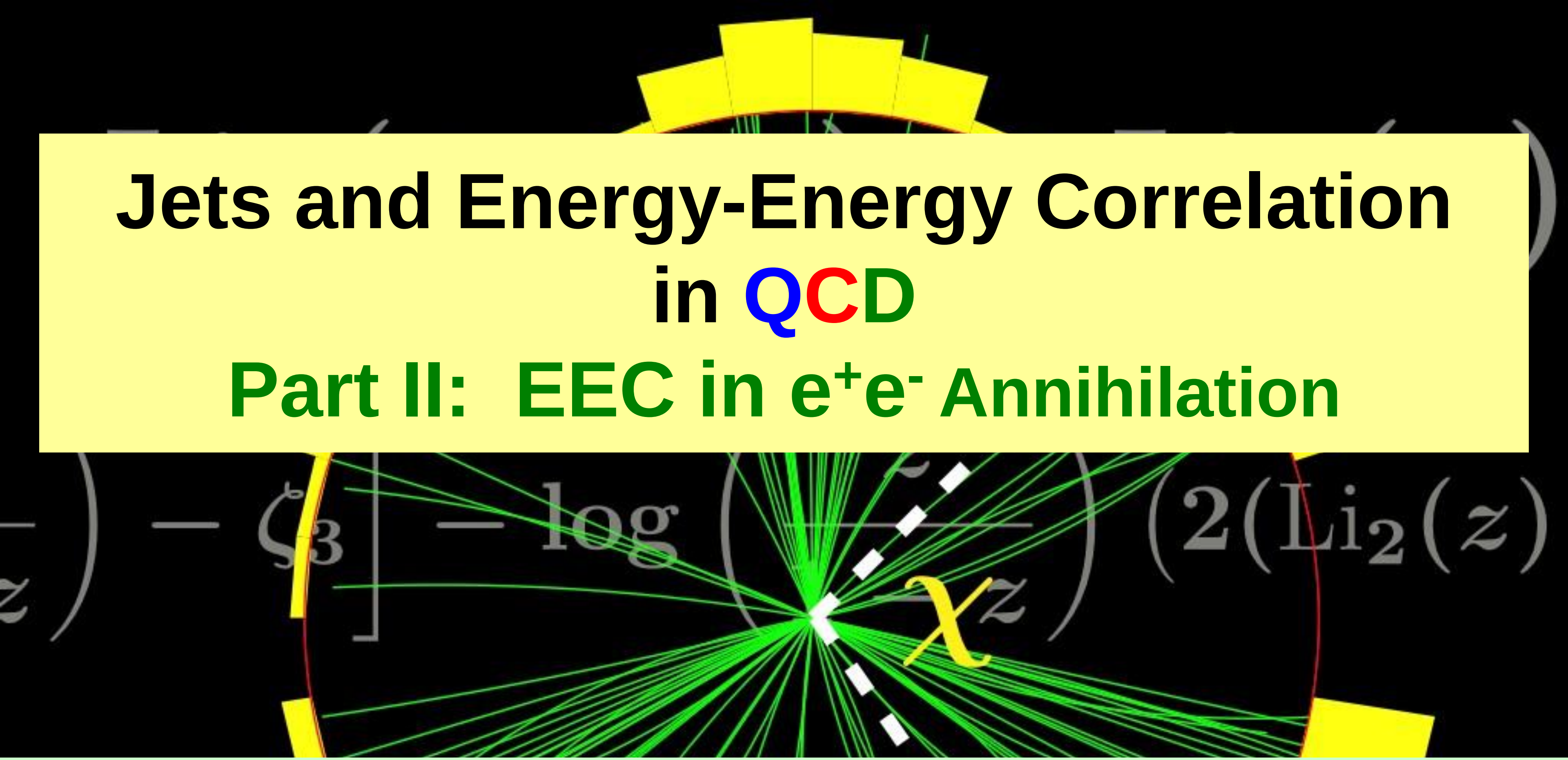


Jets and Energy-Energy Correlation in QCD

Part II: EEC in e^+e^- Annihilation

Lance Dixon (SLAC)

LD, Luo, Shtabovenko, Yang, Zhu, 1801.03219

LD, Moult, Zhu, 1904.nnnnn

University of Freiburg

4 April, 2019

SLAC
NATIONAL ACCELERATOR LABORATORY

The EEC

- Energy-energy correlation (EEC) in e^+e^- annihilation:

one of first infrared safe event-shapes defined in QCD, 40 years ago Basham, Brown, Love, S. Ellis, PRD, PRL 1978

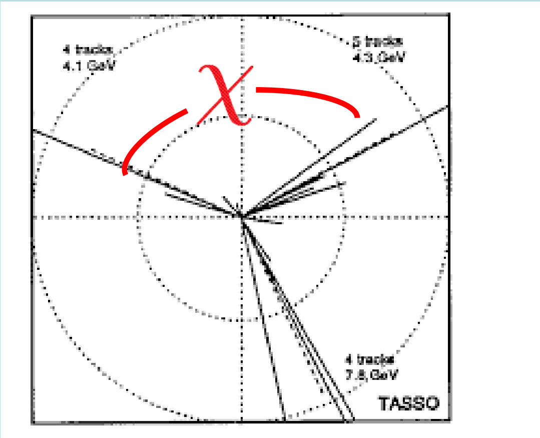
$$\boxed{\frac{d\Sigma}{d\cos\chi} = \sum_{\text{partons } i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta(\cos\theta_{ij} - \cos\chi)}$$

Collinear parton splitting

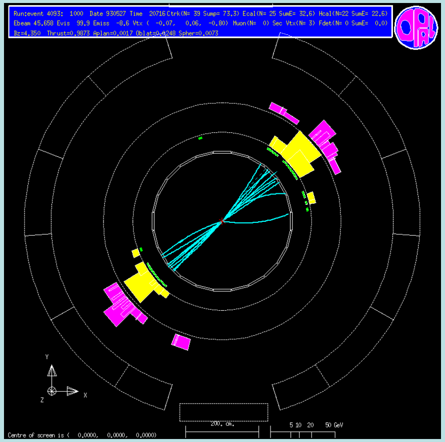
$$E_i \to x E_i + (1-x) E_i$$

preserves observable.
So does soft emission.

Data from wide range of CM energies →

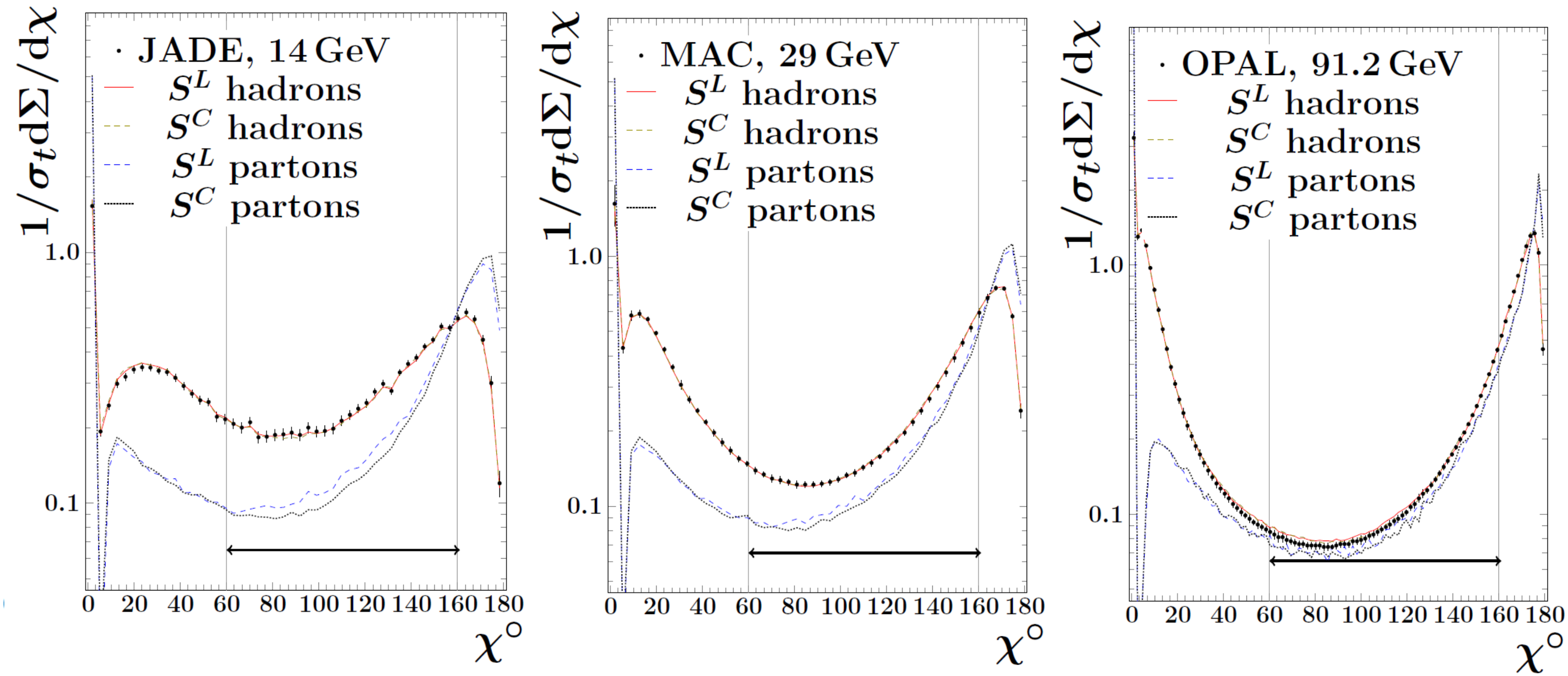


$Q \approx 22\ \text{GeV}$



$Q = 91\ \text{GeV}$

Evolution with energy clearly visible



data reviewed in Kardos et al, 1804.09146

Why the EEC?

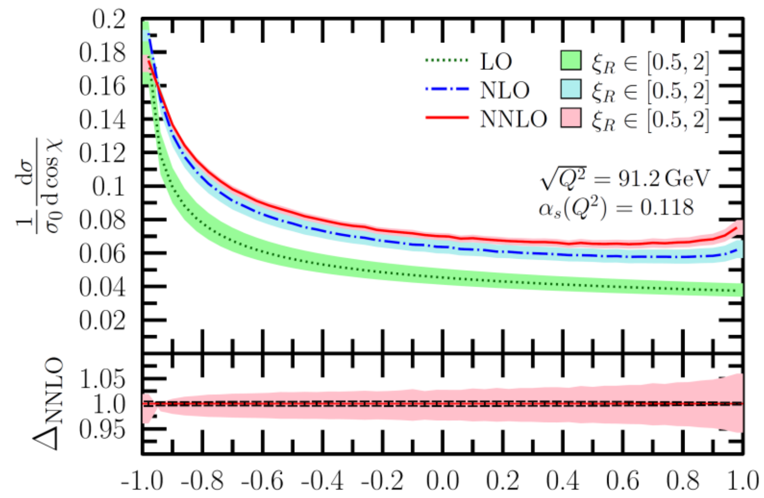
- Many event-shape variables to choose from:
  thrust, oblateness, C parameter, heavy jet mass, angularity, jet rates, …
- EEC among the simplest analytically
- Angle χ lives on a compact domain, $[0,\pi]$:
  large logarithms on **both** ends can be resummed
- As $\chi \to 0$, probe jet substructure. Can generalize to computable LHC jet substructure variables, correlating multiple small angles Moult, Necib, Thaler, 1609.07483
- **Gravitons** couple to **energy**, so AdS/CFT holography can be used to compute at strong gauge coupling
  (in planar N=4 SYM, not QCD) Hofman, Maldacena, 0803.1467

Numerical results

- EEC computed at NLO numerically in 1980s and 1990s
  Richards, WJ Stirling, Ellis, 1982, 1983; Ali, Barreiro, 1982, 1984; Schneider, Kramer, Schierholz, 1984; Falck, Kramer, 1989; Kunszt, Nason, Marchesini, Webber, LEP Yellow Book, 1989; Glover, Sutton, 1994; Clay, Ellis, 1995; Kramer, Spiesberger, 1996; Catani, Seymour, 1996 [EVENT2].
- Computed numerically at NNLO only 3 years ago

  Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, 1603.08927

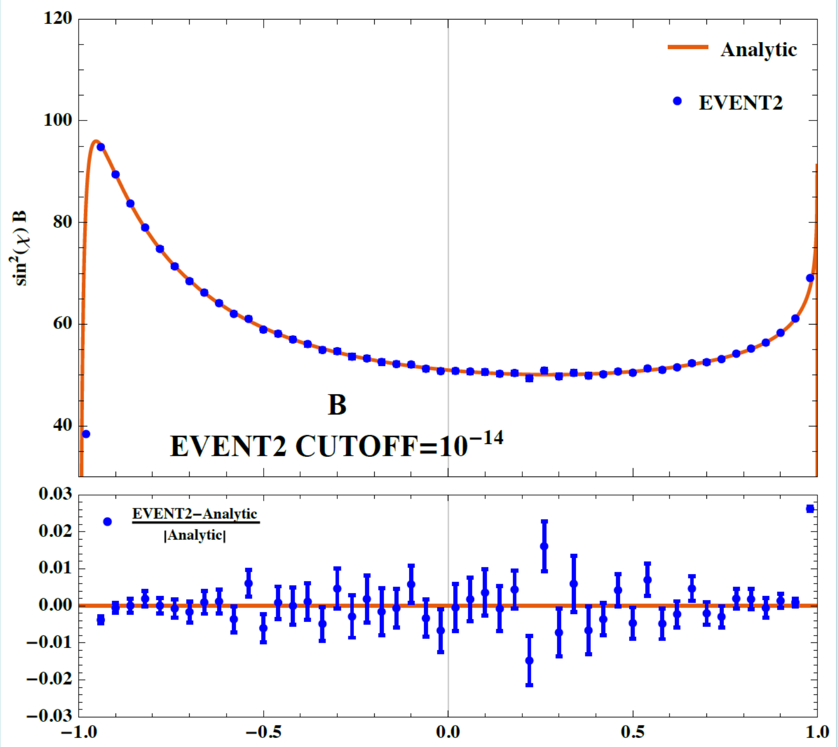
- Can now compute analytically at NLO in QCD



Why analytic?

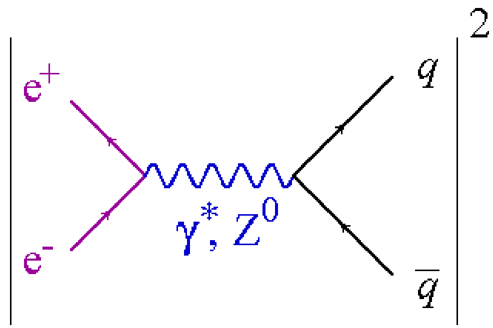
- Validate accuracy of numerical QCD results.
- Compare with analytic NLO result in N=4 SYM

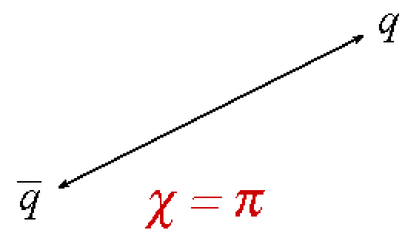
Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 1309.0769, 1309.1424, 1311.6800



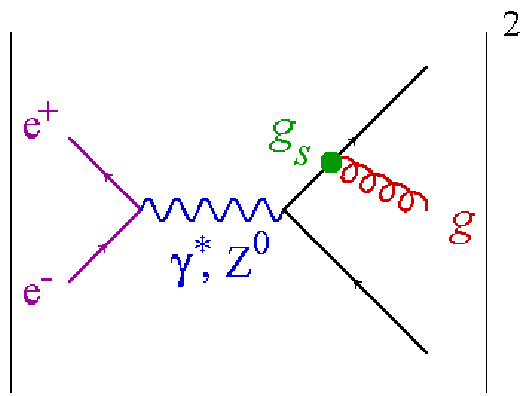
- Study limits as $\chi \to 0, \pi$ to aid resummation of large logarithms there.

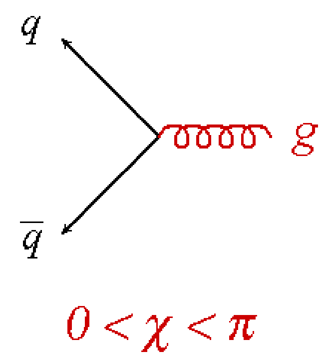
LO EEC for $0 < \chi < \pi$ is $O(\alpha_s)$





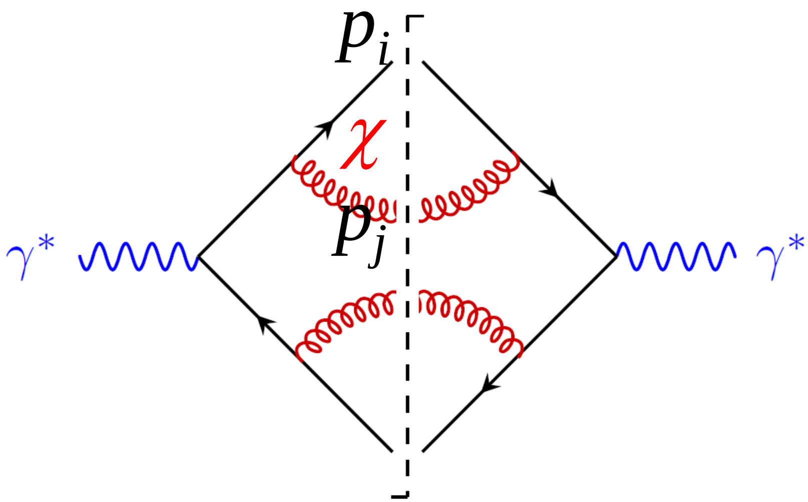
$$\alpha_s = \frac{g_s^2}{4\pi}$$





How to compute at NLO?

Sample
NLO real emission
contribution



- Use interference method with Feynman diagrams
- Reverse unitarity: Treat all momenta as loop momenta,

put all cut momenta on shell and impose $\delta(\cos\theta_{ij} - \cos\chi)$

- IBPs/Laporta algorithm Chetyrkin, Tkachov (1981), Laporta (2001)
- Differential equations for master integrals

Gehrmann, Remiddi (2000)

can all be solved in terms of polylogarithms

Structure of QCD result

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi}A(z) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \left(\beta_0 \log\frac{\mu}{Q}A(z) + B(z)\right) + \mathcal{O}(\alpha_s^3)$$

$$z = \frac{1}{2}(1-\cos\chi) \quad \in \quad [0,1]$$

LO result fits on one line:

Basham, Brown, Love, S. Ellis, 1978

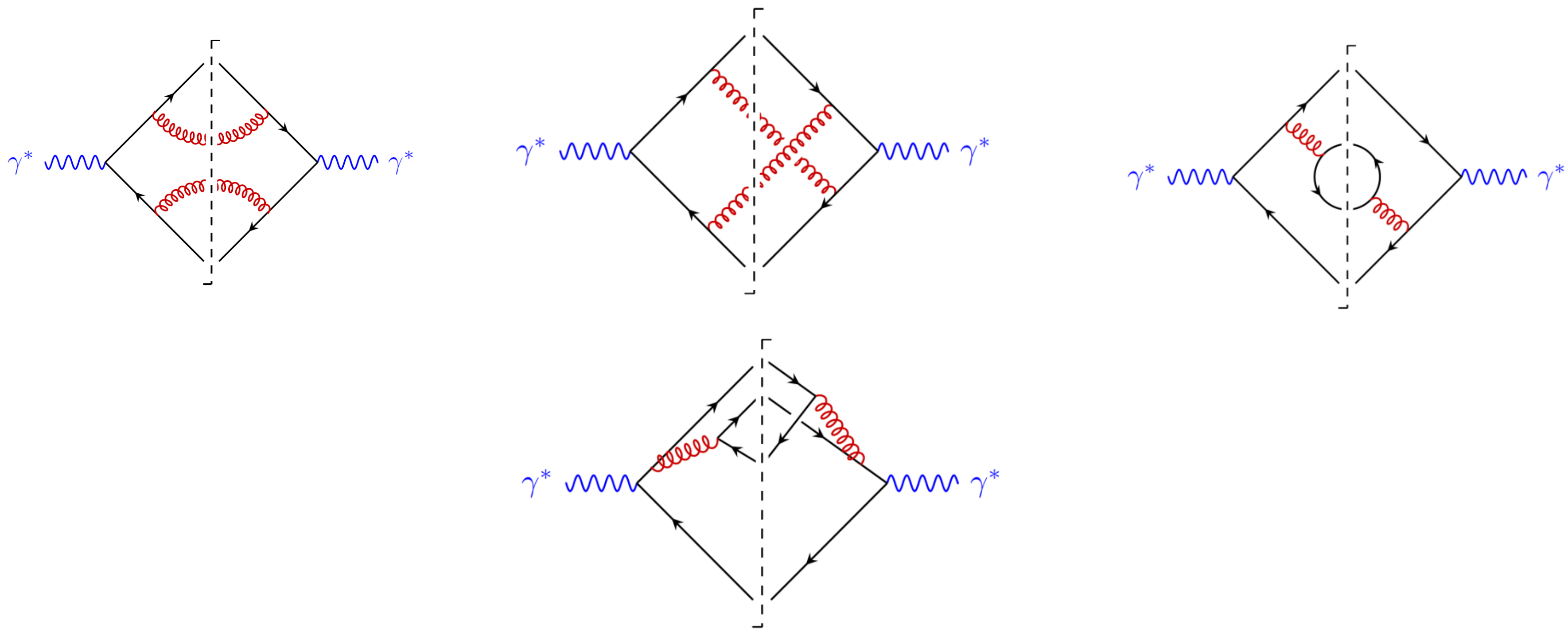
$$A(z) = C_F \frac{3-2z}{4(1-z)z^5}[3z(2-3z) + 2(2z^2-6z+3)\ln(1-z)]$$

NLO result will be expressed in terms of classical polylogarithms:

$$\mathrm{Li}_n(u) = \int_0^u \frac{dt}{t}\mathrm{Li}_{n-1}(t), \qquad \mathrm{Li}_1(t) = -\ln(1-t)$$

Color structure of NLO QCD result

$$B(z) = C_F^2 B_{\mathrm{lc}}(z) + C_F(C_A - 2C_F) B_{\mathrm{nlc}}(z) + C_F N_f T_f B_{N_f}(z)$$



Leading color coefficient fits on one page

$$
\begin{aligned}
B_{\rm lc} = &+ \frac{122400z^7 - 244800z^6 + 157060z^5 - 31000z^4 + 2064z^3 + 72305z^2 - 143577z + 63298}{1440(1-z)z^4} \\
&- \frac{-244800z^9 + 673200z^8 - 667280z^7 + 283140z^6 - 48122z^5 + 2716z^4 - 6201z^3 + 11309z^2 - 9329z + 3007}{720(1-z)z^5} g_1^{(1)} \\
&- \frac{244800z^8 - 550800z^7 + 422480z^6 - 126900z^5 + 13052z^4 - 336z^3 + 17261z^2 - 38295z + 19938}{720(1-z)z^4} g_2^{(1)} \\
&+ \frac{4z^7 + 10z^6 - 17z^5 + 25z^4 - 96z^3 + 296z^2 - 211z + 87}{24(1-z)z^5} g_1^{(2)} \\
&+ \frac{-40800z^8 + 61200z^7 - 28480z^6 + 4040z^5 - 320z^4 - 160z^3 + 1126z^2 - 4726z + 3323}{120z^5} g_2^{(2)} \\
&- \frac{1-11z}{48z^{7/2}} g_3^{(2)} - \frac{120z^6 + 60z^5 + 160z^4 - 2246z^3 + 8812z^2 - 10159z + 4193}{120(1-z)z^5} g_4^{(2)} \\
&- 2\left(85z^4 - 170z^3 + 116z^2 - 31z + 3\right) g_1^{(3)} + \frac{-4z^3 + 18z^2 - 21z + 5}{6(1-z)z^5} g_2^{(3)} + \frac{z^2+1}{12(1-z)} g_3^{(3)} \,,
\end{aligned}
$$

where

$$
g_1^{(1)} = \log(1-z)\,, \qquad g_2^{(1)} = \log(z)\,, \qquad g_1^{(2)} = 2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z)\,,
$$

$$
g_2^{(2)} = \mathrm{Li}_2(1-z) - \mathrm{Li}_2(z)\,, \qquad g_3^{(2)} = -2\,\mathrm{Li}_2\left(-\sqrt{z}\right) + 2\,\mathrm{Li}_2\left(\sqrt{z}\right) + \log\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)\log(z)\,, \qquad g_4^{(2)} = \zeta_2
$$

$$
g_1^{(3)} = -6\left[\mathrm{Li}_3\left(-\frac{z}{1-z}\right) - \zeta_3\right] - \log\left(\frac{z}{1-z}\right)\left(2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z)\right)\,,
$$

$$
g_2^{(3)} = -12\left[\mathrm{Li}_3(z) + \mathrm{Li}_3\left(-\frac{z}{1-z}\right)\right] + 6\,\mathrm{Li}_2(z)\log(1-z) + \log^3(1-z)\,,
$$

$$
g_3^{(3)} = 6\log(1-z)\left(\mathrm{Li}_2(z) - \zeta_2\right) - 12\,\mathrm{Li}_3(z) + \log^3(1-z)\,.
$$

Observations

- Other QCD color coefficients similar in complexity
- See 1801.03219 or https://www.youtube.com/watch?v=WVC1ygsjZNc
- Around both $z = 0$ and $z = 1$, expansion is in integer powers of z (and $\ln z$ or $\ln(1\text{-}z)$)
- Individual real/virtual terms have polylog argument $\frac{i\sqrt{z}}{\sqrt{1-z}}$
- Rational function prefactors have no singularities at spurious locations, but their singularities at $z = 0,\ 1,\ \infty$ are "too strong" and cancel among different terms
- Similar properties for "Higgs EEC" Luo, Shtabovenko, Yang, Zhu, 1903.07277
- N=4 SYM result (next page) is considerably simpler than QCD, but mainly in rational function prefactors, not transcendental functions

EEC for N=4 SYM at NLO

Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 1311.6800

- Correlator is for scalar source instead of electromagnetic current (but the precise source doesn't matter much)

$$F(z;a) = aF_1(z) + a^2\left[(1-z)F_2(z) + F_3(z)\right]$$

where $a = g_{\rm YM}^2 N/(4\pi^2)$

$$F_1(z) = -\ln(1-z)$$

$$F_2(z) = 4\sqrt{z}\left[\mathrm{Li}_2\left(-\sqrt{z}\right) - \mathrm{Li}_2\left(\sqrt{z}\right) + \frac{\ln z}{2}\ln\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right] + (1+z)\left[2\mathrm{Li}_2(z) + \ln^2(1-z)\right] + 2\ln(1-z)\ln\left(\frac{z}{1-z}\right) + z\frac{\pi^2}{3}$$

$$\begin{aligned}F_3(z) = \frac{1}{4}\Big\{&(1-z)(1+2z)\left[\ln^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\ln\left(\frac{1-z}{z}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}+1}\right)\right] - 4(z-4)\mathrm{Li}_3(z)\\ &+ 6(3+3z-4z^2)\mathrm{Li}_3\left(\frac{z}{z-1}\right) - 2z(1+4z)\zeta_3 + 2\left[2(2z^2-z-2)\ln(1-z) + (3-4z)z\ln z\right]\mathrm{Li}_2(z)\\ &+ \frac{1}{3}\ln^2(1-z)\left[4(3z^2-2z-1)\ln(1-z) + 3(3-4z)z\ln z\right] + \frac{\pi^2}{3}\left[2z^2\ln z - (2z^2+z-2)\ln(1-z)\right]\Big\}\end{aligned}$$

- No uniform or maximal transcendentality principle – except for $\chi \rightarrow \pi$

Belitsky et al. method for N=4 SYM

- Very different from "QCD method", which uses dimensional regularization; divergences cancel between virtual and real
- Exploit conformal invariance of 4-point function with two "energy flow operators"

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle_q = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle 0|O^\dagger(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)O(0)|0\rangle$$

$$\mathcal{E}(\vec{n}) = \int_{-\infty}^{\infty} d\tau \lim_{r\to\infty} r^2 n^i T_{0i}(t=\tau+r, r\vec{n})$$

- Analytically continue from Euclidean to physical region using double Mellin transform
- No infrared divergences at any step!
- Recently pushed to NNLO (semi-analytic): Henn, Sokatchev, Yan and Zhiboedov, 1903.05314

Analytic properties of QCD moments

- With analytic formulae, compute the integrals

$$B_N = \int_0^1 dz\ z^N B(z)$$

numerically to high accuracy, for each color coefficient

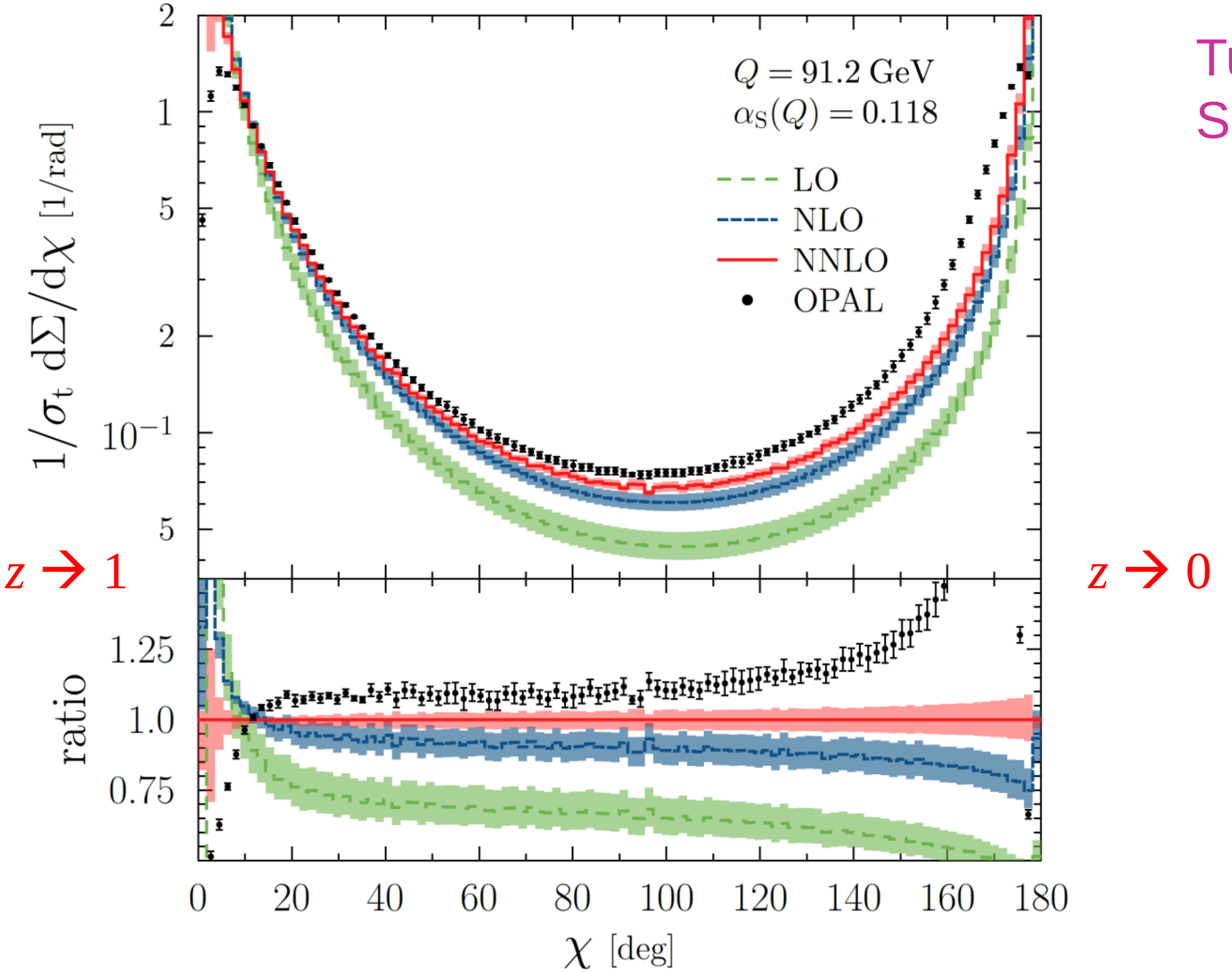
- Using PSLQ, it is always of the form

$$B_N = r_N^{(4)}\zeta(4)\ +\ r_N^{(3)}\zeta(3)\ +\ r_N^{(2)}\zeta(2)\ +\ r_N^{(0)}$$

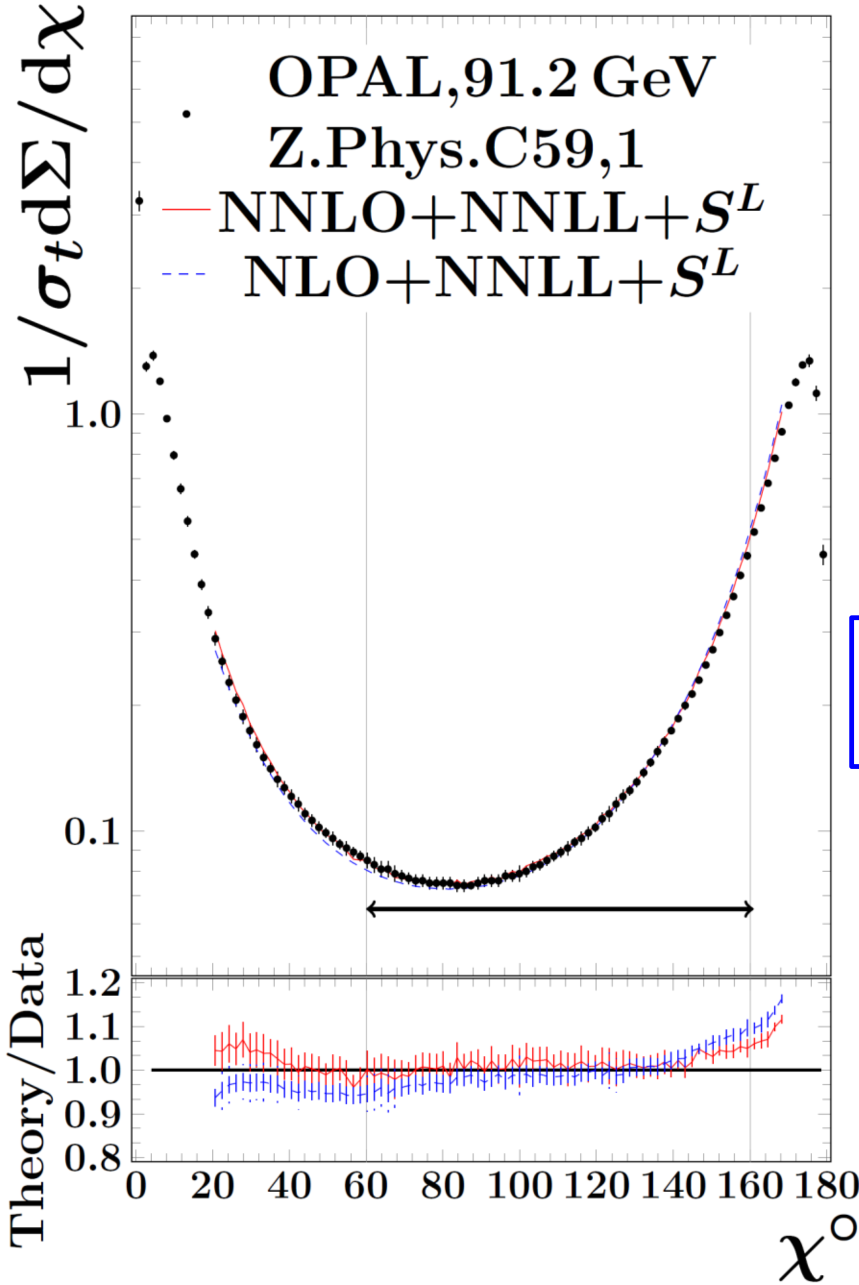
where the $r_N^{(w)}$ are rational numbers.

- E.g. $B_3(C_A) = -\frac{207}{2}\zeta(4) + \frac{14902}{35}\zeta(3) - \frac{553}{450}\zeta(2) - \frac{2369041}{5040}$
- Could they be zeta values at higher loop orders too?
- Expression for general N in terms of $\psi(N)$ functions?

Fixed order QCD vs. Z pole data



Tulipant, Kardos, Somogyi, 1708.04093



To measure strong coupling α_s :
Add NNLL $z \rightarrow 1$ resummation
+ MC estimate of
nonperturbative contributions

Kardos, Kluth, Somogyi, Tulipant, Verbytskyi, 1804.09146

$$\alpha_s(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$$

Competitive measurement of α_s

Still room for theory improvement:
→ NNNLO (approx.?)
+ NNNLL $z \rightarrow 1$ resummation
+ (N?)NLL $z \rightarrow 0$ resummation

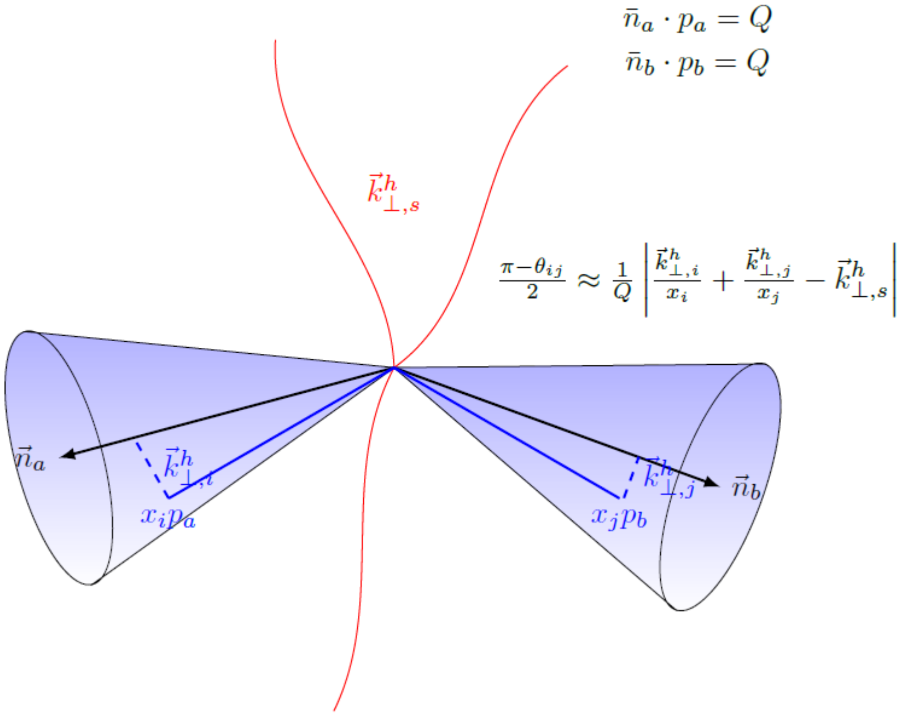
Back-to-back limit, $z \to 1$

$$
\left.\begin{aligned}
B(z) = C_F \Bigg\{ & \frac{1}{1-z}\Bigg[\frac{1}{2}C_F \ln^3(1-z) + \ln^2(1-z)\left(\frac{11C_A}{12} + \frac{9C_F}{4} - \frac{N_f T_f}{3}\right) \\
& + \ln(1-z)\left(C_A\left(\frac{\zeta_2}{2} - \frac{35}{72}\right) + C_F\left(\zeta_2 + \frac{17}{4}\right) + \frac{N_f T_f}{18}\right) \\
& + C_A\left(\frac{11\zeta_2}{4} + \frac{3\zeta_3}{2} - \frac{35}{16}\right) + C_F\left(3\zeta_2 - \zeta_3 + \frac{45}{16}\right) + N_f T_f\left(\frac{3}{4} - \zeta_2\right)\Bigg]
\end{aligned}\right\} \text{leading power}
$$

$$
\left.\begin{aligned}
& + \left(\frac{C_A}{2} + C_F\right)\ln^3(1-z) + \ln^2(1-z)\left(\frac{27C_A}{8} + \frac{13C_F}{2} - \frac{N_f T_f}{2}\right) \\
& + \ln(1-z)\left[C_A\left(22\zeta_2 - \frac{2011}{72}\right) + C_F(47 - 19\zeta_2) + N_f T_f\left(\frac{361}{36} - 4\zeta_2\right)\right] \\
& + C_A\left(\frac{6347\zeta_2}{80} - 21\zeta_2 \ln 2 - \frac{137\zeta_3}{4} - \frac{3305}{72}\right) \\
& + C_F\left(-\frac{1727\zeta_2}{20} + 42\zeta_2 \ln 2 + \frac{121\zeta_3}{2} + \frac{3437}{96}\right) \\
& + N_f T_f\left(-\frac{1747\zeta_2}{120} + 12\zeta_3 + \frac{2099}{144}\right)\Bigg\} + \mathcal{O}(1-z)
\end{aligned}\right\} \text{first subleading power}
$$

- Double log behavior, $\ln^{2L+1}(1-z)/(1-z)$ characteristic of Sudakov suppression from soft/collinear gluon emission. Collins, Soper,…
- Coefficients of leading-power terms agree precisely with NNLL resummation DeFlorian, Grazzini,hep-ph/0407241

$z \rightarrow 1$ (cont.)



Moult, Zhu, 1801.02627

Soft gluons contribute, but only via recoil, by deflecting the hard quark jet

- Factorization theorem recently proved: Relate EEC to back-to-back production of identified hadrons Collins, Soper 1981-1982
- Should allow NNNLL resummation soon

Intra-jet limit, $z \to 0$

$$\left.\begin{aligned} B(z) = C_F \Bigg\{ \frac{1}{z} \Bigg[\ln z \left(-\frac{107 C_A}{120} + \frac{25 C_F}{32} + \frac{53 N_f T_f}{240} \right) + C_A \left(-\frac{25\zeta_2}{12} + \frac{\zeta_3}{2} + \frac{17683}{2700} \right) \\ + C_F \left(\frac{43\zeta_2}{12} - \zeta_3 - \frac{8263}{1728} \right) - \frac{4913 N_f T_f}{3600} \Bigg] \end{aligned}\right\} \text{leading power}$$

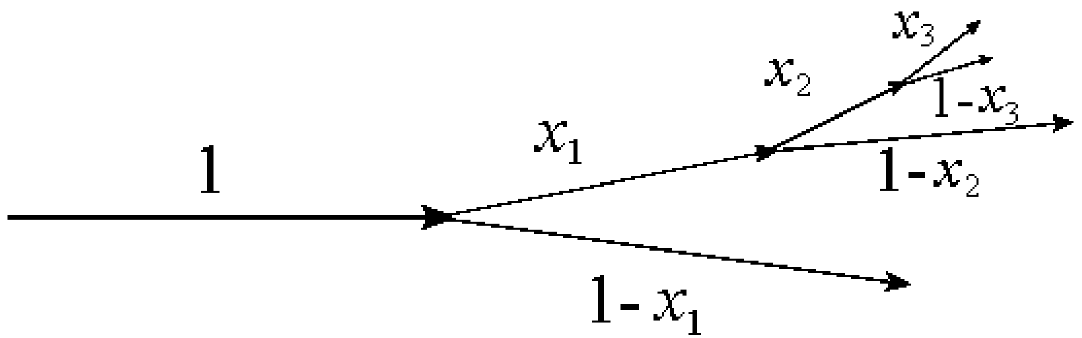
$$\left.\begin{aligned} + \ln z \left[C_A \left(\frac{33\zeta_2}{2} - \frac{703439}{25200} \right) + C_F \left(\frac{42109}{1200} - 21\zeta_2 \right) + N_f T_f \left(\frac{86501}{12600} - 4\zeta_2 \right) \right] \\ + C_A \left(\frac{213\zeta_2}{5} - \frac{101\zeta_3}{2} - \frac{26986007}{5292000} \right) + C_F \left(-\frac{1541\zeta_2}{30} + 65\zeta_3 + \frac{18563}{2700} \right) \\ + N_f T_f \left(-\frac{46\zeta_2}{3} + 12\zeta_3 + \frac{2987627}{330750} \right) \Bigg\} + \mathcal{O}(z) \end{aligned}\right\} \text{first subleading power}$$

- Single log behavior, $\ln^L z/z$ characteristic of pure collinear observable.
- Leading log (LL) resummation first performed in “jet calculus” approach Konishi, Ukawa, Veneziano, Phys.Lett.1978,1979
- Coefficients of leading-power terms agree precisely with LL result Richards, Stirling, Ellis, NPB229, 317, 1983

$z \rightarrow 0$ (cont.)

- Limit dominated by collinear emission. At leading log, only a single moment N=3 of time-like splitting function dominates

Konishi, Ukawa, Veneziano, Richards, Stirling, Ellis, Hofman, Maldacena, 0803.1467



Energy weighting →

$$\int_0^1 dx\, x(1-x)\, P_{ij}(x) \;\rightarrow\; -\int_0^1 dx\, x^2\, P_{ij}(x) \;\equiv\; \gamma_{ij}^{(N=3)}$$

Momentum sum rule controls x^1 term,
→ can drop it.

$$\int_0^1 dx\, x\, P_{ij}(x) \;\equiv\; -\gamma_{ij}^{(N=2)}$$

LL resummed formula

Richards, Stirling, Ellis, NPB229, 317, 1983

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\sqrt{z}Q)}{16\pi\ z} \sum_{i,j=q,g} \Gamma_{ij}^{(0)} \left[\frac{\alpha_s(\sqrt{z}Q)}{\alpha_s(Q)}\right]_{jq}^{-\Gamma^{(0)}/b_0}$$

$$\Gamma_{ij}^{(0)} = \begin{bmatrix} \frac{25}{6}C_F & -\frac{7}{15}N_f \\ -\frac{7}{6}C_F & \frac{14}{5}C_A + \frac{2}{3}N_f \end{bmatrix} \qquad b_0 = \frac{11}{3}C_A - \frac{2}{3}N_f$$

One-loop (LO) N=3 time-like moments

To expand back into fixed order:

$$\frac{\alpha_s(\sqrt{z}Q)}{\alpha_s(Q)} = \left[1 + b_0\frac{\alpha_s(Q)}{4\pi}\ln z\right]^{-1}$$

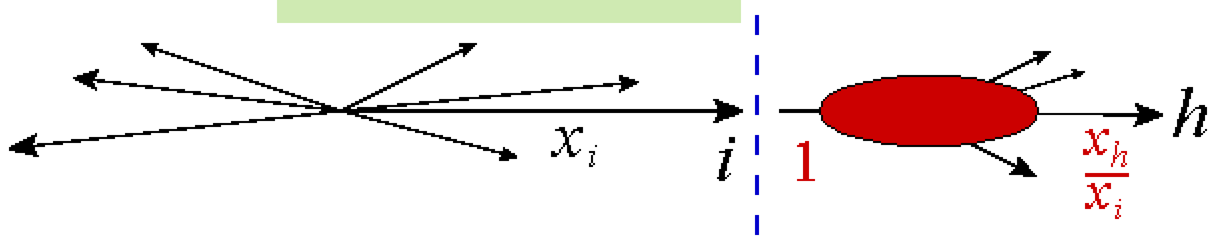
Beyond LL as $z \to 0$

LD, Moult, Zhu, to appear

- Factorize on single parton states, similar to production of identified hadrons h with momentum $p_h = x \times Q/2$

$$\frac{d\sigma(e^+e^- \to h + X)}{dx} = \sum_{i=q,g} \int_0^1 dx_i \, \underbrace{\frac{d\sigma(e^+e^- \to i + X)}{dx_i}}_{\text{perturbative hard function, computed to NNLO + evolution}} \, \underbrace{D_{i\to h}(x/x_i)}_{\text{nonperturbative fragmentation function}}$$

…, Mitov, Moch, Vogt, 2006
Moch, Vogt, 0709.3899,
Almasy, Moch, Vogt, 1107.2263



All orders factorization formula

- Cumulant $\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^z dz'\, \Sigma(z', \ln\frac{Q^2}{\mu^2}, \mu)$

$$\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}^T(\ln\frac{zx^2Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \ln\frac{Q^2}{\mu^2}, \mu)$$

- Reuses hard function $H_i = \frac{d\sigma}{dx_i}$
- Replaces nonperturbative fragmentation function with perturbative jet function J which includes the small angle EEC measurement.
- Dependence of J is on its only physical scale, $zx^2Q^2 = q_T^2$

All orders factorization formula

- Cumulant $\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^z dz' \, \Sigma(z', \ln\frac{Q^2}{\mu^2}, \mu)$

$$\boxed{\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx \, x^2 \vec{J}^T(\ln\frac{zx^2Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \ln\frac{Q^2}{\mu^2}, \mu)}$$

- Computed J directly to O(α_s) so far
  → NLL accuracy
- Reproduces coefficient of $\alpha_s^2 (\ln z)^0/z$
  in fixed order NLO result for both e^+e^- and Higgs

Evolution of jet function

- Evolution of hard function involves time-like splitting kernel, $P_T(y, \mu)$.
- Ω is RGE invariant, i.e. independent of μ
- Leads to evolution equation for J:

$$\frac{d\vec{J}^T(\ln\frac{zQ^2}{\mu^2}, \mu)}{d\ln\mu^2} = \int_0^1 dy\, y^2 \vec{J}^T(\ln\frac{zy^2Q^2}{\mu^2}, \mu) \cdot P_T(y, \mu)$$

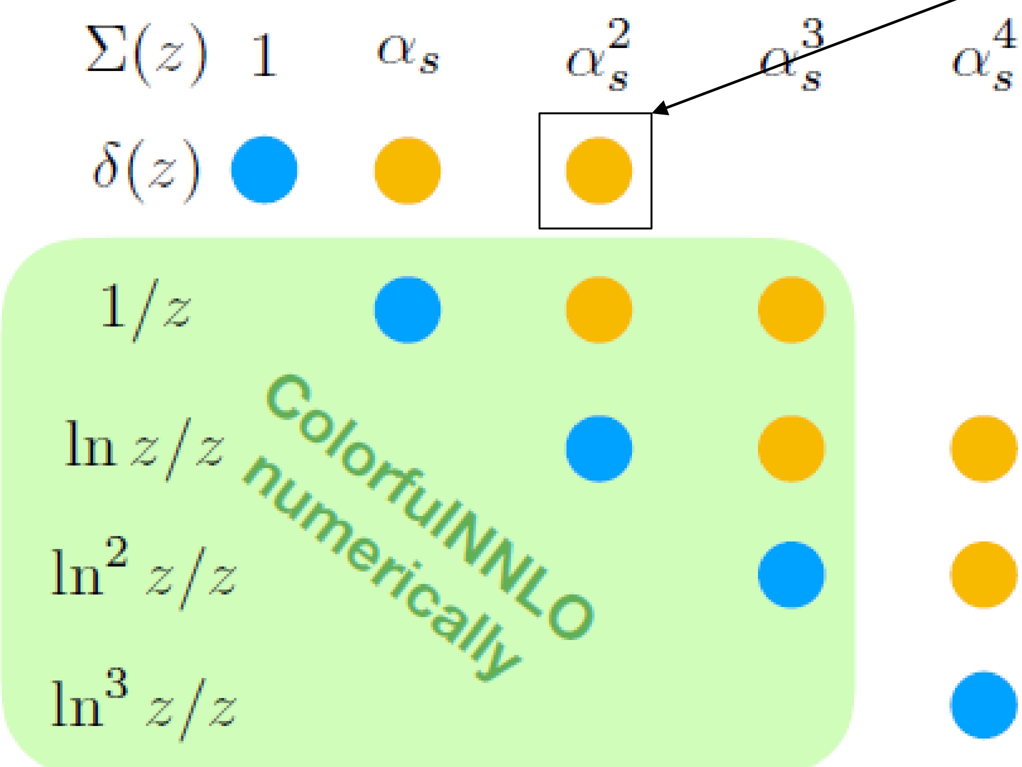
- LL evolution only uses $N=3$ time-like moments (y^2), but beyond LL, need "nearby" moments.

Counting the order

LL Konishi, Ukawa, Veneziano, 1979

NLL + NNLL Dixon, Moult, HXZ, 2019, This talk

Get this indirectly



- To get to NNLL require:
- NNLO splitting kernel
  Moch, Vermaseren, Vogt
- NNLO hard function
  Mitov, Moch, 2006;
  Almasy, Moch, Vogt, 2011
- NNLO jet function

Very challenging!

Use "unitarity" to get $\alpha_s^2\,\delta(z)$

- Get $\alpha_s^2\,\delta(1\text{-}z)$ in course of resumming $z \rightarrow 1$
- Know α_s^2 distribution for $0 < z < 1$, so we can integrate it over this range, up to the delta functions.
- Total cross section

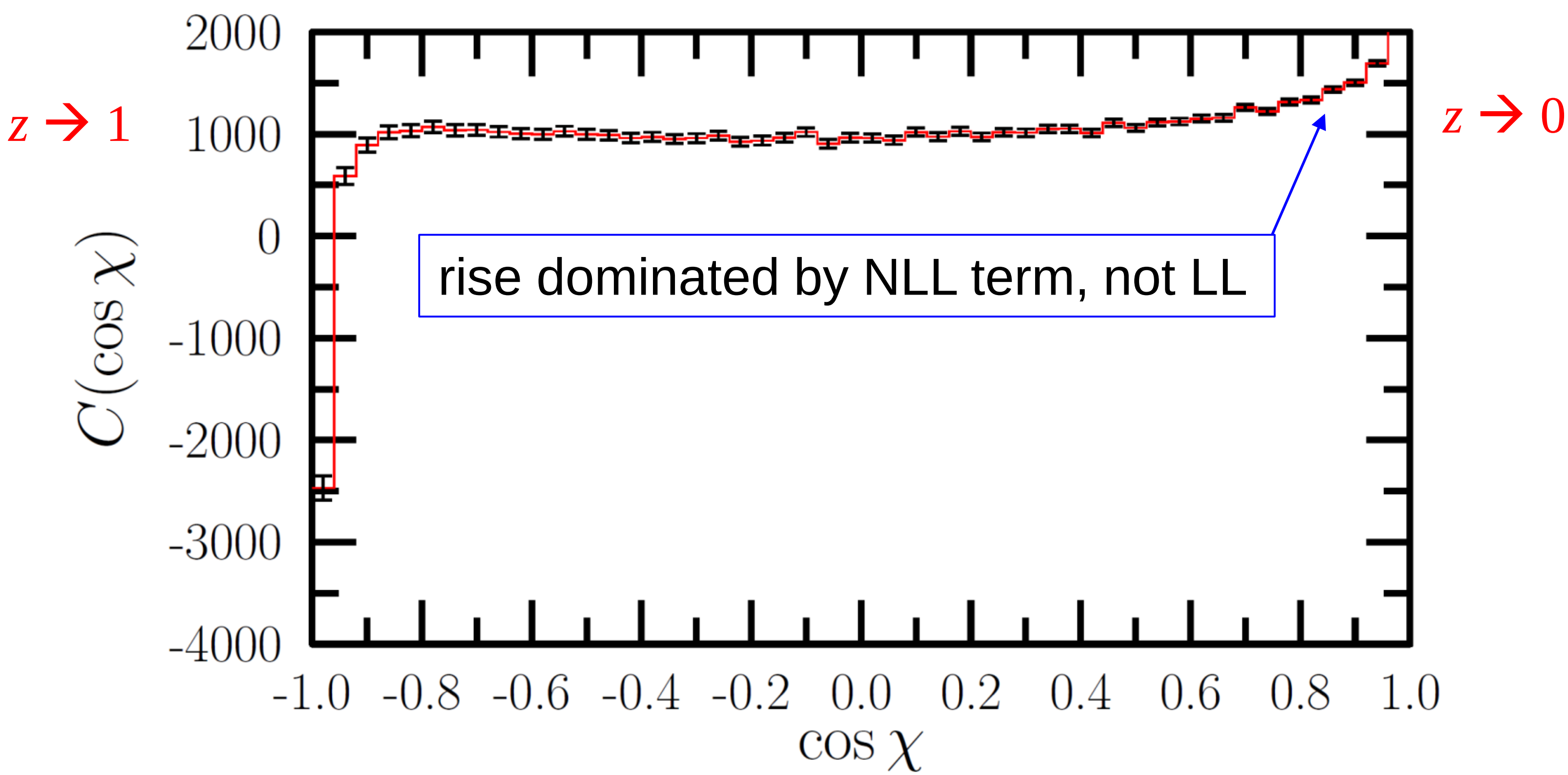
$$\sigma = \int_0^1 dz \frac{d\sigma}{dz}$$

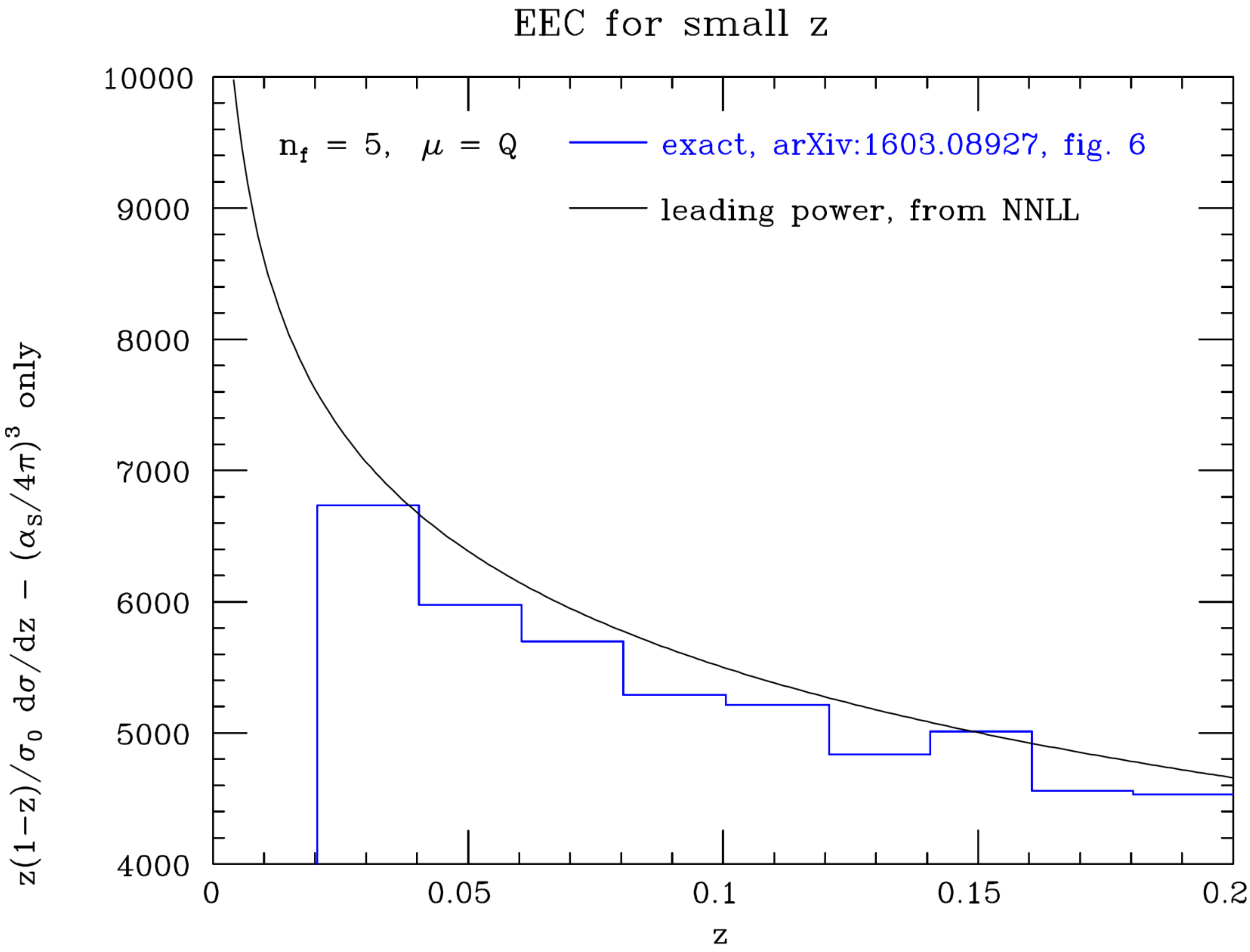
also known, for e^+e^- and Higgs, to very high order, e.g. Herzog, Ruijl, Ueda, Vermaseren, Vogt, 1707.01044

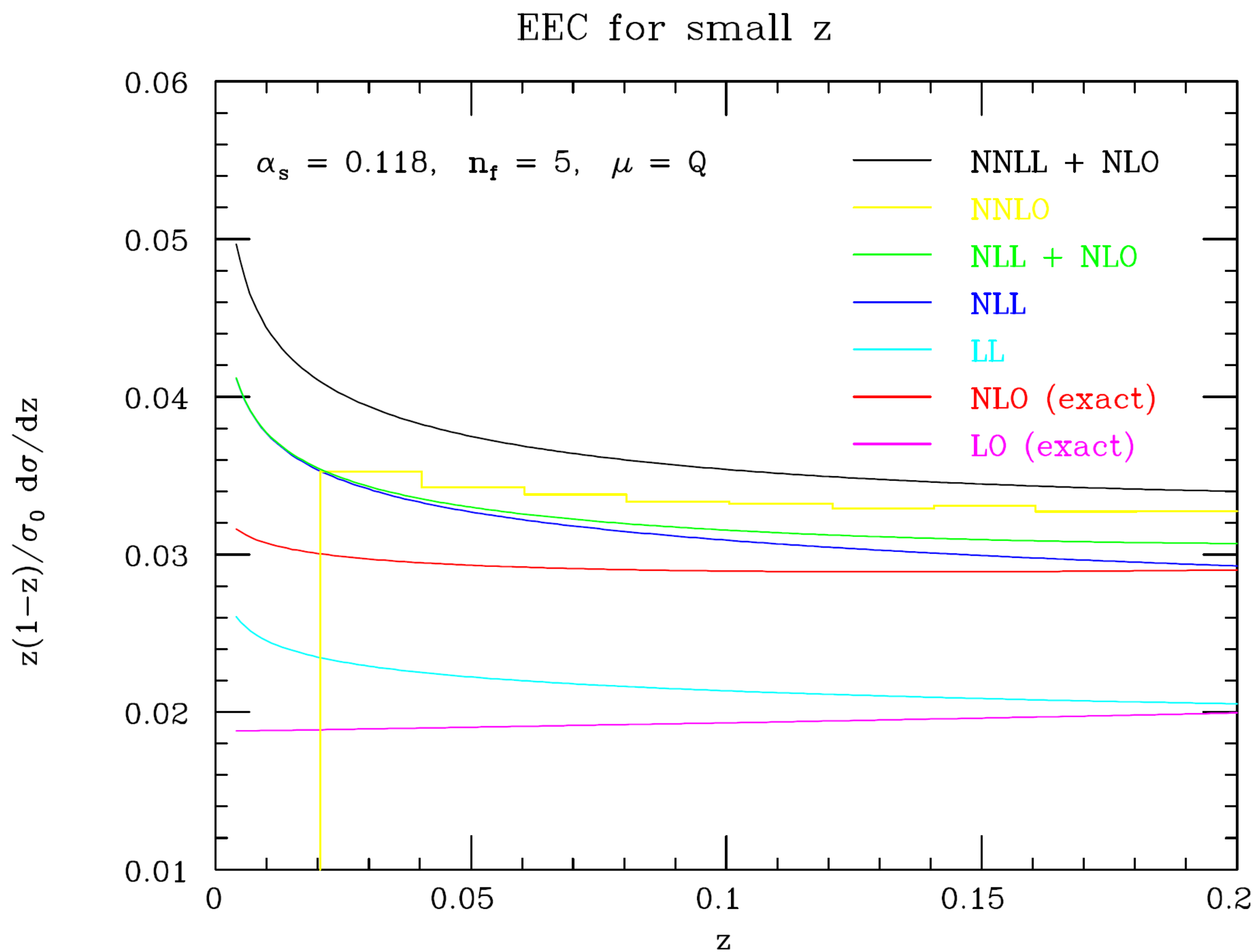
- Use the two $\delta(z)$ coefficients to fix 2-loop J_q, J_g

NNLO QCD α_s^3 coefficient

Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, 1603.08927




EEC for small z
$n_f = 5, \ \mu = Q$
exact, arXiv:1603.08927, fig. 6
leading power, from NNLL
10000
9000
8000
7000
6000
5000
4000
0
0.05
0.1
0.15
0.2
z
$z(1-z)/\sigma_0 \, d\sigma/dz - (\alpha_S/4\pi)^3$ only


EEC for small z
α_s = 0.118, n_f = 5, μ = Q
NNLL + NLO
NNLO
NLL + NLO
NLL
LL
NLO (exact)
LO (exact)
z(1−z)/σ_0 dσ/dz
z
0.06
0.05
0.04
0.03
0.02
0.01
0
0.05
0.1
0.15
0.2

Conclusions

- Analytical results possible at NLO in QCD for at least one event shape in e^+e^- annihilation, the EEC
- Transcendental structure no worse than for N=4 SYM, but rational functions considerably more complicated
- Limiting values useful for checking soft-gluon resummation for $z \rightarrow 1$ – also beyond leading power when available
  Moult, Stewart, Vita, Zhu, 1804.04665
- Also very useful in developing formalism for collinear resummation, $z \rightarrow 0$, now to NNLL   LD, Moult, Zhu, to appear
- May eventually lead to more precise value of α_s, as well as more precise jet substructure understanding at LHC

Extra Slides

Reverse unitarity

Anastasiou, Melnikov, hep-ph/0207004;
Anastasiou, LD, Melnikov, Petriello, hep-ph/0312266

- Phase space integral over final-state partons is like a loop integral with $\delta(p_i^2)$ factor for every propagator crossing the cut, and with one extra delta function, which can be turned into a fake propagator:

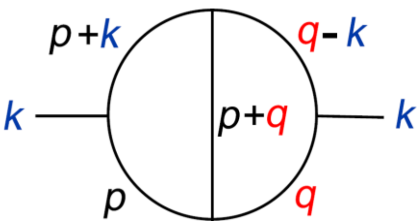
$$\delta[\mathcal{M}_{ij}(\chi)] = \frac{1}{2\pi i}\Big[\frac{1}{\mathcal{M}_{ij}(\chi) - i\varepsilon} - \frac{1}{\mathcal{M}_{ij}(\chi) + i\varepsilon}\Big]$$

where
$$\begin{aligned}\mathcal{M}_{ij}(\chi) &= (p_i\cdot Q\, p_j\cdot Q)(\vec{n}_i\cdot\vec{n}_j - \cos\chi) \\ &= (p_i\cdot Q\, p_j\cdot Q)(1-\cos\chi) - p_i\cdot p_j\end{aligned}$$

- Nonlinear in parton momenta p_i , p_j
- Sum over i,j

Integration by parts (IBP)

- Multi-loop integration technology



Chetyrkin, Tkachov (1981)

$$0 = \int d^D p\, d^D q \ldots \frac{\partial}{\partial q^\mu} \frac{k^\mu}{p^2 q^2 (p+q)^2 \ldots}$$

- Reduces problem to system of linear equations, initially solved recursively by MINCER, now by Laporta algorithm, in terms of "master integrals"

Gorishnii, Larin, Surguladze, Tkachov (1989)

Laporta, hep-ph/0102033

No-scale problem $\Longrightarrow$
like total hadronic cross section
maximal analytic simplicity:
pure numbers, Riemann zeta values

$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}$$

$$\begin{aligned}\frac{R_{e^+e^-}}{R^{(0)}} = {} & 1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-11\zeta(3) + \frac{365}{24} + n_f\left(\frac{2}{3}\zeta(3) - \frac{11}{12}\right)\right] \\ & + \left(\frac{\alpha_s}{\pi}\right)^3 \left[\frac{275}{6}\zeta(5) - \frac{1103}{4}\zeta(3) - \frac{121}{8}\zeta(2) + \frac{87029}{288}\right. \\ & + n_f\left(-\frac{25}{9}\zeta(5) + \frac{262}{9}\zeta(3) + \frac{11}{6}\zeta(2) - \frac{7847}{216}\right) \\ & \left. + n_f^2\left(-\frac{19}{27}\zeta(3) - \frac{1}{18}\zeta(2) + \frac{151}{162}\right)\right]\end{aligned}$$

EEC is "next-to-simplest case"

QCD: NLO program EVENT2 validated

M. Seymour