LABORATORIUM FÜR HOCHENERGIEPHYSIK



Sterile Neutrinos

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Standard Model of particle physics

 $\begin{aligned} \mathcal{I} &= -\frac{1}{4} F_{AV} F^{AV} \\ &+ i F \mathcal{D} \mathcal{Y} + h.c. \end{aligned}$ + X: Yis Xs\$ the $+ \left| \mathcal{D}_{g} \right|^{2} - \sqrt{g}$

 $-\frac{1}{2}\partial_{\nu}g^{\mu}_{\mu}\partial_{\nu}g^{\mu}_{\mu} - g_{s}f^{abc}\partial_{\mu}g^{b}_{\nu}g^{b}_{\nu}g^{c}_{\nu} - \frac{1}{4}g^{2}_{s}f^{abc}f^{adc}g^{b}_{\nu}g^{c}_{\nu}g^{d}_{\rho}g^{c}_{\nu} +$ $\frac{1}{2}ig_s^2(\bar{q}_s^a\gamma^a q_s^a)g_a^a + \bar{G}^a\bar{\partial}^2G^a + g_sf^{abc}\partial_a\bar{G}^aG^bg_a^c - \partial_bW_a^+\partial_bW_a^- M^2 W^+_\mu W^-_\mu - \frac{1}{2} \partial_\mu Z^0_\mu \partial_\mu Z^0_\mu - \frac{1}{3\pi} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2} \partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2} \partial_\mu H \partial_\mu H \frac{1}{5}m_b^2H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac{1}{5}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{3c^2}M\phi^0\phi^0 - \beta_b[\frac{2M^2}{c^2} +$ $\frac{2M}{2}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) + \frac{2M^4}{4}\phi_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\mu W_{\nu}^{+}W_{\mu}^{-}) - Z_{\nu}^{0}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}\partial_{\nu}W_{\mu}^{+}) + Z_{\mu}^{0}(W_{\nu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}\partial_{\mu}W_{\mu}^{-}) + Z_{\mu}^{0}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-}) + Z_{\mu}^{0}(W_{\mu}^{+}\partial_{\mu}W_{\mu}^{-}) + Z_{\mu}^{0}$ $W_{\nu}^{-}\partial_{\nu}W_{\mu}^{+})] - igs_{w}[\partial_{\nu}A_{\mu}(W_{\mu}^{+}W_{\nu}^{-} - W_{\nu}^{+}W_{\mu}^{-}) - A_{\nu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-})]$ $W^{-}_{\mu}\partial_{\nu}W^{+}_{\mu}) + A_{\mu}(W^{+}_{\nu}\partial_{\nu}W^{-}_{\mu} - W^{-}_{\nu}\partial_{\nu}W^{+}_{\mu})] - \frac{1}{3}g^{2}W^{+}_{\mu}W^{-}_{\mu}W^{+}_{\nu}W^{-}_{\nu} +$ $\frac{1}{2}g^2W^+_{\mu}W^-_{\nu}W^+_{\mu}W^-_{\nu} + g^2\epsilon^2_{\mu\nu}(Z^0_{\mu}W^+_{\mu}Z^0_{\nu}W^-_{\nu} - Z^0_{\mu}Z^0_{\mu}W^+_{\nu}W^-_{\nu}) +$ $g^{2} \delta_{w}^{2} (A_{\mu} W_{\mu}^{+} A_{\nu} W_{\nu}^{-} - A_{\mu} A_{\mu} W_{\nu}^{+} W_{\mu}^{-}) + g^{2} \delta_{w} c_{w} [A_{\mu} Z_{\nu}^{0} (W_{\mu}^{+} W_{\nu}^{-} W_{\nu}^{\dagger}W_{\mu}^{-}) - 2A_{\mu}Z_{\mu}^{0}W_{\nu}^{\dagger}W_{\nu}^{-}] - g\alpha[H^{3} + H\phi^{0}\phi^{0} + 2H\phi^{\dagger}\phi^{-}] \frac{1}{z}g^{2}\alpha_{h}[H^{4}+(\phi^{0})^{4}+4(\phi^{+}\phi^{-})^{2}+4(\phi^{0})^{2}\phi^{+}\phi^{-}+4H^{2}\phi^{+}\phi^{-}+2(\phi^{0})^{2}H^{2}]$ $gMW^+_{\mu}W^-_{\mu}H - \frac{1}{2}g^{M}_{cl}Z^0_{\mu}Z^0_{\mu}H - \frac{1}{2}ig[W^+_{\mu}(\phi^0\partial_{\mu}\phi^- - \phi^-\partial_{\mu}\phi^0) W^{-}_{\mu}(\phi^{0}\partial_{\mu}\phi^{+}-\phi^{+}\partial_{\mu}\phi^{0})]+\frac{1}{2}g[W^{+}_{\mu}(H\partial_{\mu}\phi^{-}-\phi^{-}\partial_{\mu}H)-W^{-}_{\mu}(H\partial_{\mu}\phi^{+}-\phi^{-}\partial_{\mu}H)]$ $\phi^{+}\partial_{\mu}H)] + \frac{1}{2}g \frac{1}{c_{\alpha}}(Z^{0}_{\mu}(H\partial_{\mu}\phi^{0} - \phi^{0}\partial_{\mu}H) - ig \frac{r_{0}^{2}}{c_{\alpha}}MZ^{0}_{\mu}(W^{+}_{\mu}\phi^{-} - W^{-}_{\mu}\phi^{+}) +$ $igs_{w}MA_{\mu}(W^{+}_{\mu}\phi^{-} - W^{-}_{\mu}\phi^{+}) - ig\frac{1-2a_{w}^{2}}{2c_{\mu}}Z^{0}_{\mu}(\phi^{+}\partial_{\mu}\phi^{-} - \phi^{-}\partial_{\mu}\phi^{+}) +$ $igs_wA_a(\phi^+\partial_u\phi^- - \phi^-\partial_u\phi^+) - \frac{1}{4}g^2W_a^+W_a^-[H^2 + (\phi^9)^2 + 2\phi^+\phi^-] - \frac{1}{4}g^2W_a^+W_a^-[H^2 + (\phi^9)^2 + 2\phi^+\phi^-]$ $\frac{1}{2}g^{2}\frac{1}{d^{2}}Z_{\mu}^{0}Z_{\mu}^{0}[H^{2} + (\phi^{0})^{2} + 2(2\delta_{\nu}^{2} - 1)^{2}\phi^{+}\phi^{-}] - \frac{1}{2}g^{2}\frac{\delta_{\mu}^{2}}{\delta_{\nu}}Z_{\mu}^{0}\phi^{0}(W_{\mu}^{+}\phi^{-} +$ $W^{-}_{\mu}\phi^{+}) - \frac{1}{2}ig^{2}\frac{d_{\mu}}{d_{\nu}}Z^{0}_{\mu}H(W^{+}_{\mu}\phi^{-} - W^{-}_{\mu}\phi^{+}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{0}(W^{+}_{\mu}\phi^{-} +$
$$\begin{split} W^{-}_{\mu}\phi^{+}) + \frac{1}{2}ig^{2}s_{w}\dot{A}_{\mu}H(W^{+}_{\mu}\phi^{-} - W^{-}_{\mu}\phi^{+}) - g^{2}s_{w}^{2}(2v_{w}^{2} - 1)Z^{0}_{\mu}A_{\mu}\phi^{+}\phi^{-} - g^{1}s_{w}^{2}A_{\mu}A_{\mu}\phi^{+}\phi^{-} - e^{\lambda}(\gamma\partial + m_{k}^{\lambda})e^{\lambda} - D^{\lambda}\gamma\partial\nu^{\lambda} - \bar{u}_{j}^{\lambda}(\gamma\partial + m_{k}^{\lambda})u_{j}^{\lambda} - \bar{d}_{j}^{\lambda}(\gamma\partial + m_{k}$$
 $m_{2}^{\lambda}d_{j}^{\lambda} + igs_{\psi}A_{\mu}[-(\bar{e}^{\lambda}\gamma e^{\lambda}) + \frac{2}{3}(\bar{u}_{i}^{\lambda}\gamma u_{i}^{\lambda}) - \frac{1}{3}(\bar{d}_{i}^{\lambda}\gamma d_{j}^{\lambda})] + \frac{ig}{4\pi}Z_{\mu}^{0}[(D^{\lambda}\gamma^{\mu}(1 + igs_{\psi}A_{\mu}) + igs_{\psi}A_{\mu}] + \frac{ig}{4\pi}Z_{\mu}^{0}[(D^{\lambda}\gamma^{\mu}(1 + igs_{\psi}A_{\mu}) + igs_{\psi}A_{\mu}]] + \frac{ig}{4\pi}Z_{\mu}^{0}[(D^{\lambda}\gamma^{\mu}(1 + igs_{\psi}A_{\mu})] + \frac{ig}{4\pi}Z_{\mu}^{0}[(D^{\lambda}\gamma^{\mu}(1 + igs_{\psi}A_{\mu})] + \frac{ig}{4\pi}Z_{\mu}^{0}[(D^{\lambda}\gamma^{\mu}(1 + igs_{\psi}A_{\mu})]] + \frac{ig}{4\pi}Z_{\mu}^{0}[(D^{\lambda}\gamma^{\mu}(1 + igs_{\psi}A_{\mu})] + \frac{ig}{4\pi}Z$ $\gamma^{5}(\nu^{\lambda}) + (\bar{e}^{\lambda}\gamma^{\mu}(4s_{w}^{2} - 1 - \gamma^{5})e^{\lambda}) + (\bar{u}_{j}^{\lambda}\gamma^{\mu}(\frac{4}{3}s_{w}^{2} - 1 - \gamma^{5})u_{j}^{\lambda}) +$ $(d_j^{\lambda}\gamma^{\mu}(1-\frac{8}{3}s_w^2-\gamma^5)d_j^{\lambda})]+\frac{\omega}{2\sqrt{2}}W^+_{\mu}[(\mathcal{P}^{\lambda}\gamma^{\mu}(1+\gamma^5)e^{\lambda})+(u_j^{\lambda}\gamma^{\mu}(1+\gamma^5)e^{\lambda})]$ $\gamma^{\overline{a}}$ $)C_{\lambda\nu}d\overline{j}$ $] + \frac{\partial}{\partial \beta}W^{-}_{\mu}[(\overline{e}^{\lambda}\gamma^{\mu}(1+\gamma^{\overline{b}})\nu^{\lambda}) + (\overline{d}_{j}C^{\dagger}_{\lambda\nu}\gamma^{\mu}(1+\gamma^{\overline{b}})u^{\lambda})] +$ $\frac{ig}{2\sqrt{2}}\frac{m^{\lambda}}{M}\left[-\phi^{+}(\bar{\nu}^{\lambda}(1-\gamma^{5})e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}(1+\gamma^{5})\nu^{\lambda})\right]-\frac{g}{2}\frac{m^{\lambda}}{M}\left[H(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}(1+\gamma^{5})\nu^{\lambda})\right]-\frac{g}{2}\frac{m^{\lambda}}{M}\left[H(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}(1+\gamma^{5})\nu^{\lambda})\right]-\frac{g}{2}\frac{m^{\lambda}}{M}\left[H(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}(1+\gamma^{5})\nu^{\lambda})\right]-\frac{g}{2}\frac{m^{\lambda}}{M}\left[H(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}(1+\gamma^{5})\nu^{\lambda})\right]-\frac{g}{2}\frac{m^{\lambda}}{M}\left[H(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}e^{\lambda})\right]-\frac{g}{2}\frac{m^{\lambda}}{M}\left[H(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}e^{\lambda})+\phi^{-}(\bar{e}^{\lambda}e^{\lambda})\right]$ $i\phi^{\bar{0}}(\bar{e}^{\lambda}\gamma^{\bar{5}}e^{\lambda})] + \frac{ig}{2M\sqrt{2}}\phi^{+}[-m_{b}^{a}(\bar{u}_{j}^{\lambda}C_{\lambda b}(1-\gamma^{\bar{5}})d_{j}^{b}) + m_{b}^{\lambda}(\bar{u}_{j}^{\lambda}C_{\lambda b}(1+$ $\gamma^{5} dt_{j}^{e} + \frac{ig}{2M_{*}S} \phi^{-}[m_{d}^{\lambda}(\tilde{d}_{j}^{\lambda}C_{\lambda W}^{\dagger}(1 + \gamma^{5})u_{j}^{e}) - m_{u}^{e}(\tilde{d}_{j}^{\lambda}C_{\lambda W}^{\dagger}(1 - \gamma^{5})u_{j}^{e}]$ $g \frac{m_h^\lambda}{U} H(\bar{u}_i^\lambda u_i^\lambda) - \frac{g \frac{m_h^\lambda}{U}}{U} H(\bar{d}_i^\lambda d_i^\lambda) + \frac{ig \frac{m_h^\lambda}{U}}{U} \phi^0(\bar{u}_i^\lambda \gamma^5 u_i^\lambda) - \frac{ig \frac{m_h^\lambda}{U}}{U} \phi^0(\bar{d}_i^\lambda \gamma^5 d_i^\lambda) +$ $\bar{X}^{+}(\partial^{2} - M^{2})X^{+} + \bar{X}^{-}(\partial^{2} - M^{2})X^{-} + \bar{X}^{0}(\partial^{2} - \frac{M^{2}}{d^{2}})X^{0} + \bar{Y}\partial^{2}Y +$ $igc_w W^+_u (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + igs_w W^+_u (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) +$ $igc_w W^-_{\mu}(\partial_{\mu} X^- X^0 - \partial_{\mu} X^0 X^+) + igs_w W^-_{\mu}(\partial_{\mu} X^- Y - \partial_{\mu} Y X^+) +$ $igc_w Z^0_\mu(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igs_w A_\mu(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) \tfrac{1}{2}gM[\tilde{X}^+X^+H+\tilde{X}^-X^-H+\tfrac{1}{c_0}\tilde{X}^0X^0H]+\tfrac{1-2c_0^2}{2c_0}igM[\tilde{X}^+X^0\phi^+ \bar{X}^- X^0 \phi^-] + \frac{1}{2m} - igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-]$ $\hat{X}^{0}X^{+}\phi^{-}$] + $\frac{1}{2}igM[\hat{X}^{+}X^{+}\phi^{0} - \hat{X}^{-}X^{-}\phi^{0}]$

Are we done? No !

- Gravity is not included
- Matter-antimatter asymmetry?
- Large tuning needed to obtain the measured low Higgs mass (natural ?)
- Unification of electroweak and strong forces?
- Dark matter and dark energy?
- Origin of neutrino masses?
- Additional neutrino states ?



Many scientists and projects addressing these questions I will discuss neutrinos today, focusing on sterile neutrinos

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Neutrino oscillations





$$P(\mathcal{N}_{a} \to \mathcal{N}_{b}) = \sin^{2} 2\mathcal{Q}_{ij} \cdot \sin^{2} \left(1.27 \, \mathrm{D}m_{ij}^{2} \frac{L}{E} \right)$$

- Three angles θ_{12} , θ_{23} and θ_{13}
- One phase δ_{CP}
- Two mass-squared differences Δm_{21}^2 and Δm_{23}^2



$$\begin{split} P(\bar{\mathbf{v}}_e \to \bar{\mathbf{v}}_e) &\approx 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{13}^2 L}{4E} \right) \\ &- \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{12}^2 L}{4E} \right), \end{split}$$

A global experimental effort

KamLAND, Double Chooz, Daya Bay, RENO, Gösgen, SNO, SK, T2K, MINOS, NOvA, OPERA, ICARUS, IceCube, ANTARES, ...

Parameter	best-fit	3σ	
$\Delta m_{21}^2 \ [10^{-5} \text{ eV}^2]$	7.37	6.93 - 7.96	
$\Delta m^2_{31(23)} \ [10^{-3} \text{ eV}^2]$	2.56(2.54)	2.45 - 2.69 (2.42 - 2.66)	
$in^2 \theta_{12}$	0.297	0.250 - 0.354	_
$ in^2 \theta_{23}, \Delta m^2_{31(32)} > 0 $	0.425	0.381 - 0.615	017
$\sin^2 \theta_{23}, \Delta m^2_{32(31)} < 0$	0.589	0.384 - 0.636	2 J
$\sin^2 \theta_{13}, \Delta m^2_{31(32)} > 0$	0.0215	0.0190 - 0.0240	e D
$\sin^2 \theta_{13}, \Delta m^2_{32(31)} < 0$	0.0216	0.0190 - 0.0242	dat
δ/π	1.38(1.31)	2σ : (1.0 - 1.9)	an (
		$(2\sigma: (0.92-1.88))$	

LSND

The Liquid Scintillator Neutrino Detector at Los Alamos National Lab 1993 -- 1998









4th neutrino mass state

Constraints





Cosmology

ACDM is sensitive to N_v (Large scale structures (BAO) and light nuclei abundance.) Best fit is also consistent with $N_v = 3$ active neutrinos.





• High rates, statistics to measure cross sections

MiniBooNE @Fermilab (2002--)





A.A. Aguilar-Arevalo et al. [MiniBooNE Collaboration], arXiv:1805.12028

More hints, experiments, searches

- Investigations, also with new technologies (liquid argon TPC)
- "Reactor anomaly"
- Source experiments
- Disappearance analyses at long baseline experiments
- Dedicated reactors neutrino experiments

Investigating LSND



(Antineutrino, Pion Decay At Rest DAR)



Also DAEδALUS and IsoDAR

Explore the MiniBooNE excess of events with a high resolution liquid Argon TPC





LArTPC

Anode wire planes

Cathode



















M. Del Tutto

High-Definition 3D imaging detector and calorimeter









Time with respect to the BNB Trigger Time [µs]

Towards the low energy oscillation result















22

CCv_{μ} double differential cross section

arXiv:1905.09694



23



$v_{\mu} \rightarrow v_{e}$?

NuMI DATA: RUN 10811, EVENT 2549. APRIL 9, 2017.

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17 cm

Sterile neutrinos: more hints, experiments, searches

- Source experiments
- "Reactor anomaly"
- Dedicated reactors neutrino experiments
- Disappearance analyses at long baseline experiments



Radioactive source experiments

Electron





Reactor experiments:

Pure $\overline{\nu}_{e}$ source produced by fissions in ²³⁵U, ²³⁸U, ²³⁹Pu, ²⁴¹Pu



$\overline{\nu}_{e}$ disappearance

baseline ~m











At short baselines experiments see fewer neutrinos than expected (red line)



G. Mention, M. Fechner, Th. Lasserre, Th. A. Mueller, D.Lhuillier, M. Cribier, and A. Letourneau, Phys. Rev. D 83, 073006 (2011)



Experiment	Type	Channel	Significance
LSND	DAR	$\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e \text{ CC}$	3.8σ
MiniBooNE	SBL accelerator	$\nu_{\mu} \rightarrow \nu_{e} \text{ CC}$	4.5σ
MiniBooNE	SBL accelerator	$\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e} \text{ CC}$	2.8σ
GALLEX/SAGE	Source - e capture	ν_e disappearance	2.8σ
Reactors	Beta-decay	$\bar{\nu}_e$ disappearance	3.0σ

Taken individually, each anomaly is not significant enough to be convincing....

> most commonly interpreted as hint for one or more new **"sterile**" neutrino.

The reactor anomaly can be explained



Daya Bay (and RENO) results suggest that the neutrino flux from ²³⁵U ~3 sigma below what expected from fission models so far

→ Need experiments at Highly Enriched Uranium reactors (20-90% ²³⁵U), i.e. research reactors to thoroughly test this



Compact reactors Highly enriched ²³⁵U









More reactor sterile neutrino searches





 10^{-2}

Phys.Lett. B787 (2018) 56-63

Ratio Dn / Up

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Reactor Core

Tendon Gallery

10 meter

10

 10^{-3}





Neutrino-4



Phys.Lett. B782 (2018) 13-21

sin²20a

10-1

Sterile Neutrinos and long baseline experiments





$$\nu_{\mu} \to \nu_{\mu} : |U_{\mu4}|^2 = \sin^2 \theta_{24} \cos^2 \theta_{14}$$

Different oscillation signatures in near and far detectors



 $v_e \rightarrow$

 $\Delta m^2_{41} [eV^2]$

 10^{-1}

DaR

 10^{-3}

 $|U_{e4}|^{2}$





 10^{-2}

 $\sin^2 2\theta_{\mu e}$

 10^{-1}

3 + 1

 $|U_{\mu 4}|^2$







Three-detector setup with LArTPCs











- Neutrinos are interesting (known physics BSM)
- Anomalies point to possible additional states (Sterile neutrinos)
- Follow up experiments with new technology underway
- Sterile neutrinos are still viable



- CP violation in the lepton sector
- Sign of Δm^2_{31} , "Mass hierarchy"
- Majorana or Dirac neutrinos
- Absolute neutrino masses



BNB DATA : RUN 5211 EVENT 1225. FEBRUARY 29, 2016

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Takaaki Kajita, Neutrino '98





In den 2020er...





LArTPC development in Bern



ArgonTUBE 2009-2015



- Greinacher circuit: 125 stages, input 4 kV AC
- COMSOL finite element analysis software to optimize the geometry of the field-shaping rings
- Goal: drift field of 1 kV/cm
- Reached 170 kV (0.34 kV/cm)







MicroBooNE in USA (2015 -)



















Overview of KArlsruhe TRItium Neutrino Experiment



$$P(\nu_{\mu} \rightarrow \nu_{\mu}) \sim 1 - \cos^{4}(\theta_{13}) \sin^{2}(2\theta_{23}) \sin^{2}(\Delta m_{31}^{2} L/4E) -\sin^{2}(2\theta_{13}) \sin^{2}(\theta_{23}) \sin^{2}(\Delta m_{31}^{2} L/4E)$$

Interaction regime



Rev. Mod. Phys. 84, 1307–1341 (2012)



Typical neutrino beams



Nucleus





Mixing (rotation) matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

 $|v_e(0)\rangle = cos\theta_{12} \cdot |v_1\rangle + sin\theta_{12} \cdot |v_2\rangle$

Next, introduce time evolution (propagation in space)

$$|v_e(t)\rangle = cos\theta_{12} \cdot e^{-iE_1t}|v_1\rangle + sin\theta_{12} \cdot e^{-iE_2t}|v_2\rangle$$

$$P(n_a \to n_b) = \sin^2 2q_{ij} \cdot \sin^2 \left(1.27 \, \mathrm{D}m_{ij}^2 \frac{L}{E} \right)$$