How strong are the strong interactions?

Rainer Sommer

John von Neumann Institute for Computing, DESY
&
Humboldt Universität zu Berlin

Freiburg by zoom, January 27, 2021

Particle physics - the quest for the fundamental theory

energy

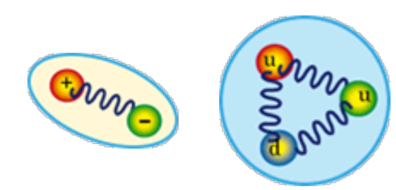
today's frontier

dark matter
matter/anti-matter asy
(strings, extra dimensions, …)

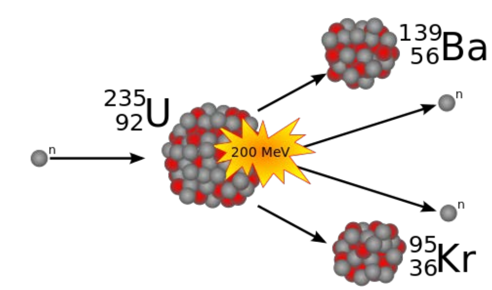
Standard Model

(+ neutrino mass terms)

= strong + weak
+ electromagnetic



our focus



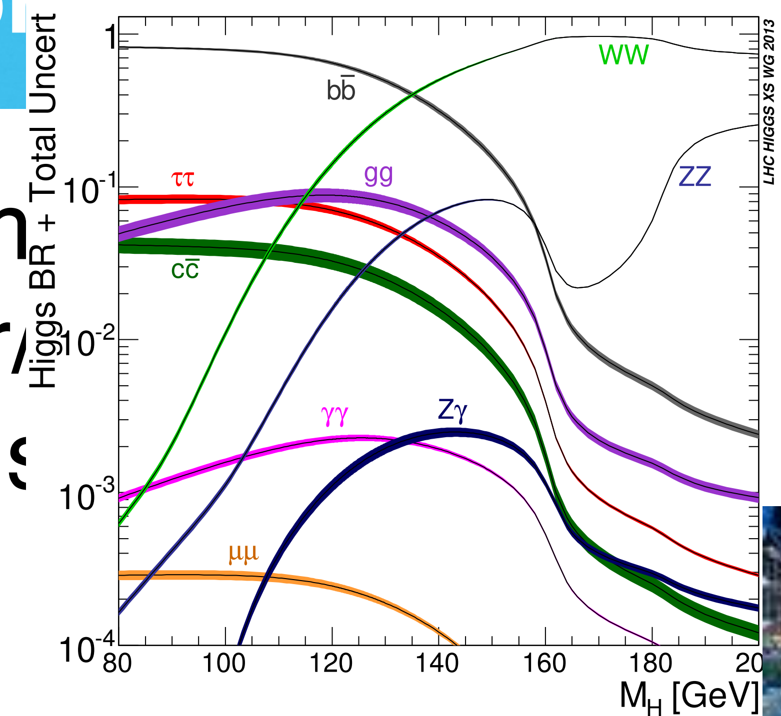
electromagnetic
+ Fermi-theory

Particle physics - the quest for the fundamental theory

today's frontier

energy

dark m
matter/
(strings
…)

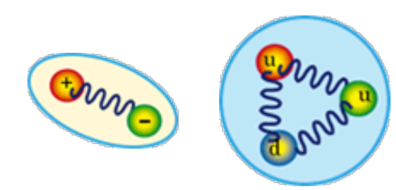


s,

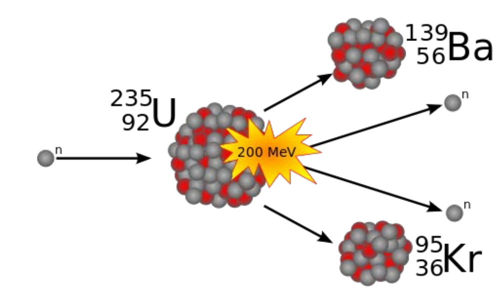
Standard Model

(+ neutrino mass terms)

= strong + weak
+ electromagnetic



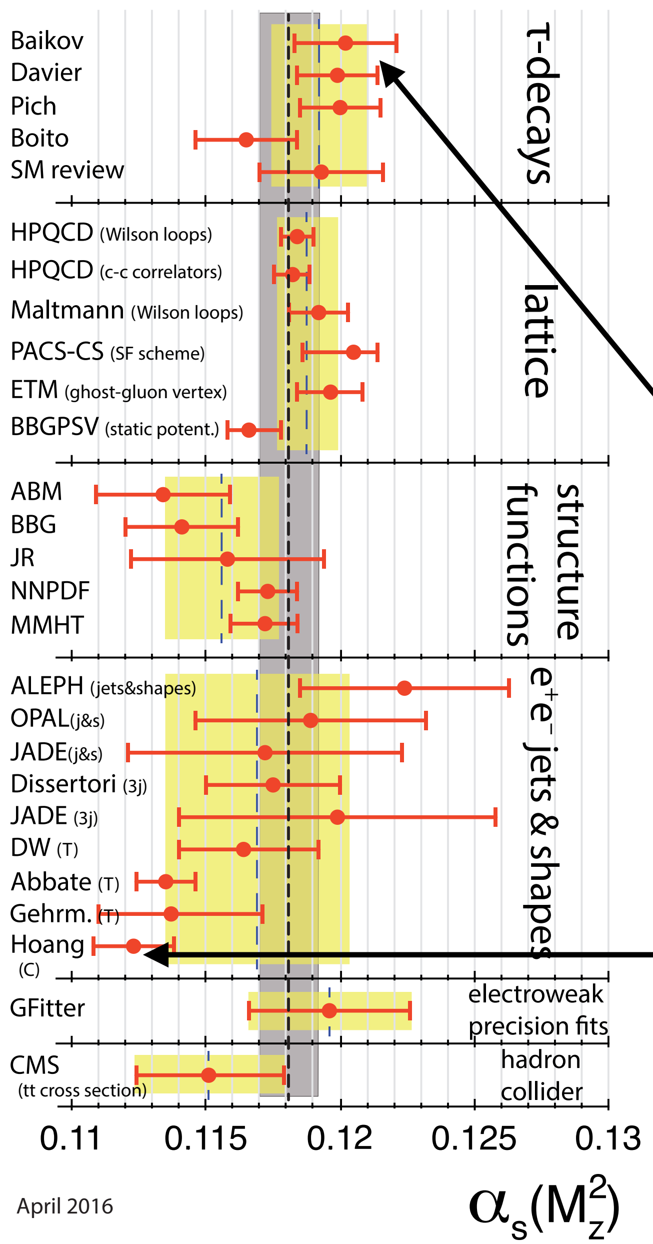
our focus

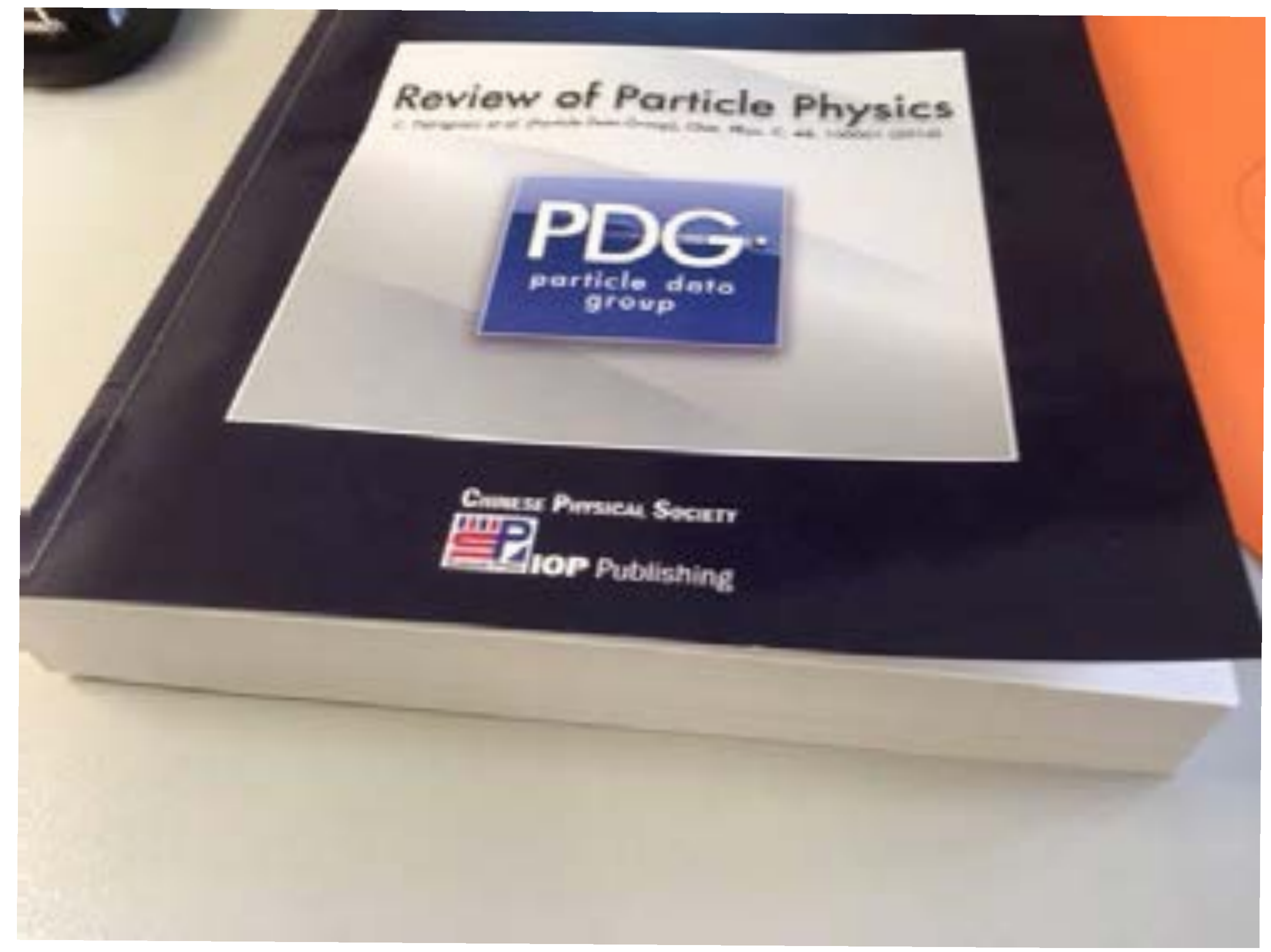


electromagnetic
+ Fermi-theory

QCD and the Particle Data Group review

The strong coupling





How strong are the strong interactions?

QCD

- Theory of strong interactions
- Quantum Field theory with Lagrangian

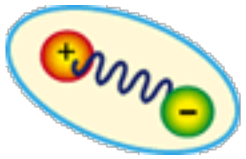
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2{g_0}^2}\,\mathrm{tr}\;F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_\mathrm{f}} \overline{\psi}_f\,\{D + m_{0f}\}\,\psi_f$$

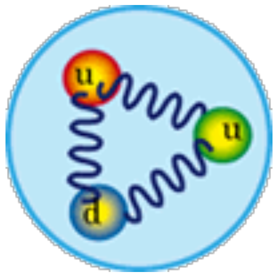
- Fields: gluons and quarks

name	Char	mass in M
up	2/3	5
down	-1/3	10
charm	2/3	1000
strange	-1/3	100
top	2/3	175000
bottom	-1/3	42000

- But particles: hadrons

  p, n, π, K,… confinement!





- Definition of coupling is not straight forward (we do e.g. not want the π-π coupling)

QCD coupling

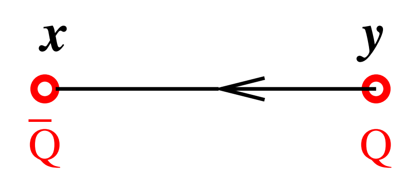
- Theorists:

  take dimensions
  subtract poles in … ← no physics

QCD coupling

Analogous to $$F_{pe}(r) = \alpha_{\rm em}\frac{1}{r^2}$$ $$r = |\mathbf{x}-\mathbf{y}|$$

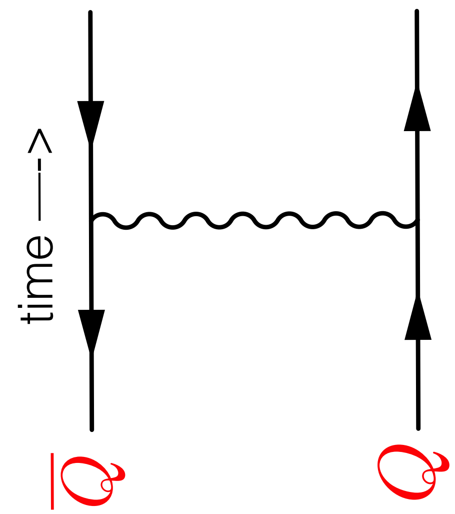
Quark as test charge Q with $m_Q \to \infty$



force in PT: $$F_{Q\bar{Q}}(r) = \alpha_{\overline{\rm MS}}(\mu)\frac{4}{3}\frac{1}{r^2} + \mathrm{O}(\alpha^2_{\overline{\rm MS}})$$

define: $$\boxed{\alpha_{\rm qq}(\mu) \equiv \frac{3r^2}{4}F_{\rm Q\bar{Q}}(r)\,,\quad \mu = \frac{1}{r}}$$

no corrections



.

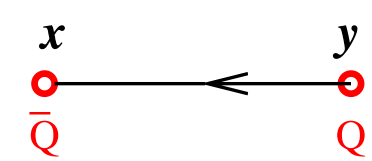
$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1\alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

$$c_1 = \frac{1}{(4\pi)^2}\left\{\frac{35}{3} - 22\gamma_E - (\frac{2}{9} - \frac{4}{3}\gamma_E)N_{\rm f}\right\} = \mathrm{O}(1)$$

[Billoire; Fishler]

QCD coupling

$r = |\mathbf{x} - \mathbf{y}|$



$$\alpha_{\rm qq}(\mu) \equiv \frac{3r^2}{4} F_{Q\bar{Q}}(r)\,, \qquad \mu = \frac{1}{r}$$

then

$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1 \alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

always
(non-perturbatively)
defined
physics!

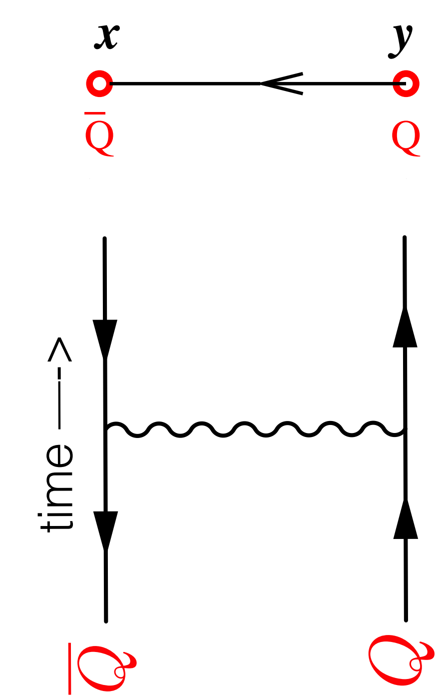
perturbatively defined
by such relations

makes sense for $\alpha \ll 1$

QCD coupling

$$r = |\mathbf{x} - \mathbf{y}|$$

$$\alpha_{\mathrm{qq}}(\mu) \equiv \frac{3r^2}{4} F_{Q\bar{Q}}(r)\,, \quad \mu = \frac{1}{r}$$



then

$$\alpha_{\mathrm{qq}}(\mu) = \alpha_{\overline{\mathrm{MS}}}(\mu) + c_1 \alpha^2_{\overline{\mathrm{MS}}}(\mu) + \ldots$$

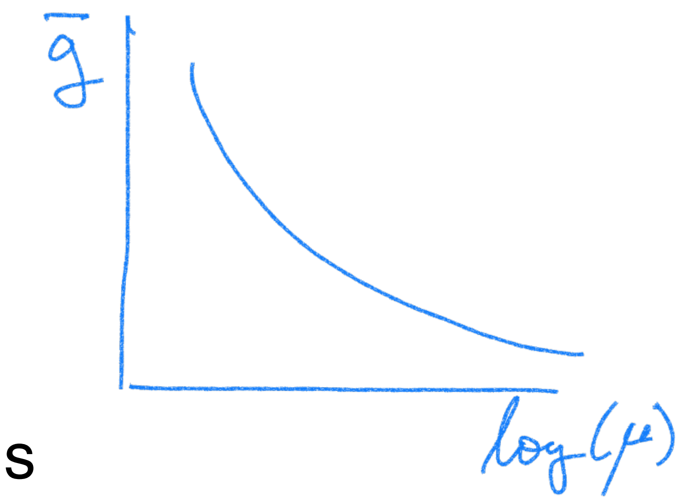
always (non-perturbatively) defined physics!

perturbatively defined by such relations

makes sense for $\alpha \ll 1$

Energy dependence: Asymptotic freedom

$$\mu\frac{\partial}{\partial\mu}\bar{g}_s(\mu) = \beta_s(\bar{g}_s) = -\bar{g}_s^3\,(b_0 + b_1\bar{g}_s^2 + \ldots)$$



> 0, independent of scheme(=definition) s

- Taylor series in $\alpha_s = \bar{g}_s^2/(4\pi)$ is reliable at large energy μ
- Reach large energy, with precision
- Determine α_s in some scheme s
- Use PT —> predictions for high energy processes in terms of perturbative series, e.g. for LHC

A look at phenomenology, e.g. R_{e+e-}

total cross section for $e^+e^- \to$ hadrons at center-of-mass energy Q

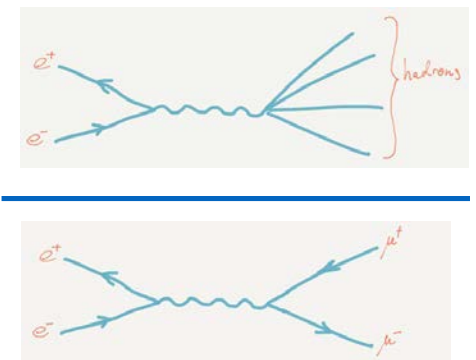
$$R_{e^+e^-}(Q) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\frac{\sigma(e^+e^- \to \text{hadrons}, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} \equiv R(Q) = R_{\mathrm{EW}}(Q)(1 + \delta_{\mathrm{QCD}}(Q))$$

$$\delta_{\mathrm{QCD}}(Q) = \sum_{n=1}^{\infty} c_n \cdot \left(\frac{\alpha_s(Q^2)}{\pi}\right)^n + \mathcal{O}\left(\frac{\Lambda^4}{Q^4}\right)$$

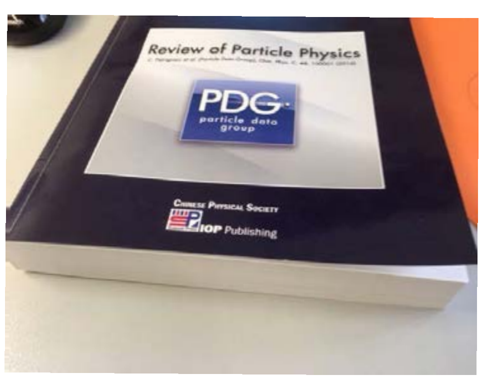
determine $\alpha_s(\mu = Q)$

$R_{e^+e^-}$

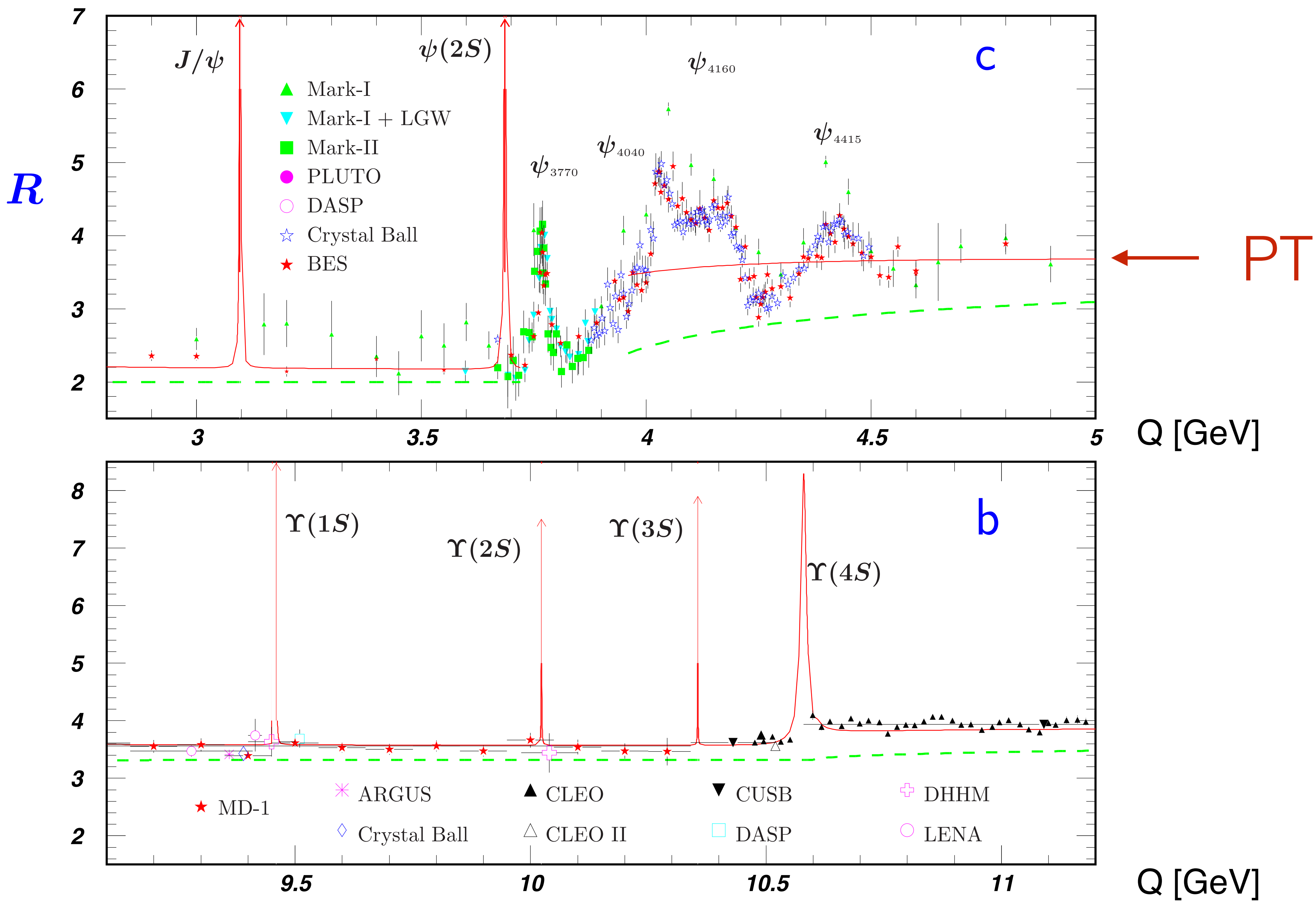


e^+
e^-
hadrons
e^+
e^-
μ^+
μ^-

see

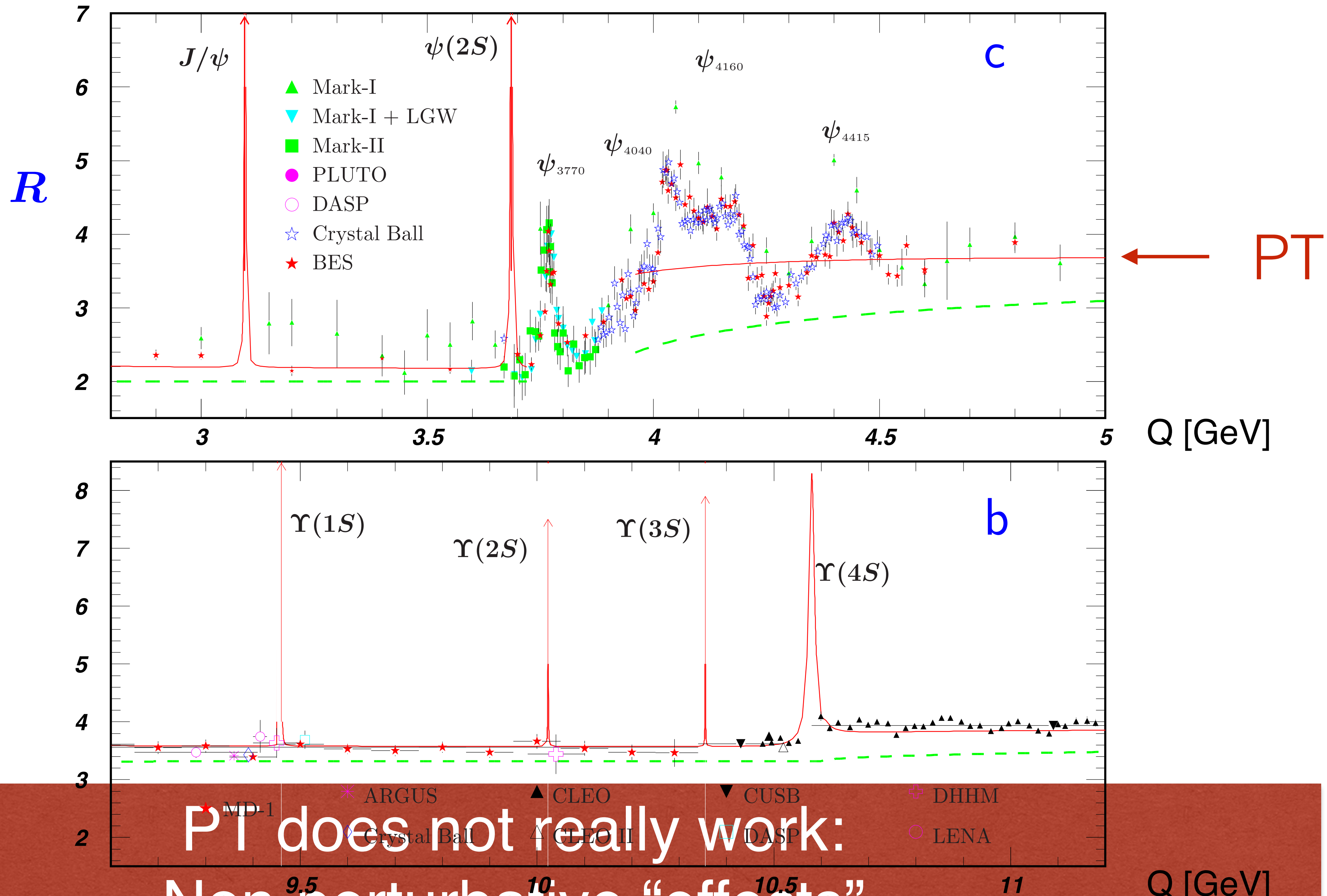


Review of Particle Physics
PDG
particle data group
Chinese Physical Society
IOP Publishing

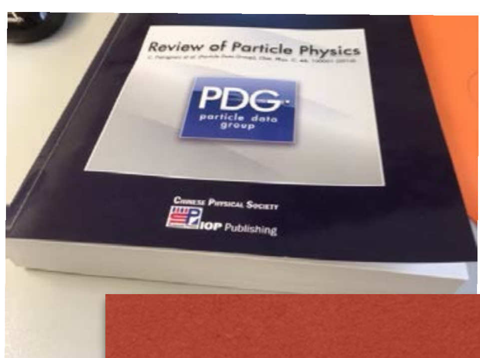


R
J/ψ
$\psi(2S)$
ψ_{4160}
ψ_{4040}
ψ_{3770}
ψ_{4415}
c
Mark-I
Mark-I + LGW
Mark-II
PLUTO
DASP
Crystal Ball
BES
PT
Q [GeV]
$\Upsilon(1S)$
$\Upsilon(2S)$
$\Upsilon(3S)$
$\Upsilon(4S)$
b
MD-1
ARGUS
Crystal Ball
CLEO
CLEO II
CUSB
DASP
DHHM
LENA
Q [GeV]

$R_{e^+e^-}$



see



PT does not really work:

Non-perturbative "effects"

particle (= hadrons) — production

partial solution: go to Euclidean region (smearing, moments)

Determinations of α_s

- high energy experiment + phenomenology is very challenging (as we just saw)

- alternative:
  low energy experiment + “simulation”
  = MC-evaluation of discretized path integral

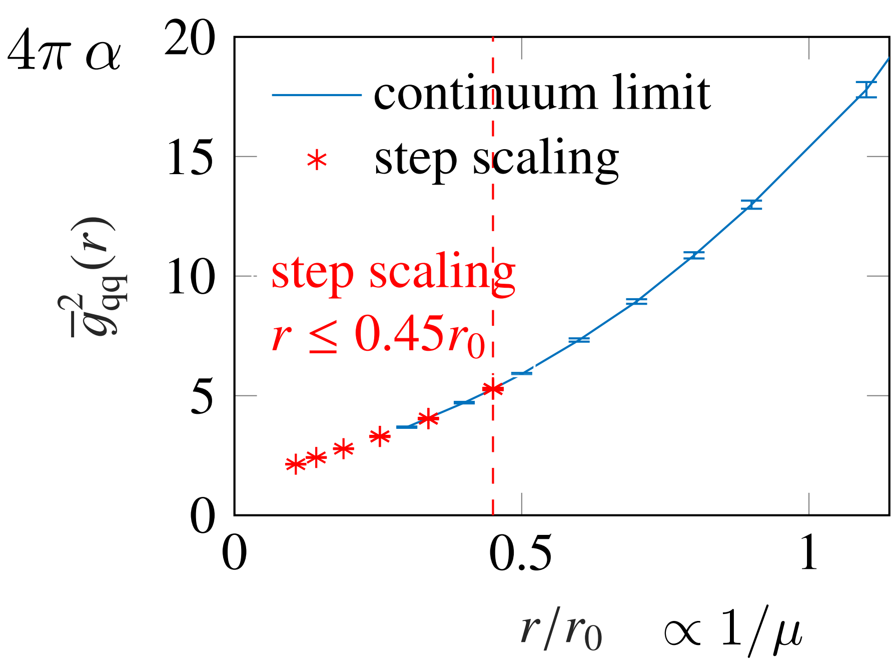
Lattice QCD

hadron masses / properties

↓

parameters of theory

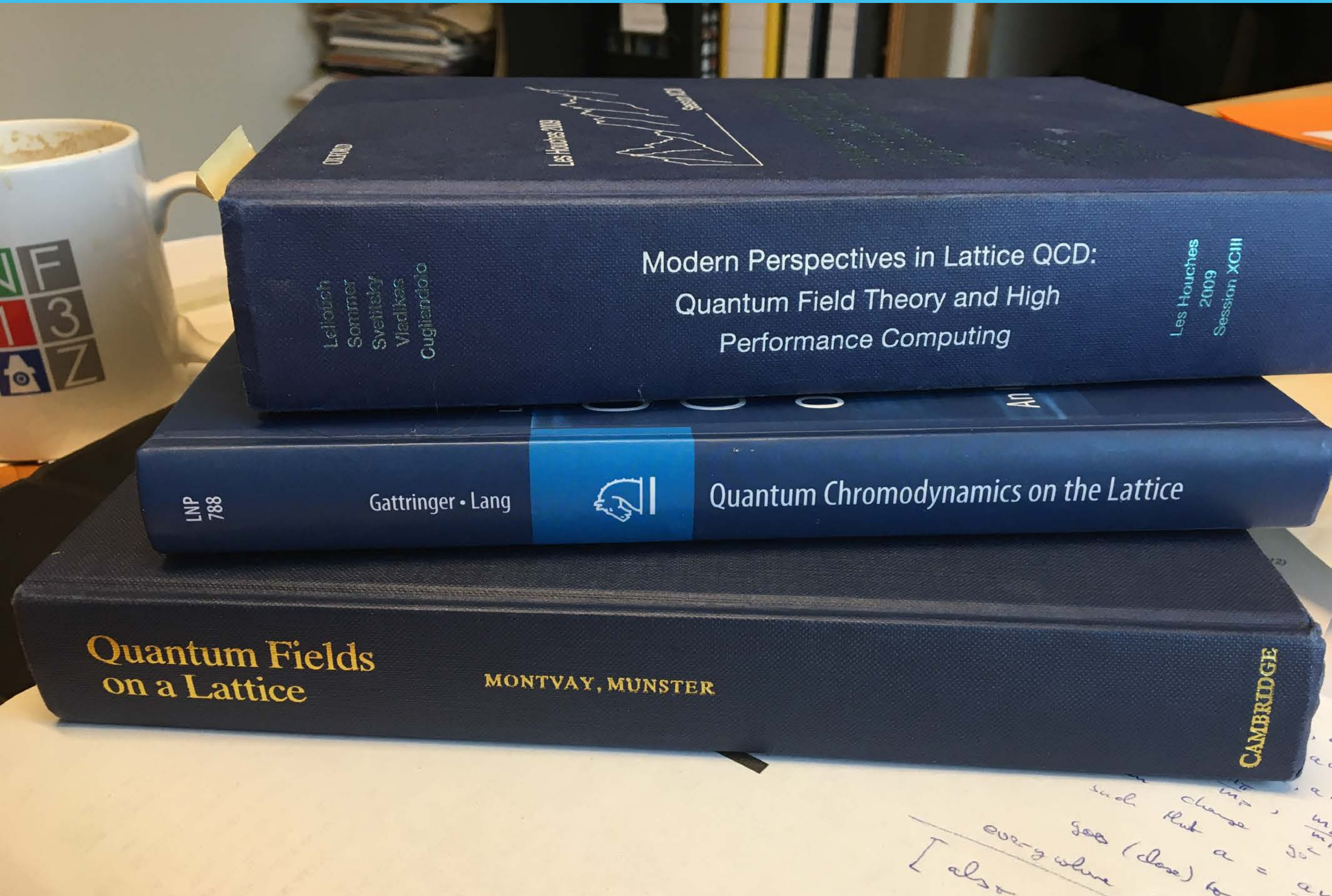
↓



$$\alpha_{\rm qq}(\mu) \equiv \frac{3r^2}{4} F_{Q\bar{Q}}(r)\,, \qquad \mu = \frac{1}{r}$$

example: pure gauge theory, very fine lattice; Husung, Koren, Krah, S. 2017

Lattice QCD in a nutshell

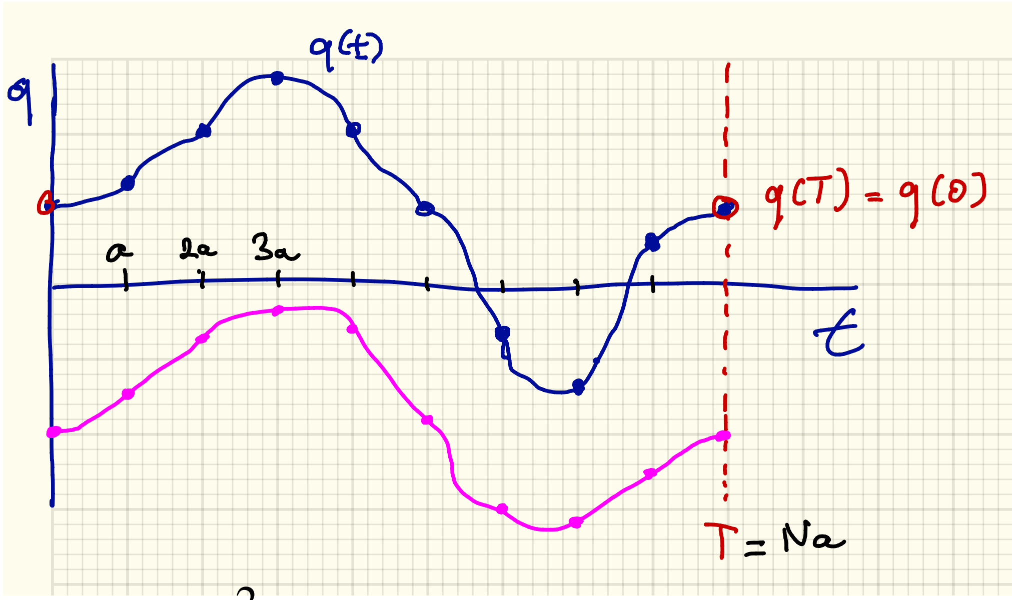


“Simulating” Quantum Mechanics

- Euclidean Green functions

$$G(\tau) = \langle 0\,|\,\hat{q}\,\mathrm{e}^{-\hat{H}\tau}\,\hat{q}\,|\,0\rangle = \sum_n |\alpha_n|^2\mathrm{e}^{-(E_n-E_0)\tau}\,, \quad \alpha_n = \langle n\,|\,\hat{q}\,|\,0\rangle\,, \quad \hat{H} = V(\hat{q}) + \frac{\hat{p}^2}{2m}$$

—> access to energy levels E_n and matrix elements α_n



$$G(\tau) = \lim_{T\to\infty} \frac{\int [\prod_i \mathrm{d}q_i]\mathrm{e}^{-S[q]}\, q_n\, q_0}{\int [\prod_i \mathrm{d}q_i]\mathrm{e}^{-S[q]}}\,, \tau = n\,a$$

$$S[q] = \sum_{i=0}^{N-1} V(q_i) + \frac{m}{2}\left(\frac{q_{i+1}-q_i}{a}\right)^2 = \int_0^T \mathrm{d}t\,[V(q(t)) + \frac{m}{2}\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)^2] + \mathrm{O}(a^2)$$

$$q_j = q(j\,a)$$

- Euclidean lattice path integral: ordinary integral over variables $q_j, j = 1,\ldots, N-1\,, N = T/a$

- Monte Carlo integration called “Simulation”, importance sampling, get low lying spectrum and more

QCD

- quarks: 3-vectors in color space

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \end{pmatrix} \in \mathbb{C}$$

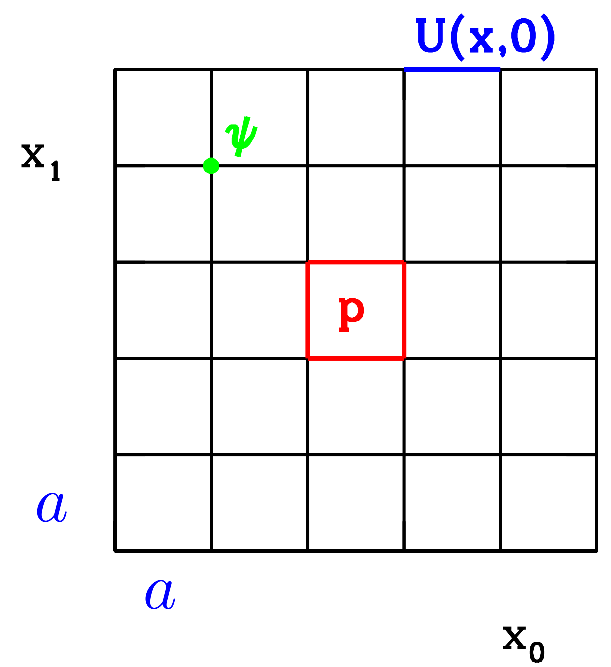
- gauge invariance

$$\Lambda(x) \in \mathrm{SU}(3) \ : \psi(x) \rightarrow \psi^{\Lambda}(x) = \Lambda(x)\,\psi(x)$$

- together with locality, unitarity, causality this fixes almost (θ-term) entirely the Lagrangian
  gluons come through gauge invariance (minimal coupling)

- apart from quark masses only one parameter: strong coupling α_s

Simulating QCD



discrete space-time, spacing a, hyper-cubic lattice

gluons: $U(x,\mu) = \mathscr{P}\exp\left\{a\int_0^1 \mathrm{d}s\, A_\mu(x+a(1-s)\hat\mu)\right\} \in \mathrm{SU}(3)$

on links

Euclidean action: $S = S_G + S_F$

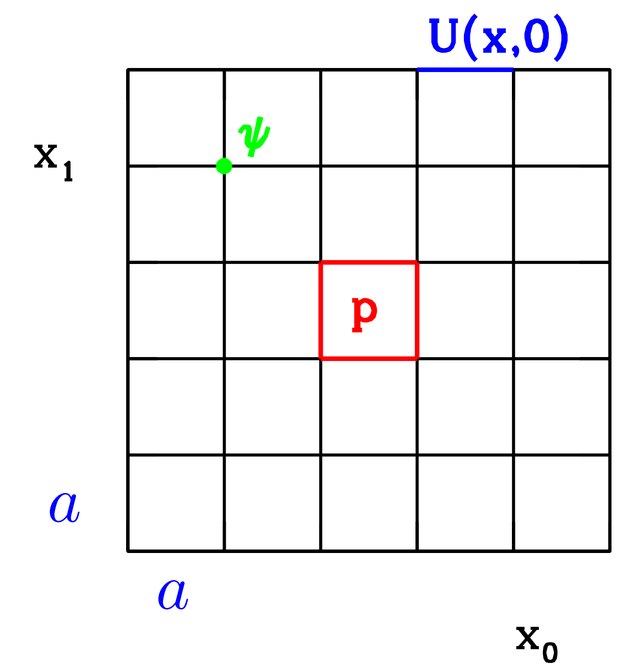
$$S_G = \frac{1}{g_0^2}\sum_p \operatorname{tr}\{1 - U(p)\},$$

$$S_F = a^4 \sum_x \bar\psi(x)\,(D(U)+m)\,\psi(x)\;,\quad D(U):\ \text{discretized Dirac operator}$$

Path integral expectation values, $\langle O\rangle = Z^{-1}\int_{\text{fields}} O\,\mathrm{e}^{-S}$ by MC integration

take $a \to 0$: THE definition of QCD

Simulating QCD



hadron Green function

$$G_\pi(\tau) = \langle 0 | \hat{\pi}\, e^{-\hat{H}\tau}\, \hat{\pi}^\dagger | 0 \rangle = \sum_n |\alpha_n|^2 e^{-(E_n(\pi)-E_0)\tau}$$

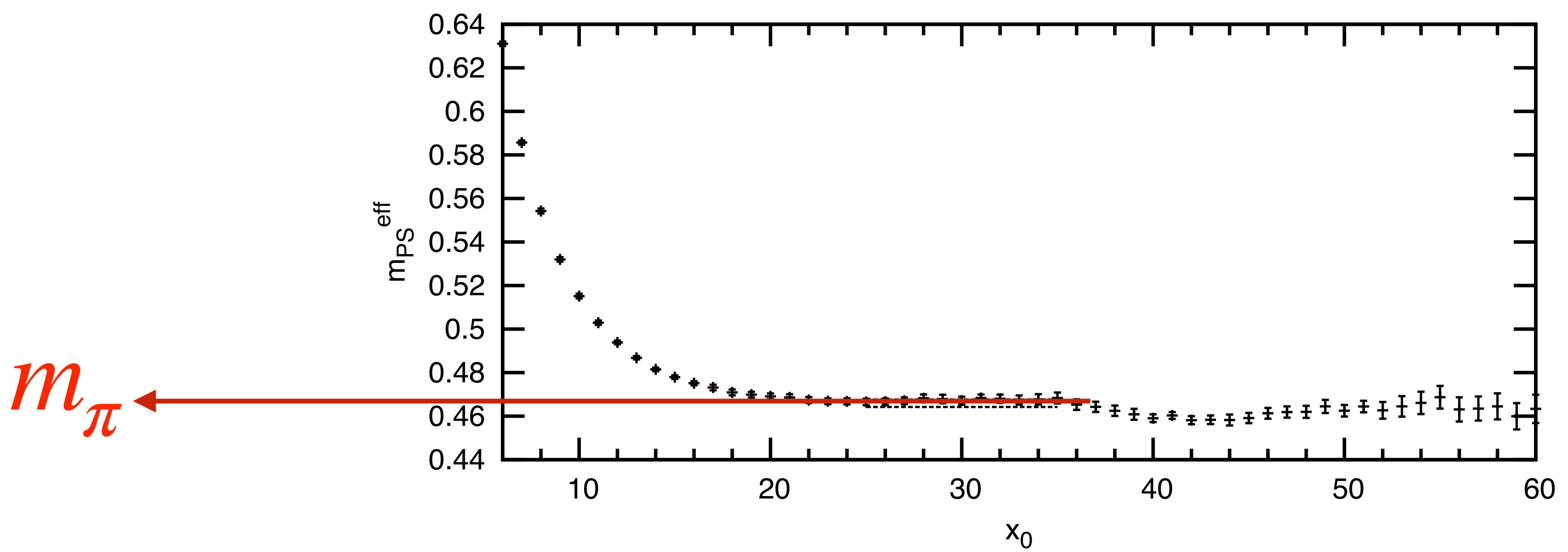
$\hat{H} =$ latt. QCD hamiltonian $\quad E_1(\pi) - E_0 = m_\pi$

evaluated by MC, importance sampling, only ~1000 MC "events"="configurations"

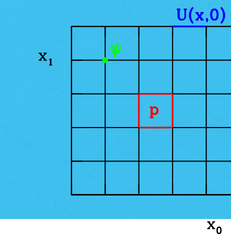
error $\propto 1/\sqrt{\text{\# configs}}$, computer time $\propto (L/a)^3 T/a \times (a\Lambda)^{-z}$

$z \approx 2$ algorithmic, "critical slowing down"

$$am_\pi^{\rm eff}(\tau) \equiv \log(G_\pi(\tau)/G_\pi(\tau+a)) = m_\pi + \mathrm{O}(e^{-(E_2(\pi)-m_\pi)\tau})$$



"Solving" (lattice) QCD (logics)



simulate with $\{am_u = am_d,\, am_s,\, g_0\}$ = (bare) parameters

compute $am_\pi,\, am_K,\, am_{\text{proton}}$

tune parameters until $$\left\{\frac{am_\pi}{am_{\text{proton}}},\, \frac{am_K}{am_{\text{proton}}}\right\} = \left\{\frac{m_\pi}{m_{\text{proton}}},\, \frac{m_K}{m_{\text{proton}}}\right\}_{\text{experimental}}$$

repeat with smaller and smaller g_0

smaller and smaller $a\, m_{\text{proton}}$

compute e.g.

$$\alpha_{\text{qq}}\left(r = \text{const}\,./m_{\text{proton}}\right)$$

extrapolate to $a\, m_{\text{proton}} \to 0$ continuum limit

"Solving" (lattice) QCD (logics)

simulate with $\{am_u = am_d,\ am_s,\ g_0\}$ = (bare) parameters

compute $am_\pi,\ am_K,\ am_{\rm proton}$

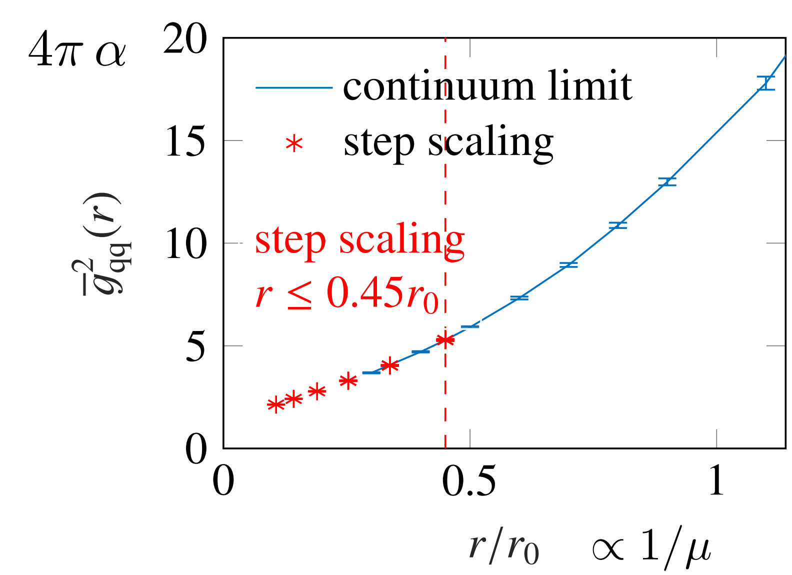
tune parameters until
$$\left\{\frac{am_\pi}{am_{\rm proton}},\ \frac{am_K}{am_{\rm proton}}\right\} = \left\{\frac{m_\pi}{m_{\rm proton}},\ \frac{m_K}{m_{\rm proton}}\right\}_{\rm experimental}$$

repeat with smaller and smaller g_0

smaller and smaller $a\, m_{\rm proton}$

compute e.g.
$\alpha_{\rm qq}\left(r = {\rm const}\,./m_{\rm proton}\right)$
extrapolate to

$a\, m_{\rm proton} \to 0$ continuum limit



A lot of progress in recent years

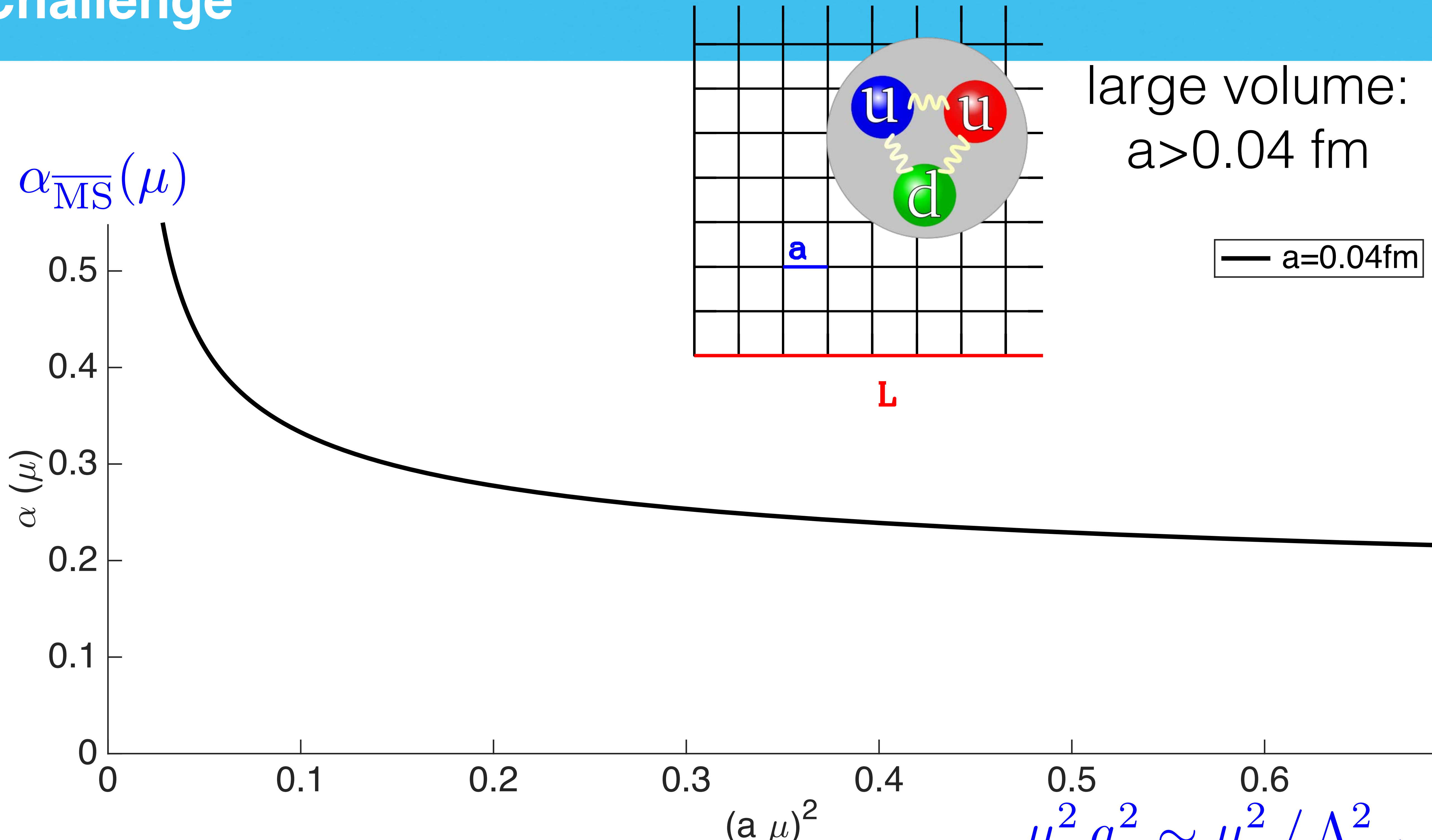
- a lot of progress in recent years
  - concepts
  - algorithms

year	**Cost** to generate one 96x48^3 configuration [hours on 512 cores]	
2001	17000	"Berlin wall"
2015	5	Hasenbusch preconditioning, multigrid/deflation, open BC

  - computers
- precise results are possible
- but $\alpha(\mu)$ is a challenge

Challenge

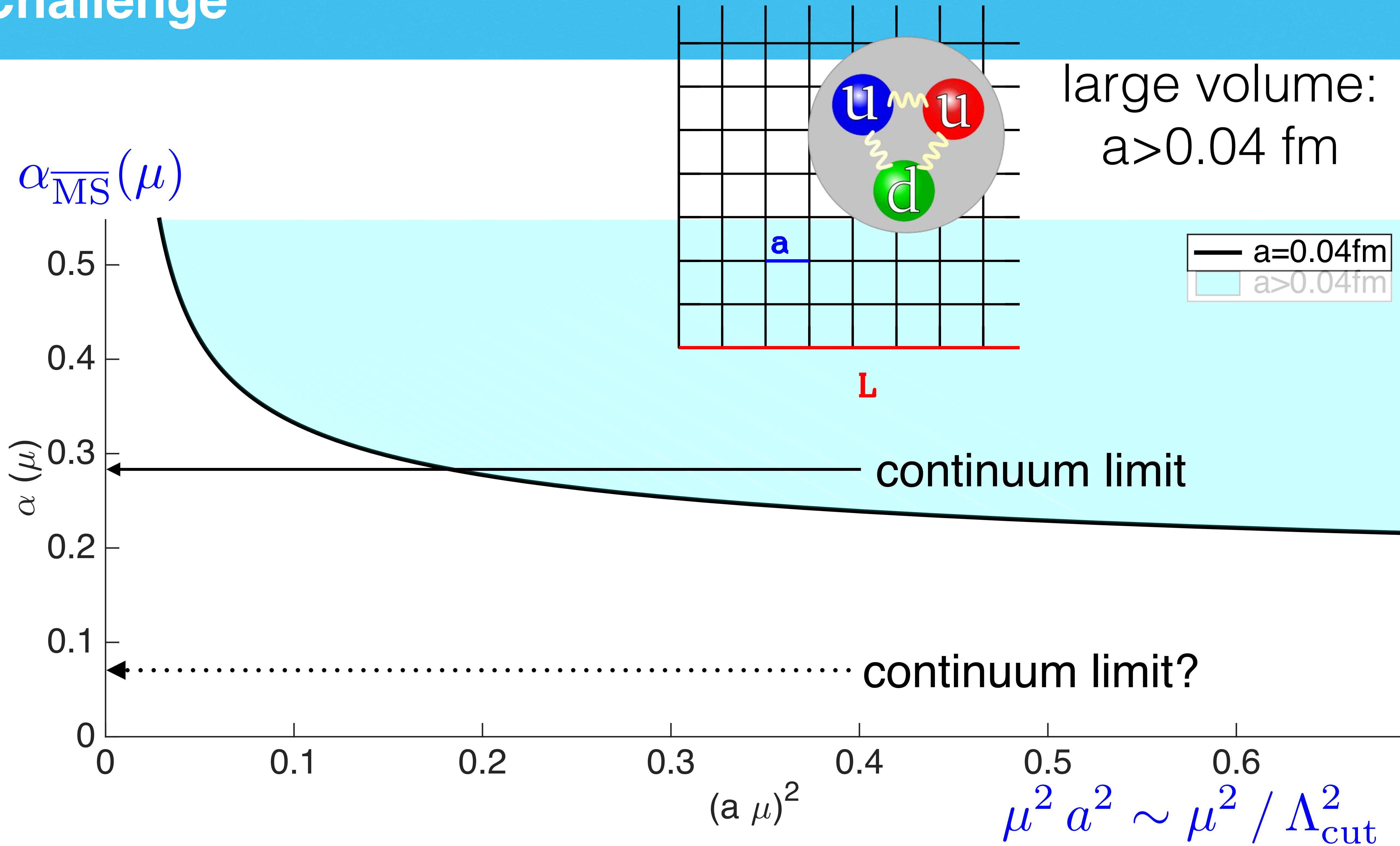
large volume:
a>0.04 fm



$$\mu^2\, a^2 \sim \mu^2 \,/\, \Lambda_{\mathrm{cut}}^2$$

Challenge

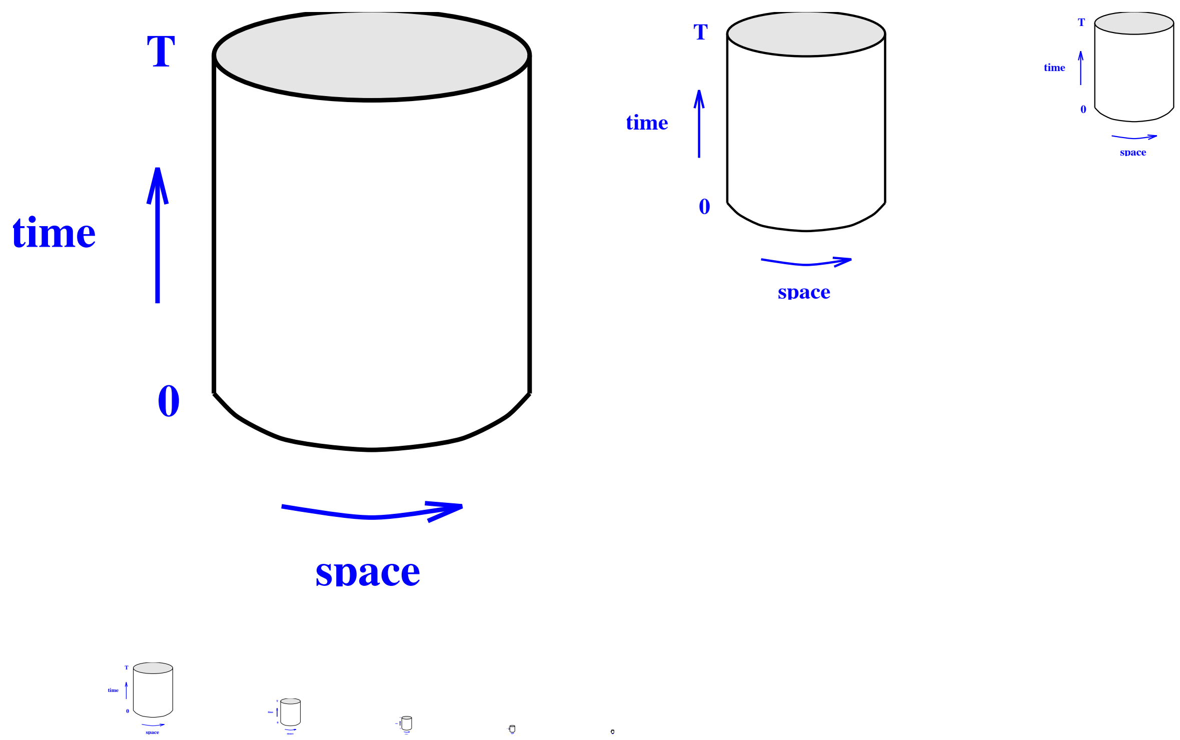
large volume:
a>0.04 fm

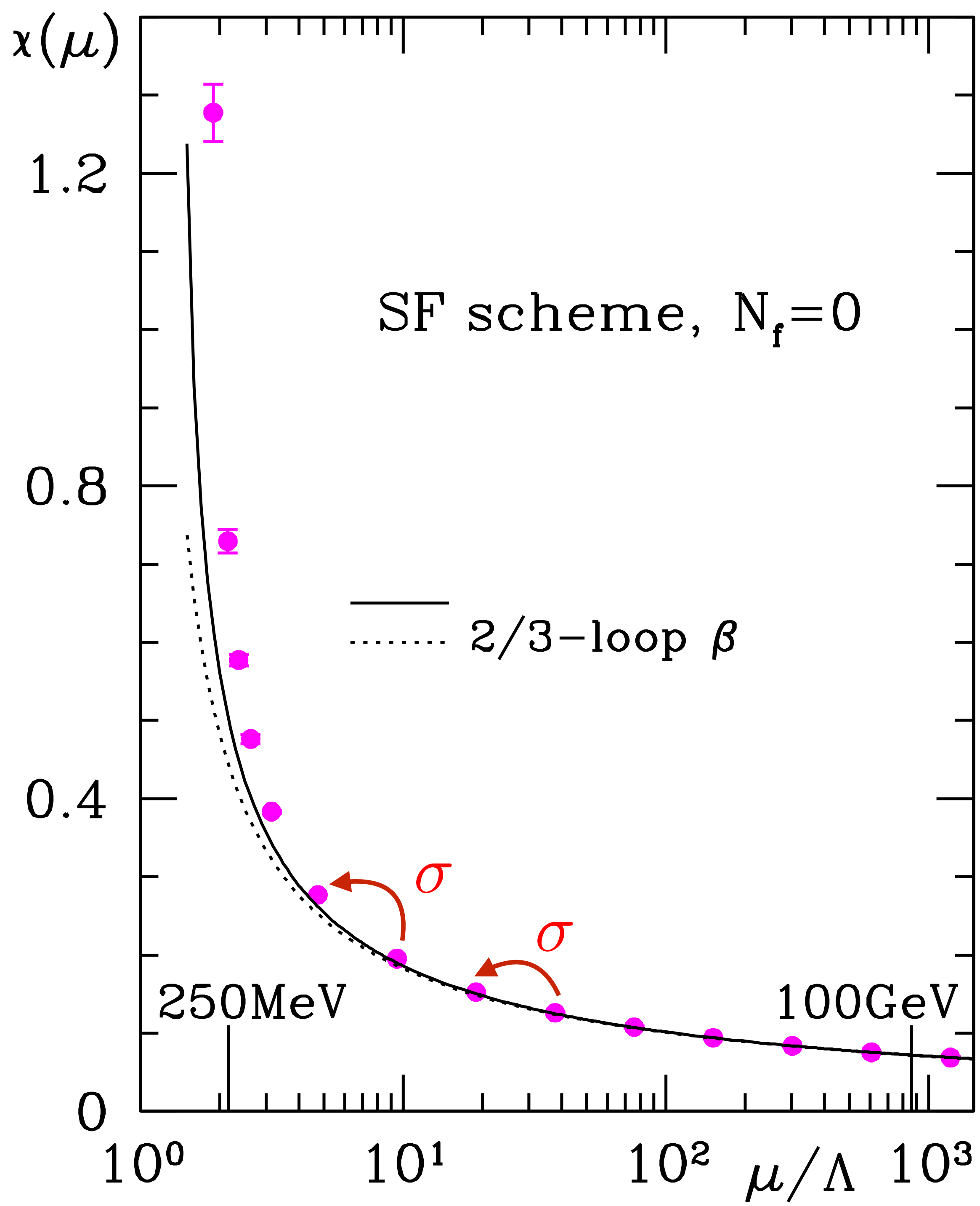


$a^2\mu^2 \ll 1$ or strong assumptions to take continuum limit

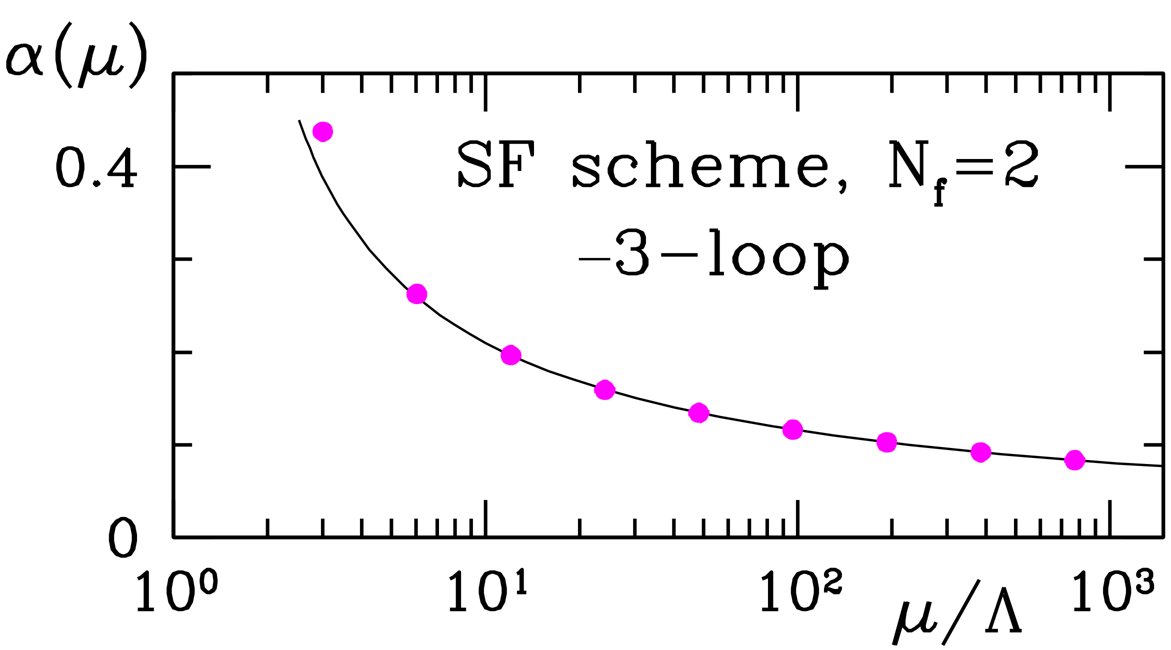
Solution: finite volume $\mu = 1/L$

Running from Observables in finite volume





[ALPHA Collaboration, 2001]



[ALPHA Collaboration, 2005]

Step Scaling Function: Connects L → 2L

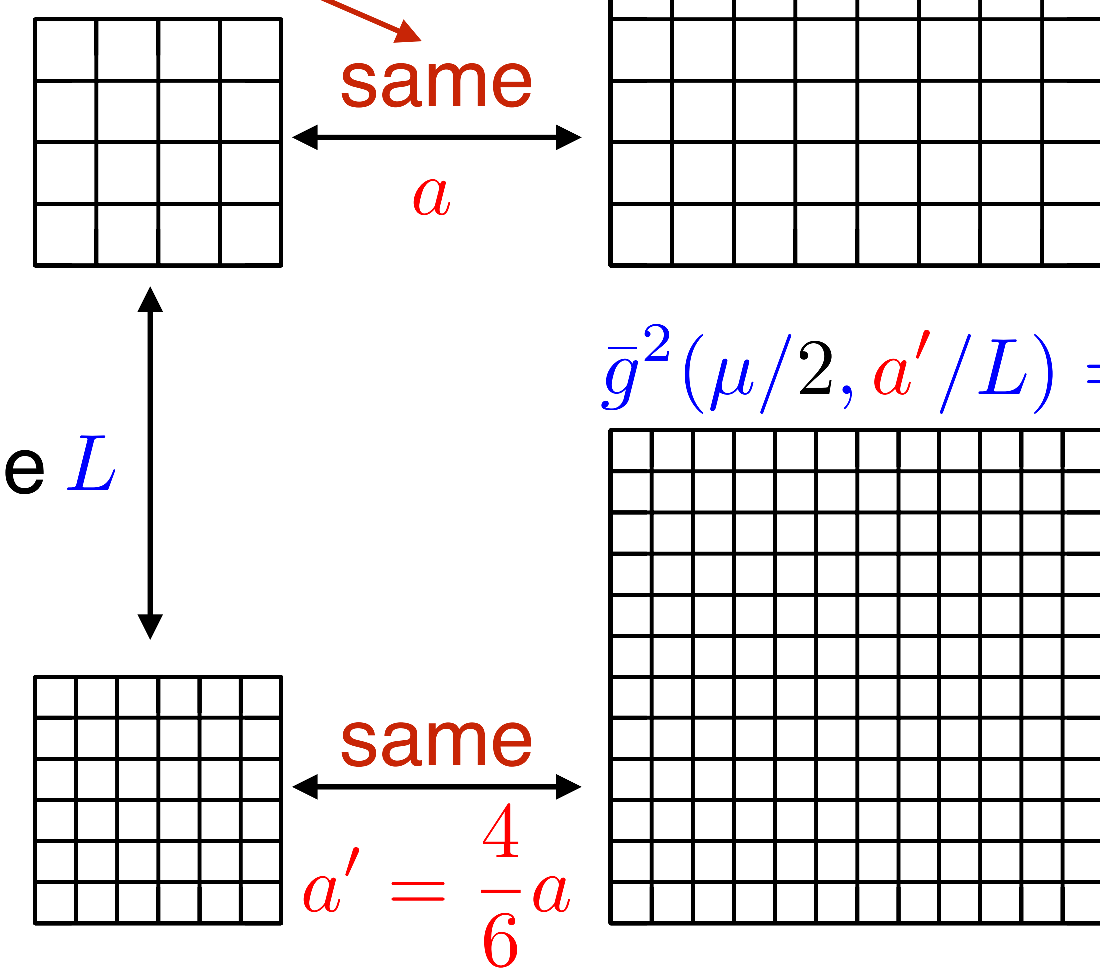
ALPHA Collaboration

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2{g_0}^2}\,\mathrm{tr}\;F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_\mathrm{f}} \overline{\psi}_f\,\{D + m_{0f}\}\,\psi_f$$

$$\bar{g}^2(\mu/2, a/L) = \bar{g}^2(1/(2L), a/L)$$

1/4

$$\bar{g}^2(\mu) = \bar{g}^2(1/L)$$



$$\bar{g}^2(\mu/2, a'/L) = \bar{g}^2(1/(2L), a'/L)$$

1/6

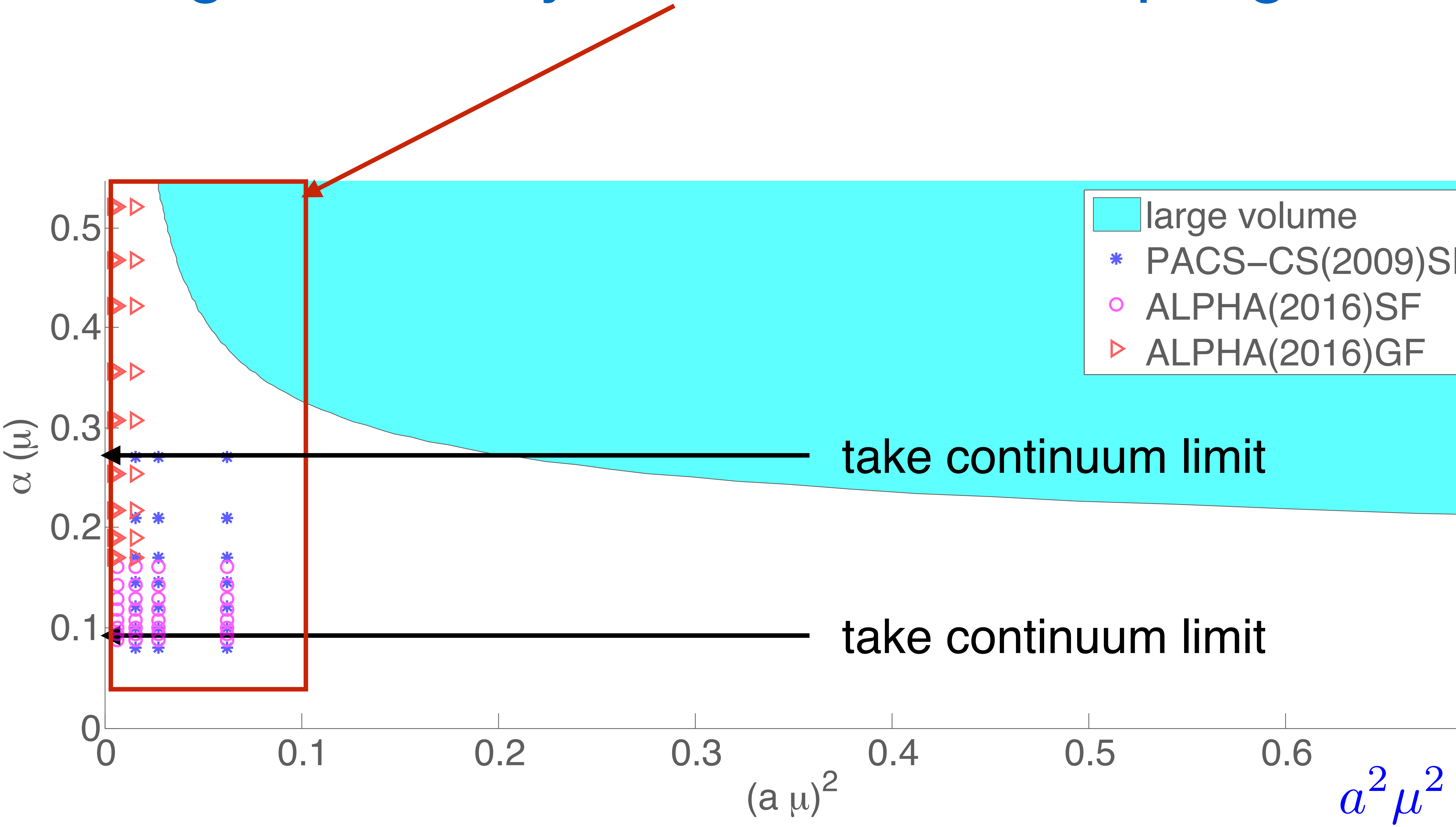
$$\bar{g}^2(\mu) = \bar{g}^2(1/L)$$

extrapolate

$$\bar{g}^2(\mu/2,\,0\,) = \sigma(\,\bar{g}^2(\mu)\,)$$

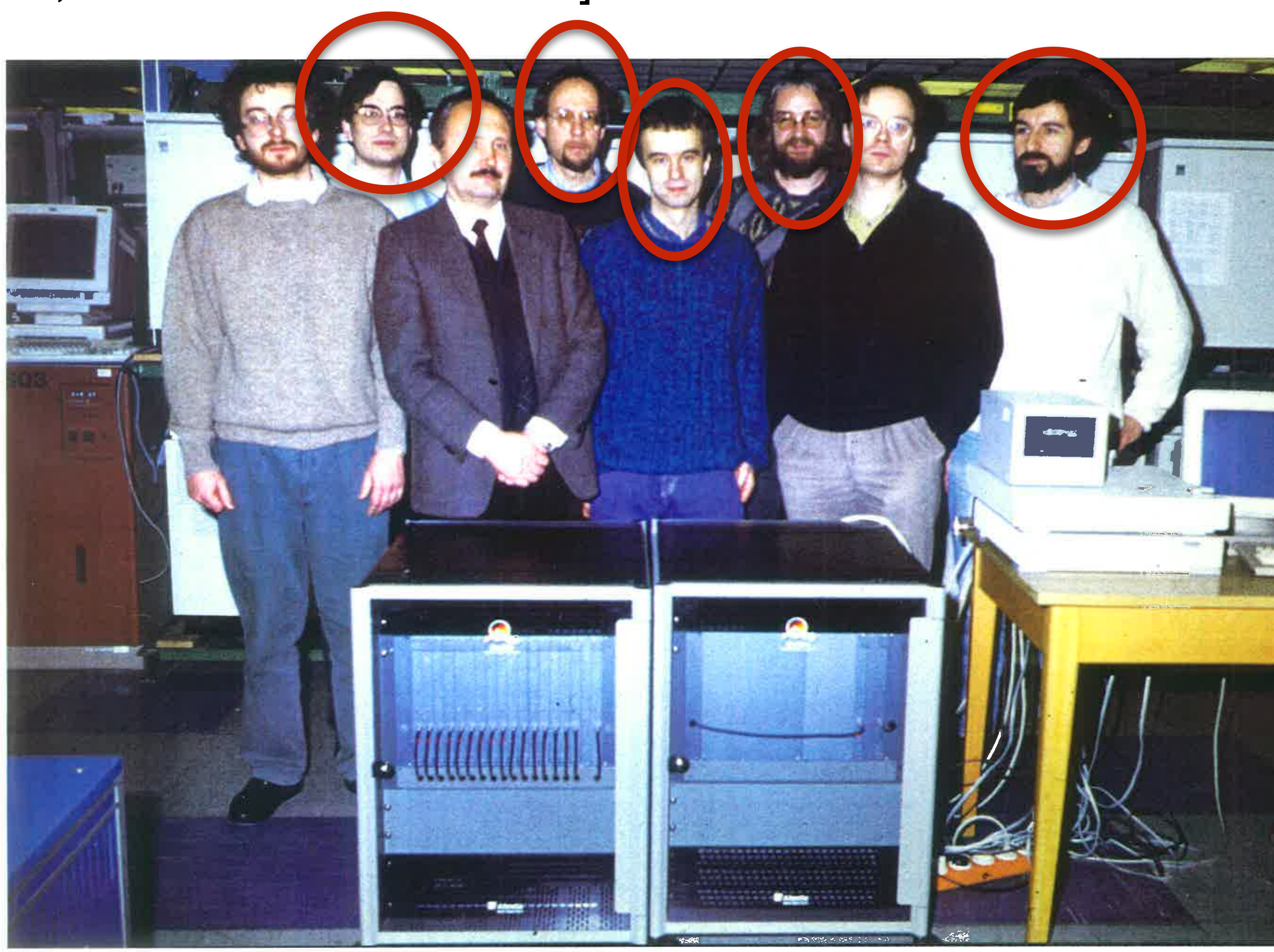
$\sigma =$ **continuum** step scaling function

Challenge is met by finite volume couplings



History of finite volume couplings

- 1991 2-d sigma model [LüWeWo]
- 1992 Schrödinger functional [LüNaWeWo, Si]
- 1992-95 SU(2) YM coupling [LüSoWeWo, DiFrGuLüPeSoWeWo]
- 1993 DESY gets an APE-computer (
- 1994 SU(3) YM coupling [LüSoWeWo
- 2000 3-loop β for SF coupling [BoWeW
- 2001-05 N_f=2 coupling [BoFrGeHaHe
- 2009 N_f=3 coupling S. Aoki et al. (PAC



Bode, dellaBrida, Bruno, Frezzotti, Divitiis, Fritzsch, Gehrmann, Guagnelli, Hasenbusch, Heitger, Jansen, Kurth, Korzec, Lüscher, dellaMorte, Narayanan, Neuberger, Petronzio, Ramos, Rolf, Schaefer, Simma, Sint, Sommer, Weisz, Wittig, Wolff

History of finite volume couplings

- 1991 2-d sigma model [LüWeWo]
- 1992 Schrödinger functional [LüNaWeWo, Si]
- 1992-95 SU(2) YM coupling [LüSoWeWo, DiFrGuLüPeSoWeWo]
- 1993 DESY gets an APE-computer (< speed of my laptop)
- 1994 SU(3) YM coupling [LüSoWeWo]
- 2000 3-loop β for SF coupling [BoWeWo]
- 2001-05 N_f=2 coupling [BoFrGeHaHeJaKuRoSimSinSoWeWiWo]
- 2009 N_f=3 coupling S. Aoki et al. (PACS-CS)
- 2010-2013 Gradient flow coupling [NaNe, Lü, LüWe, RaFr, SinRa]

Bode, dellaBrida, Bruno, Frezzotti, Divitiis, Fritzsch, Gehrmann, Guagnelli, Hasenbusch, Heitger, Jansen, Kurth, Korzec, Lüscher, dellaMorte, Narayanan, Neuberger, Petronzio, Ramos, Rolf, Schaefer, Simma, Sint, Sommer, Weisz, Wittig, Wolff

History of finite volume couplings

- 1991 2-d sigma model [LüWeWo]
- 1992 Schrödinger functional [LüNaWeWo, Si]
- 1992-95 SU(2) YM coupling [LüSoWeWo, DiFrGuLüPeSoWeWo]
- 1993 DESY gets an APE-computer (< speed of my laptop)
- 1994 SU(3) YM coupling [LüSoWeWo]
- 2000 3-loop β for SF coupling [BoWeWo]

- 2010
- 2017 N_f=3 coupling [BriBruFrKoRaSchSimSinSo] with good precision

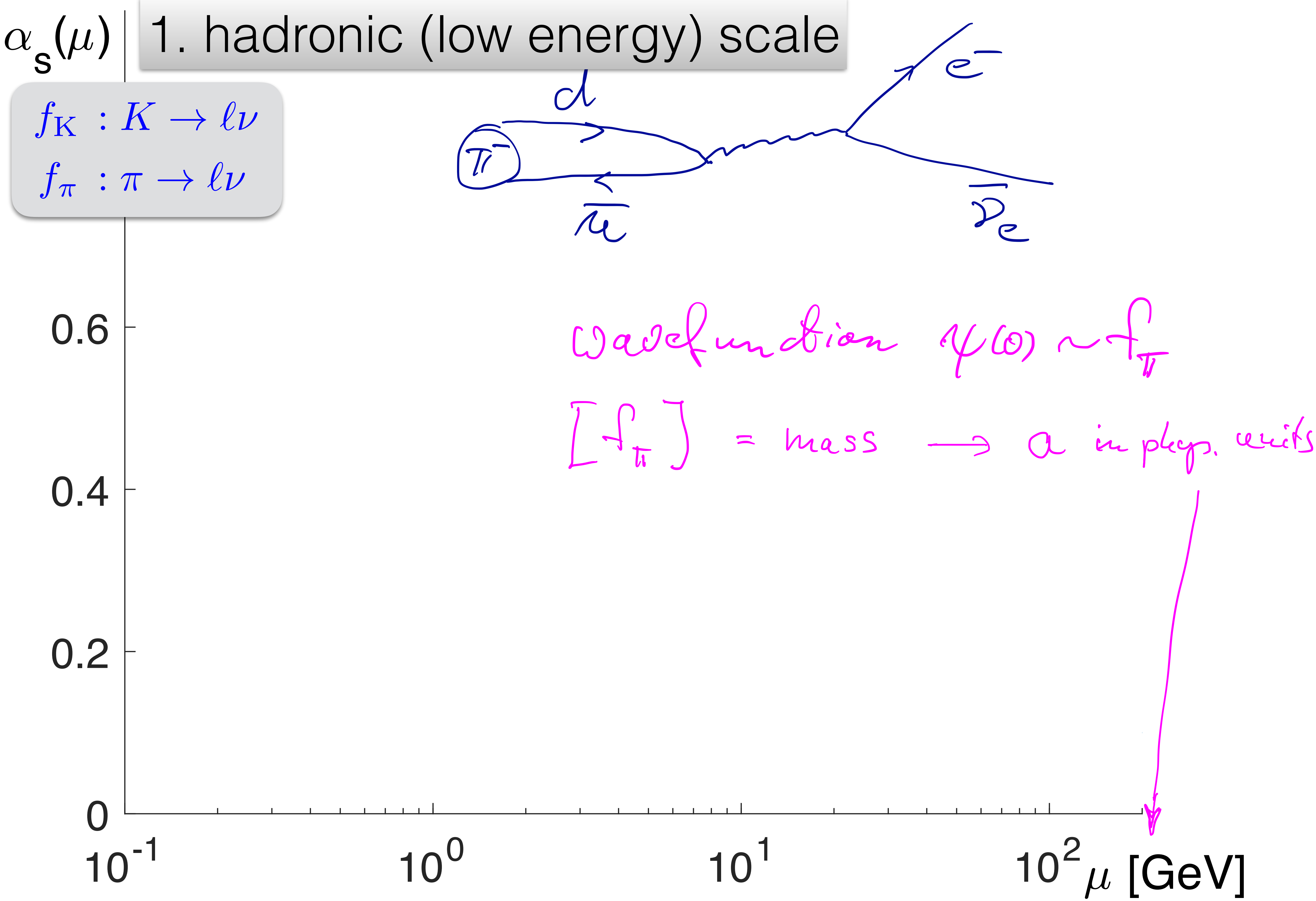
Bode, dellaBrida, Bruno, Frezzotti, Divitiis, Fritzsch, Gehrmann, Guagnelli, Hasenbusch, Heitger, Jansen, Kurth, Korzec, Lüscher, dellaMorte, Narayanan, Neuberger, Petronzio, Ramos, Rolf, Schaefer, Simma, Sint, Sommer, Weisz, Wittig, Wolff

Overall strategy

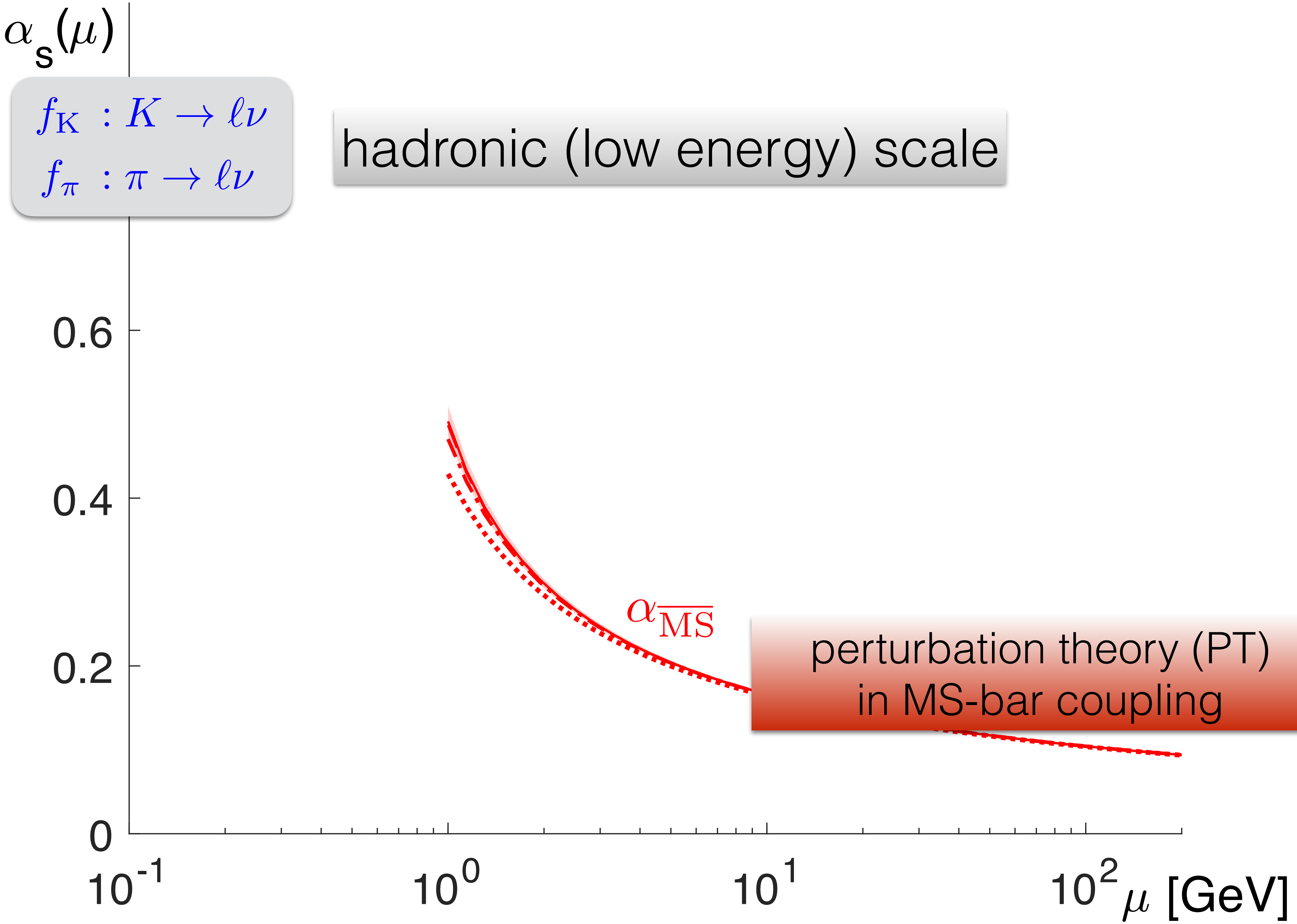
1. hadronic (low energy) scale

f_K : $K \to \ell\nu$

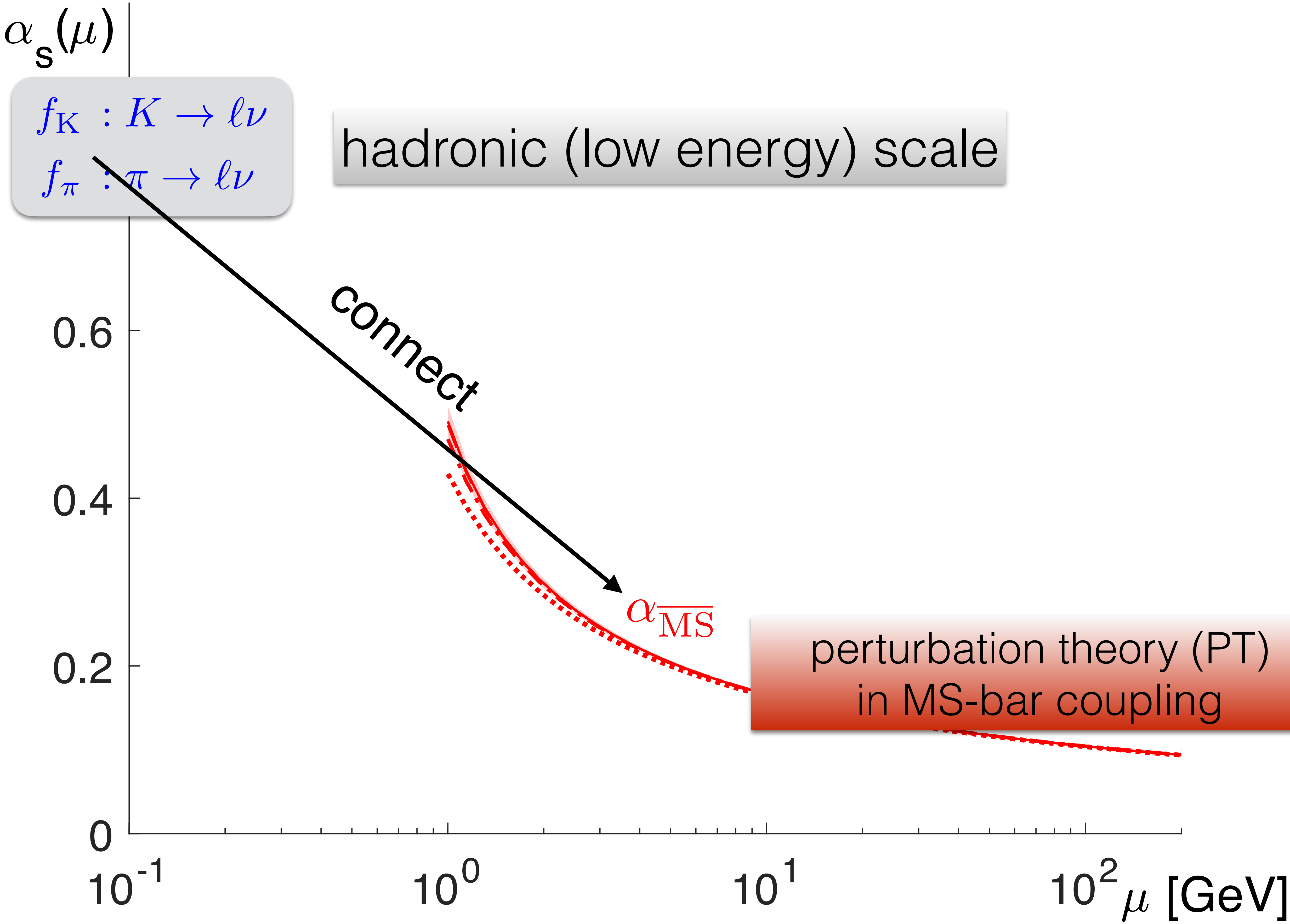
f_π : $\pi \to \ell\nu$



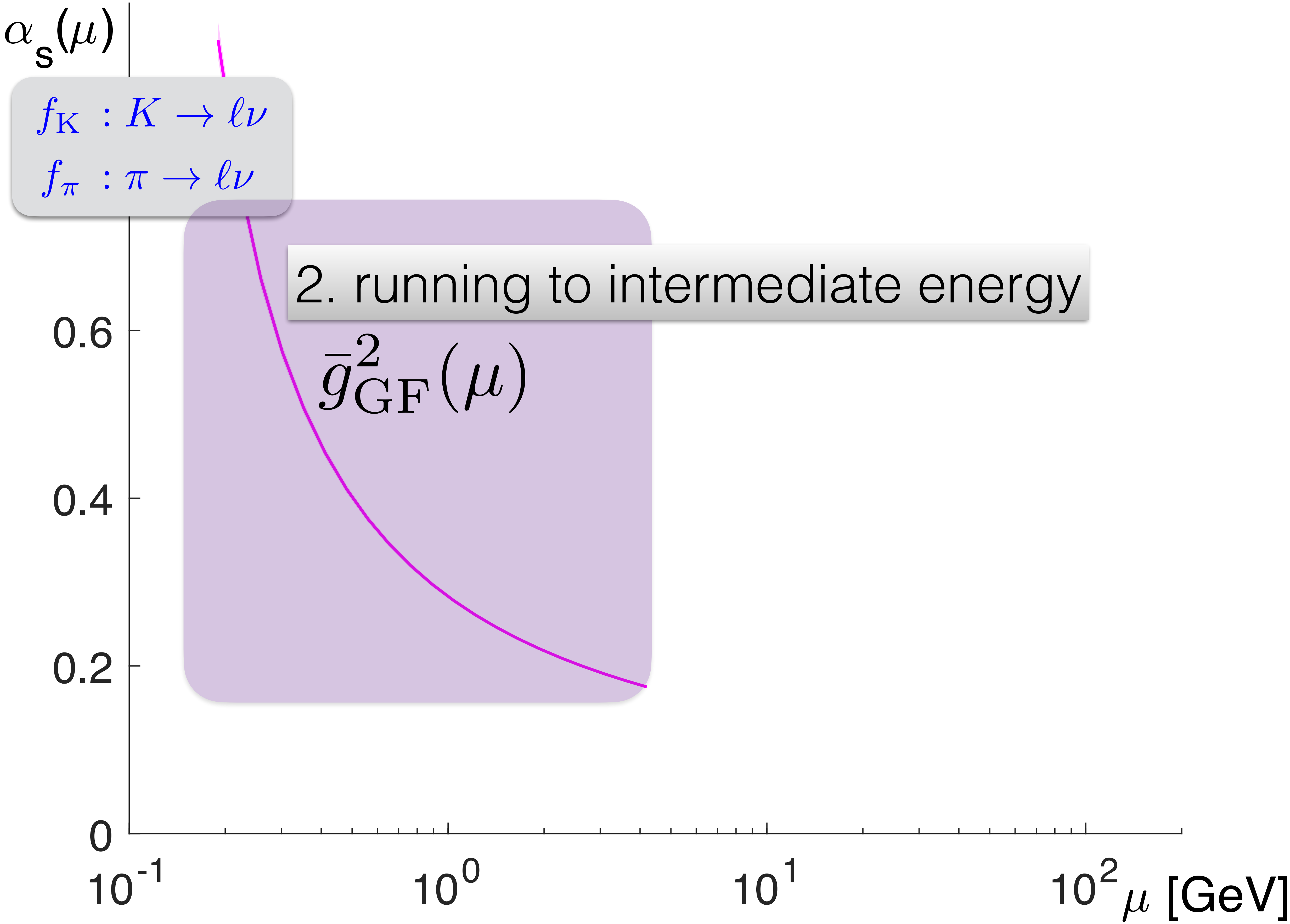
Overall strategy



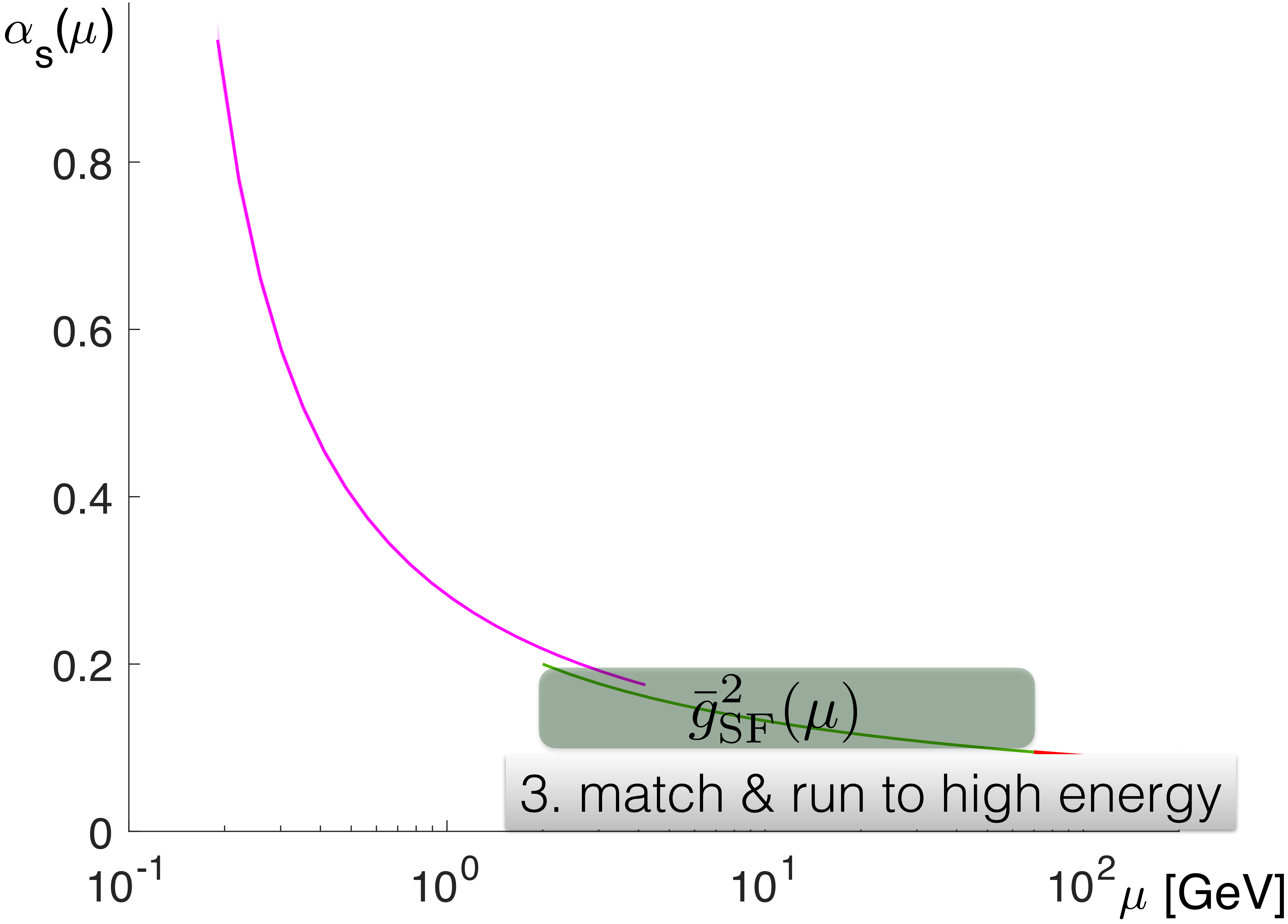
Overall strategy



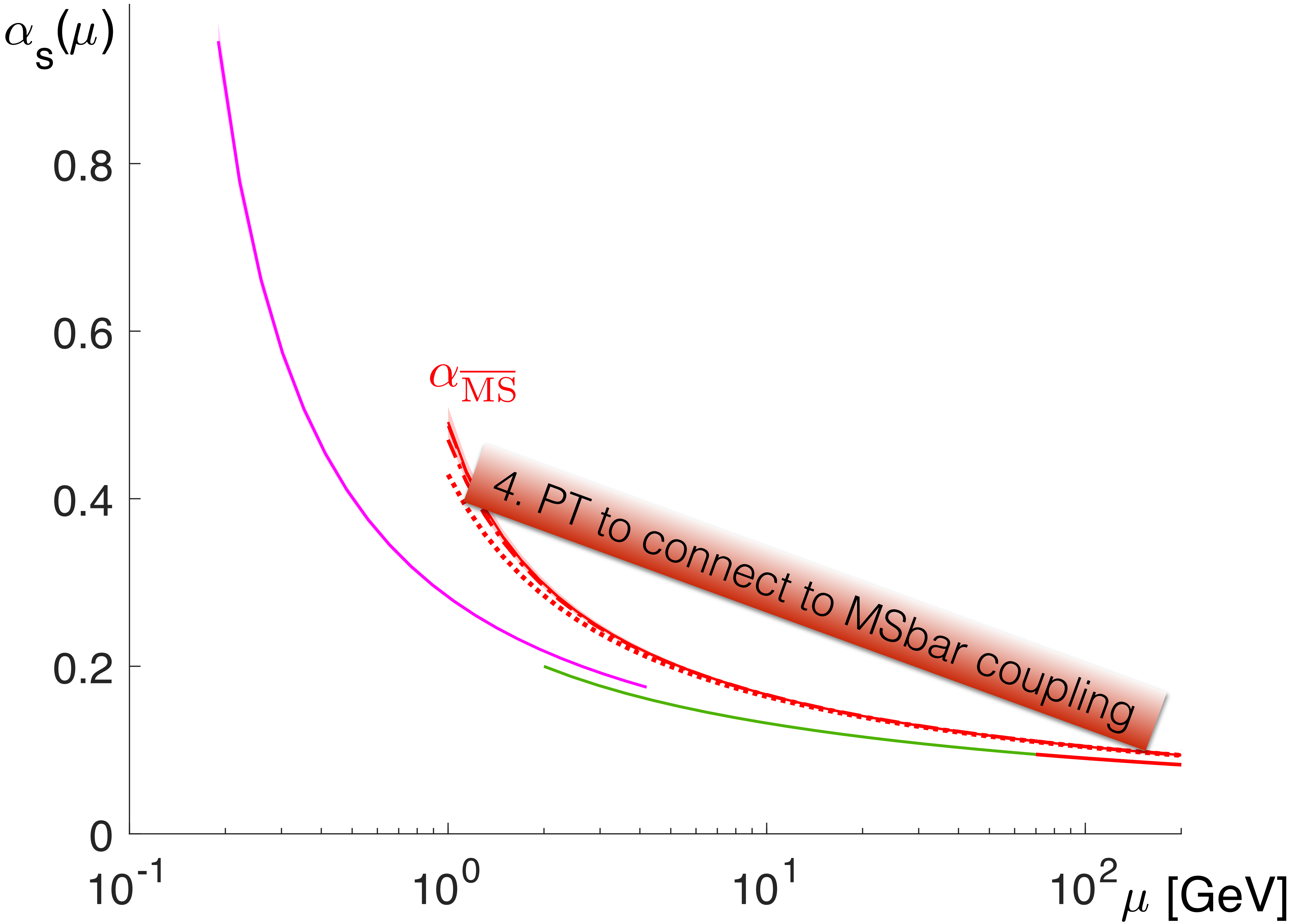
Overall strategy



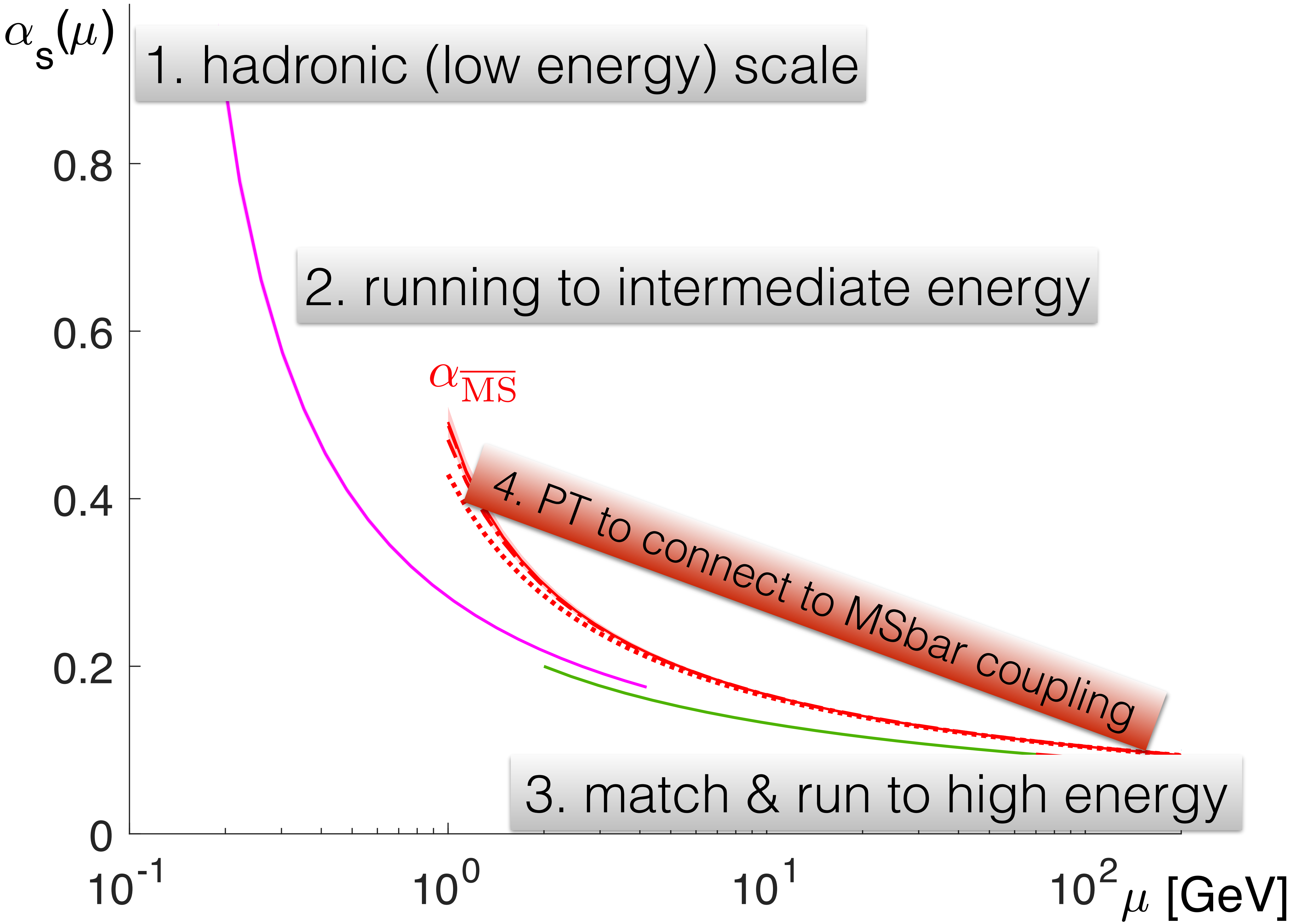
Overall strategy



Overall strategy

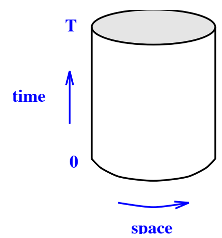


Overall strategy



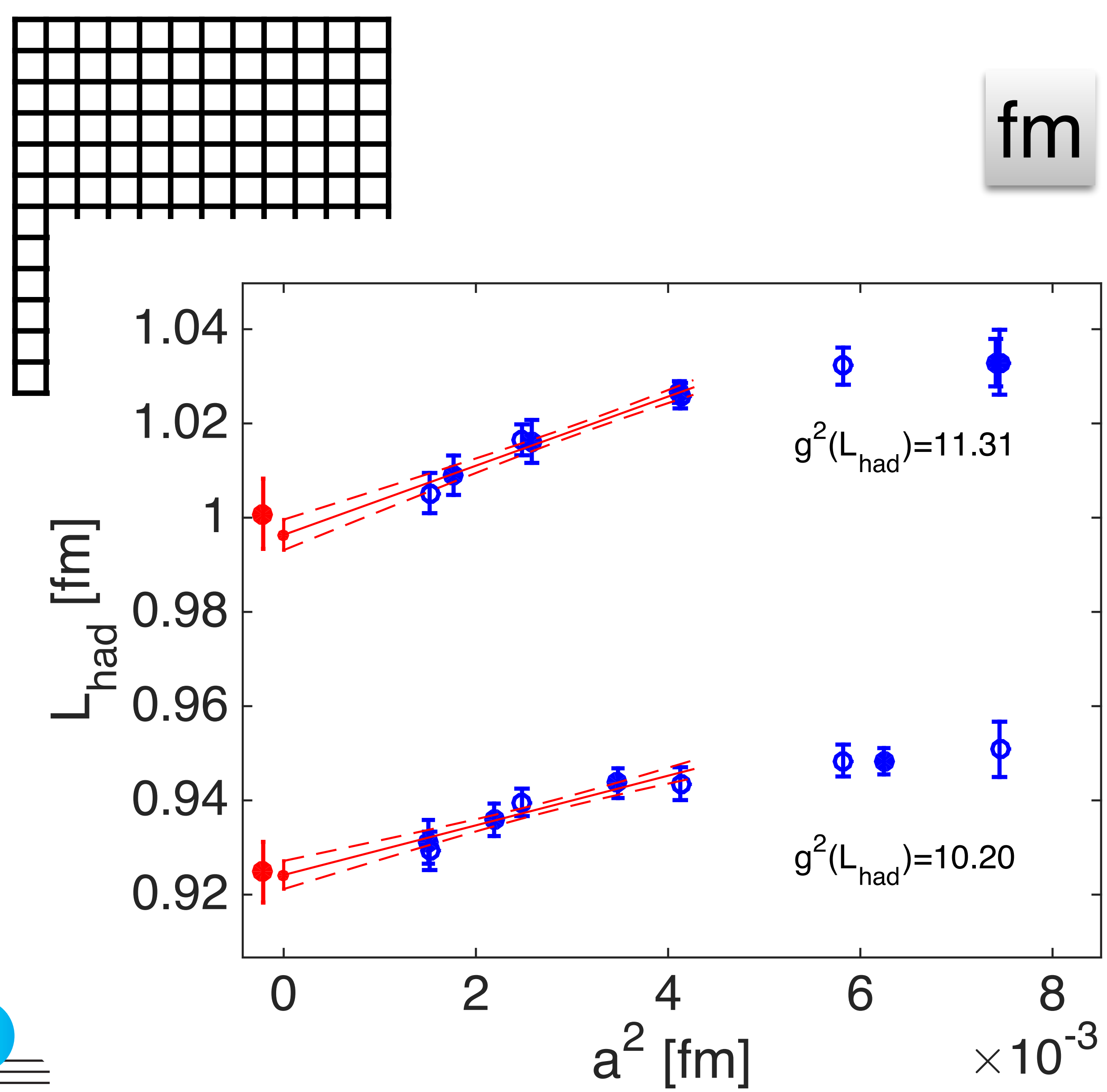
1. Determination of hadronic scale: CLS Ensembles

Bruno et al, 1411.3982
Bruno, Korzec, Schaefer, 1608.089000

- finite L 

large L

simulated at common g_0 ⇔ common lattice spacing a

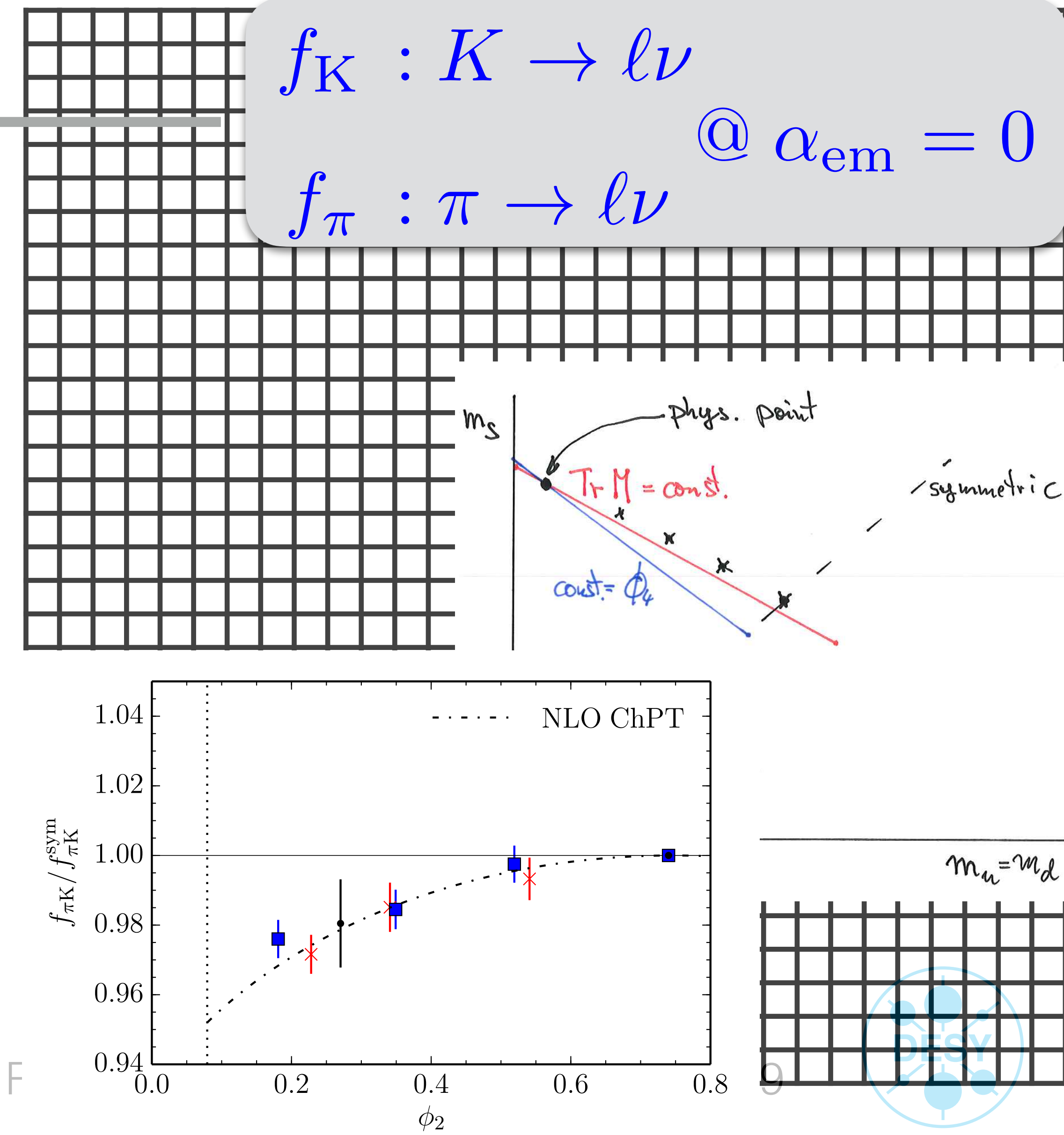


fm

$f_K \; : K \to \ell\nu$

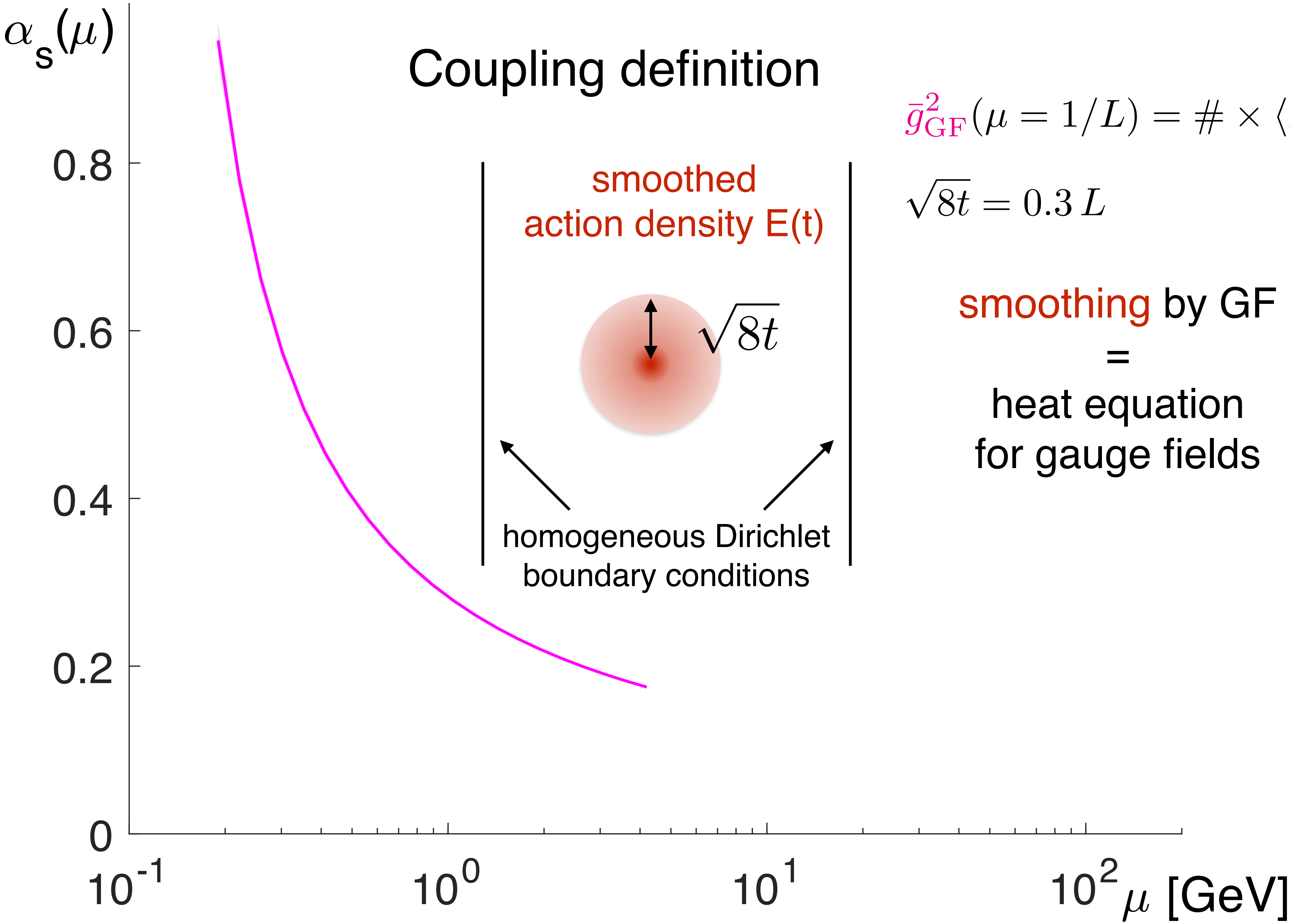
$@ \; \alpha_{em} = 0$

$f_\pi \; : \pi \to \ell\nu$



2. Running to intermediate energy

Coupling definition

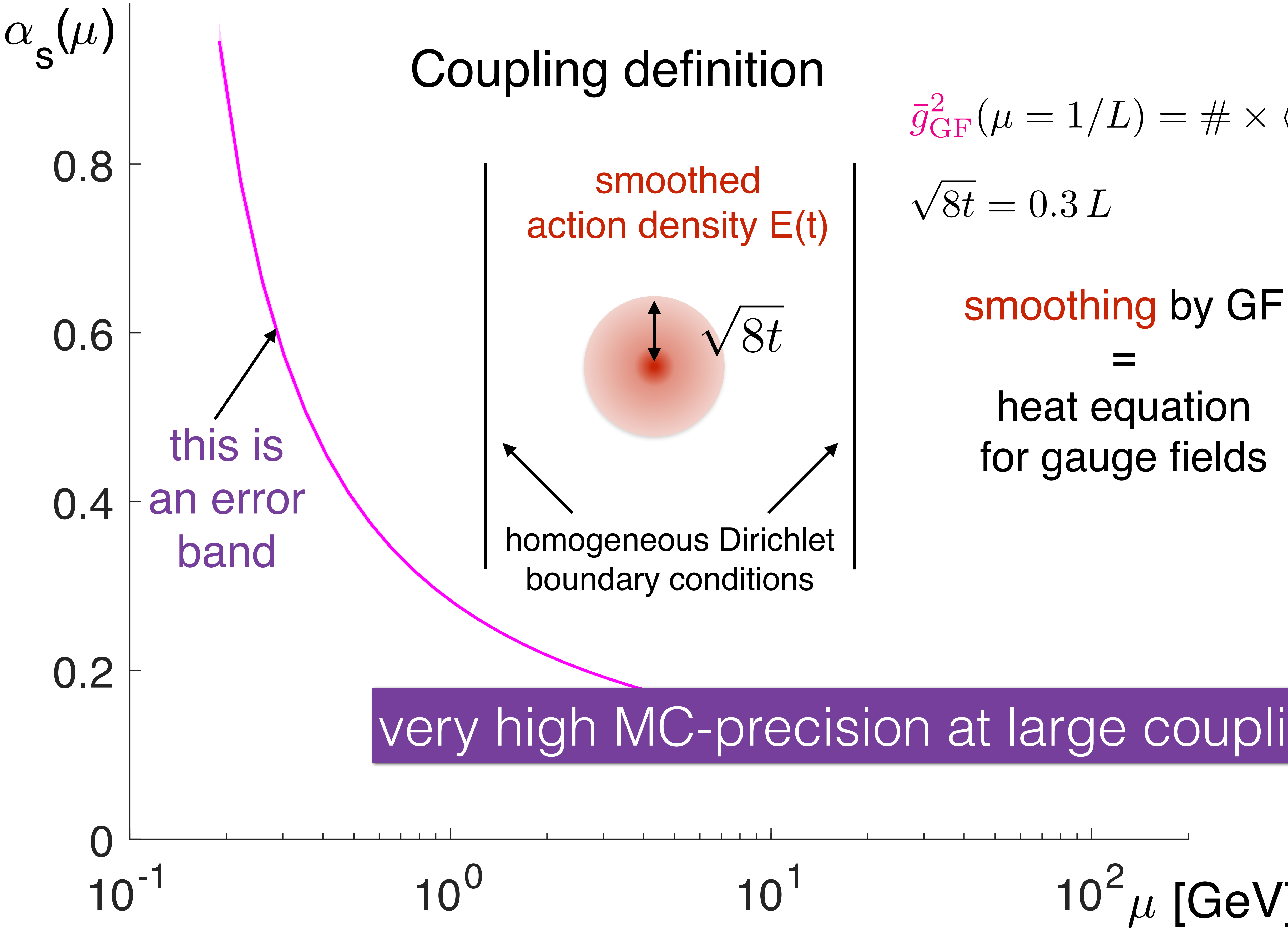


$$\bar{g}^2_{\mathrm{GF}}(\mu = 1/L) = \# \times \langle E(t) \rangle$$

$$\sqrt{8t} = 0.3\,L$$

smoothing by GF
=
heat equation
for gauge fields

2. Running to intermediate energy



Coupling definition

$$\bar{g}^2_{\mathrm{GF}}(\mu = 1/L) = \# \times \langle E(t) \rangle$$

$$\sqrt{8t} = 0.3\,L$$

smoothed action density E(t)

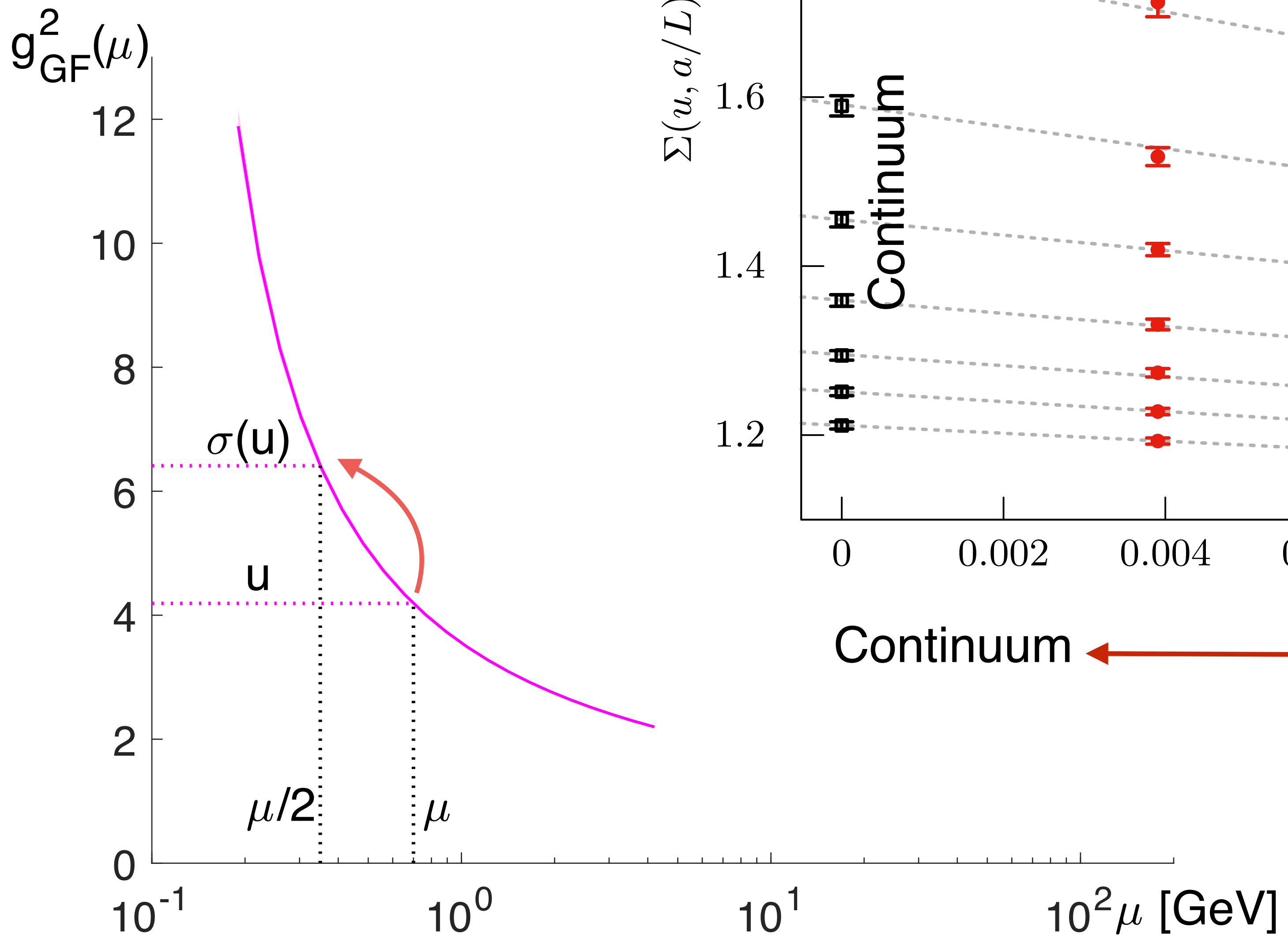
$\sqrt{8t}$

homogeneous Dirichlet boundary conditions

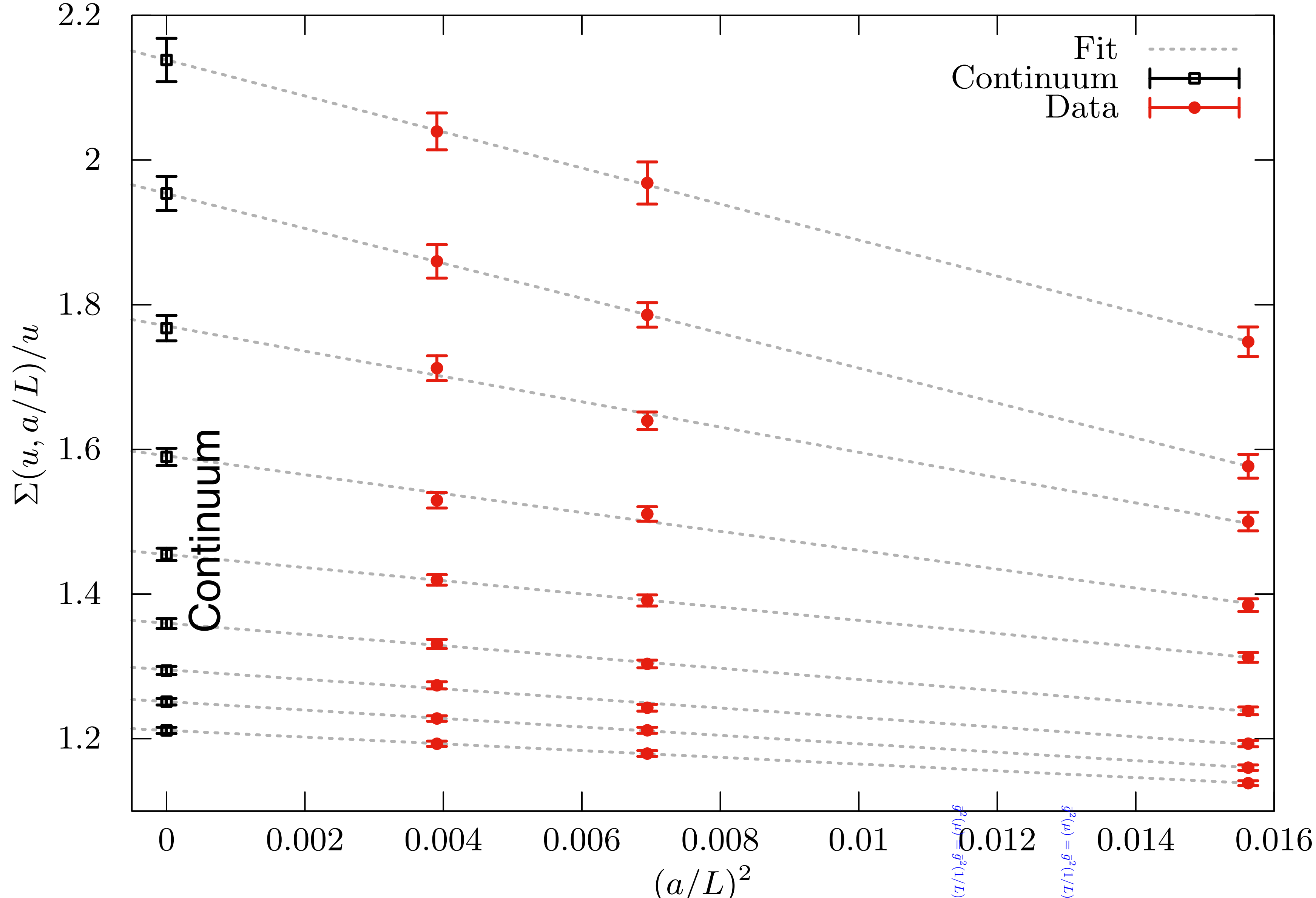
smoothing by GF
=
heat equation for gauge fields

very high MC-precision at large coupling

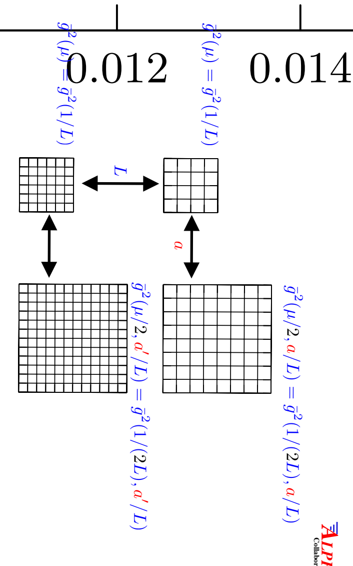
2. Running to intermediate energy



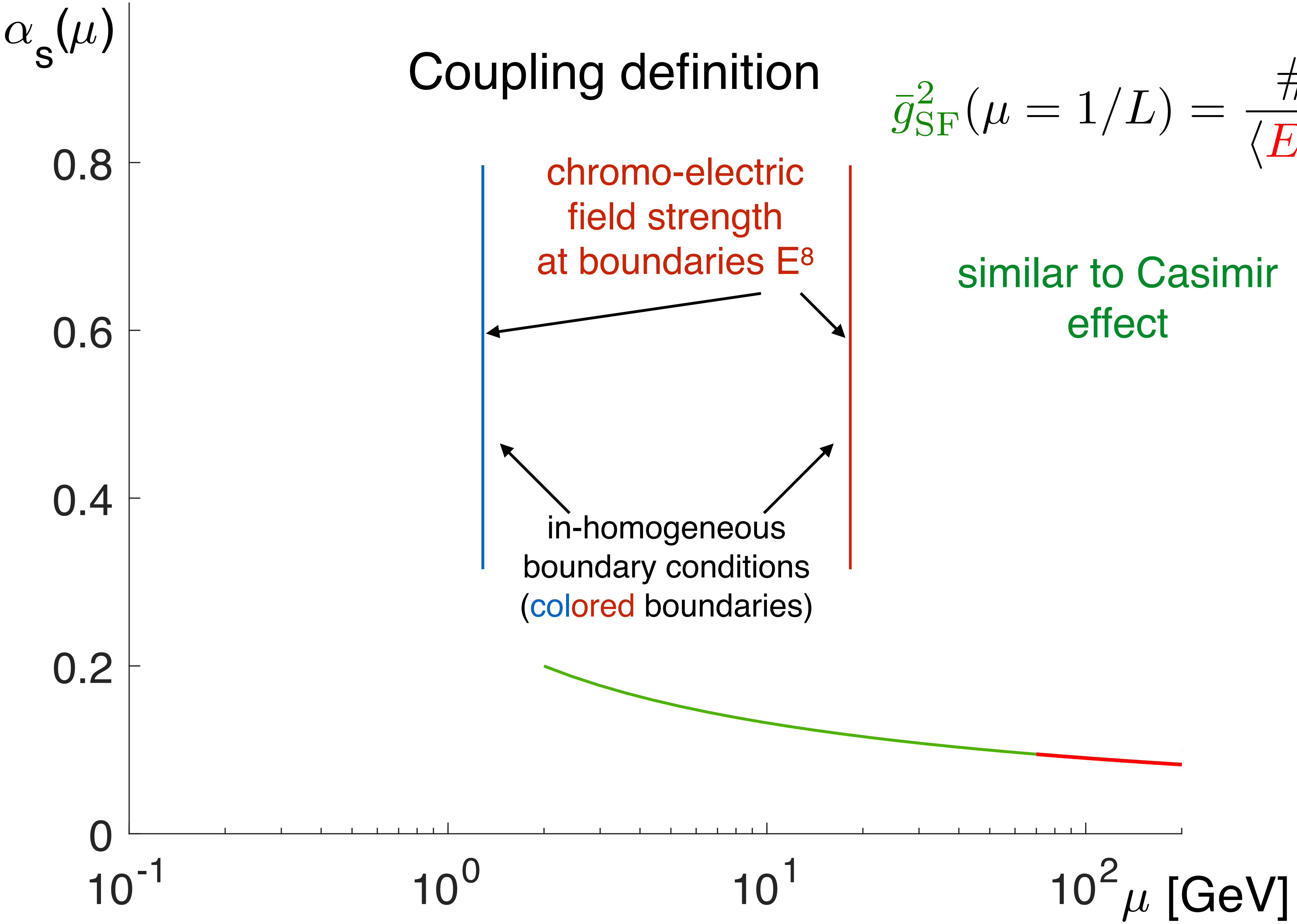
Continuum extrapolations of σ(u)=Σ(u,0)



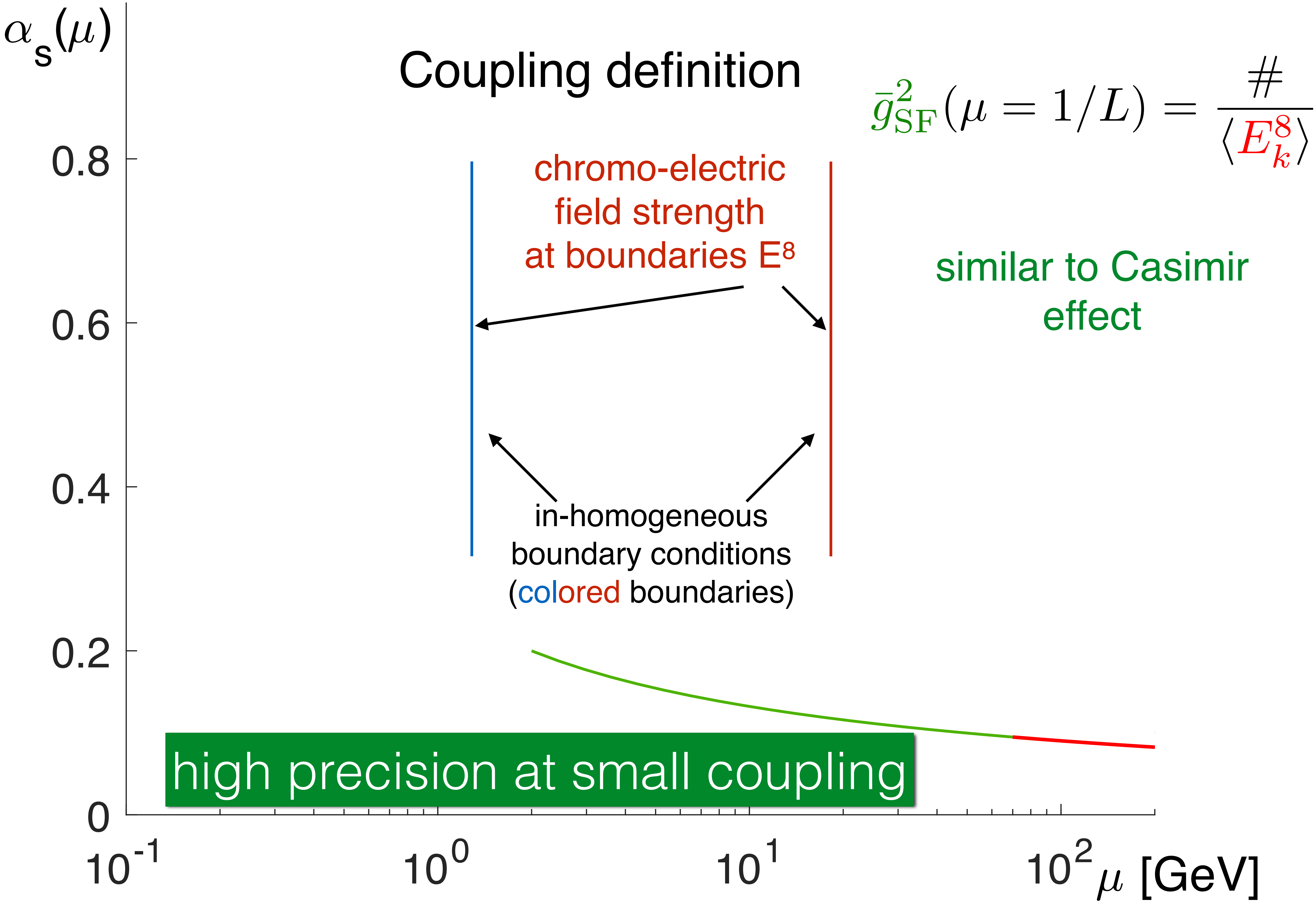
Continuum



3. Running to large energy



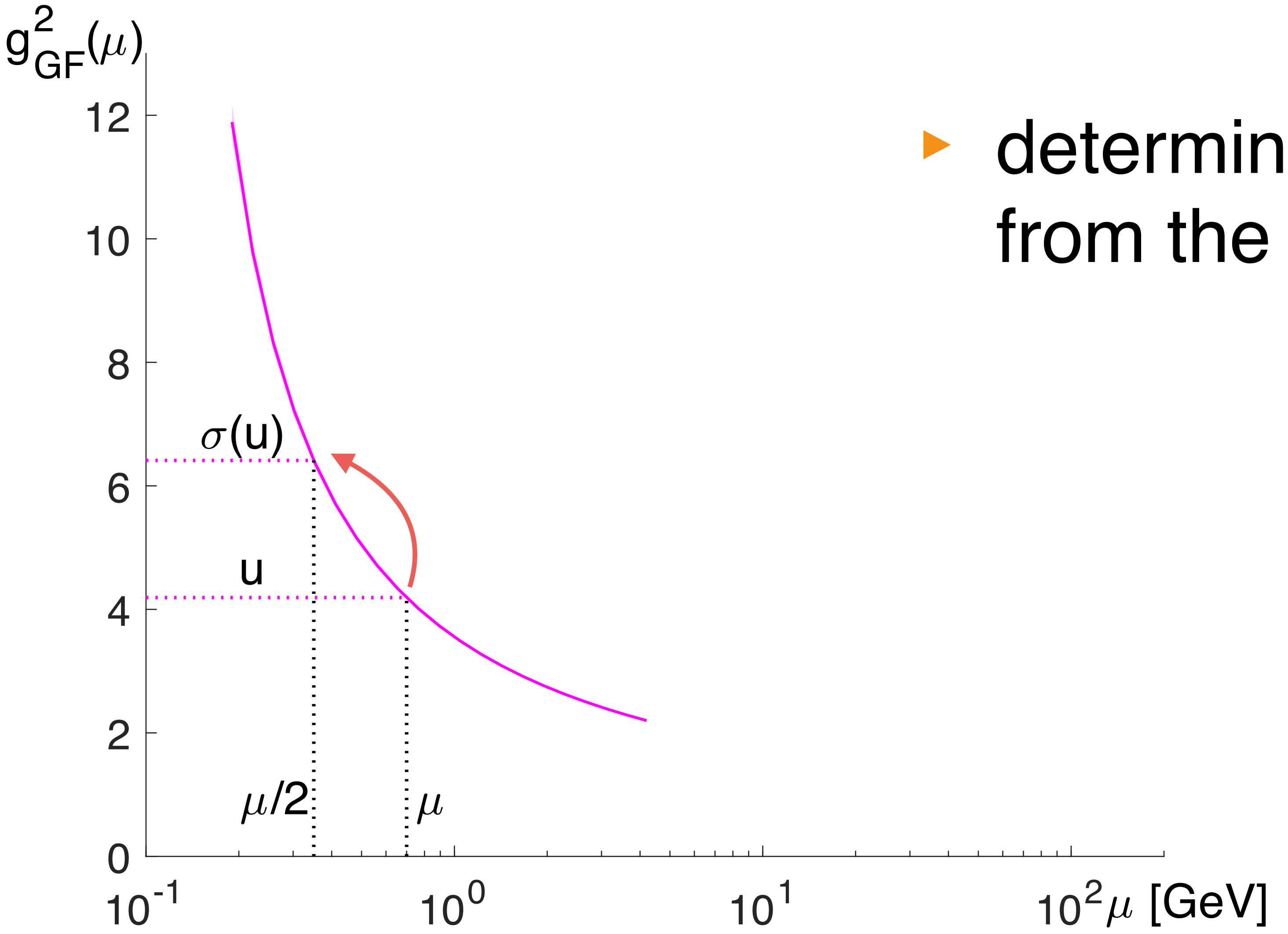
3. Running to large energy



The β-function from the step scaling function

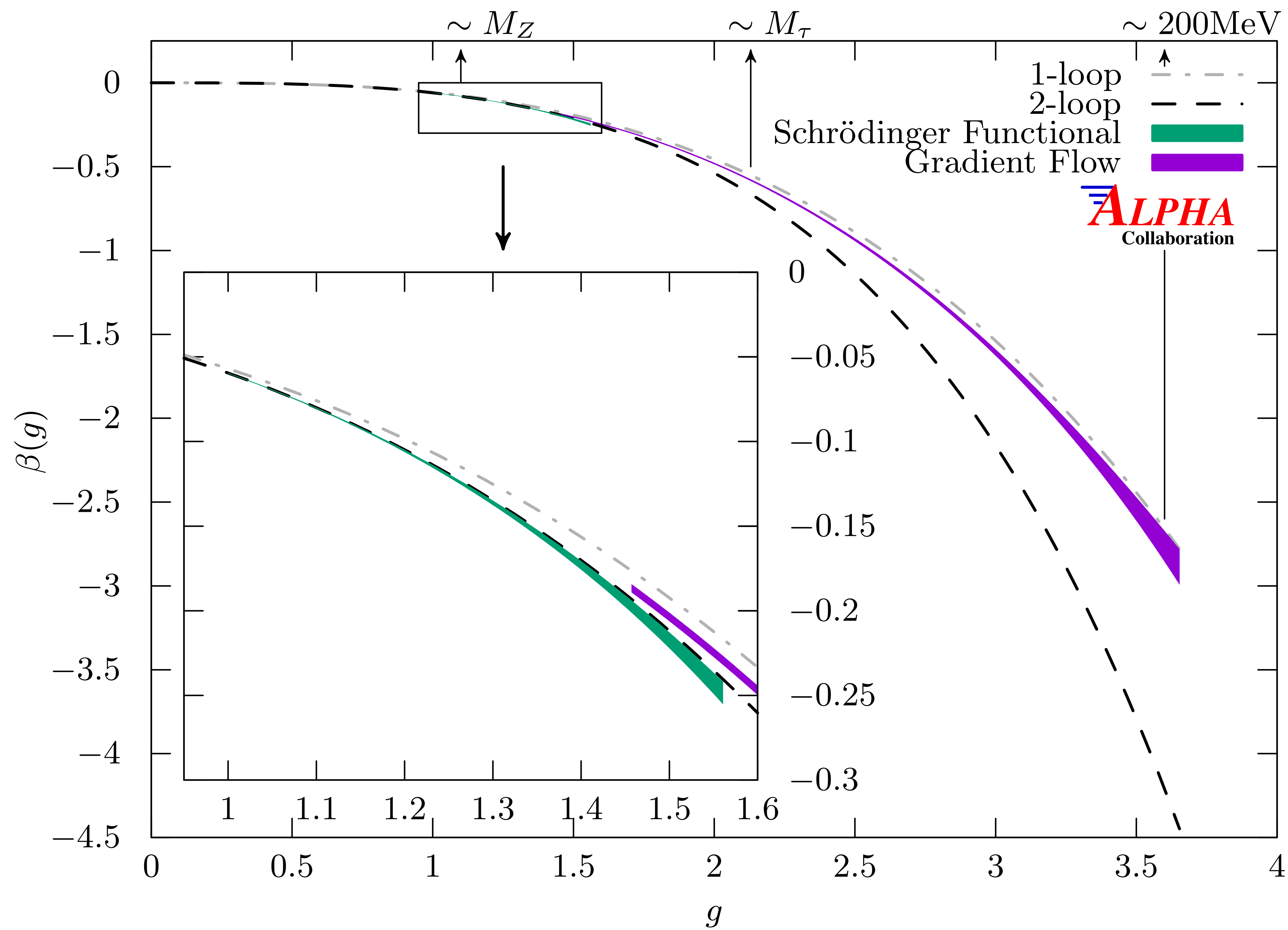
$$\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{-1}{\beta(x)} = \log\, 2$$

- smooth fit function for β(x)
- determine parameters in fit fct from the data points σ(u)



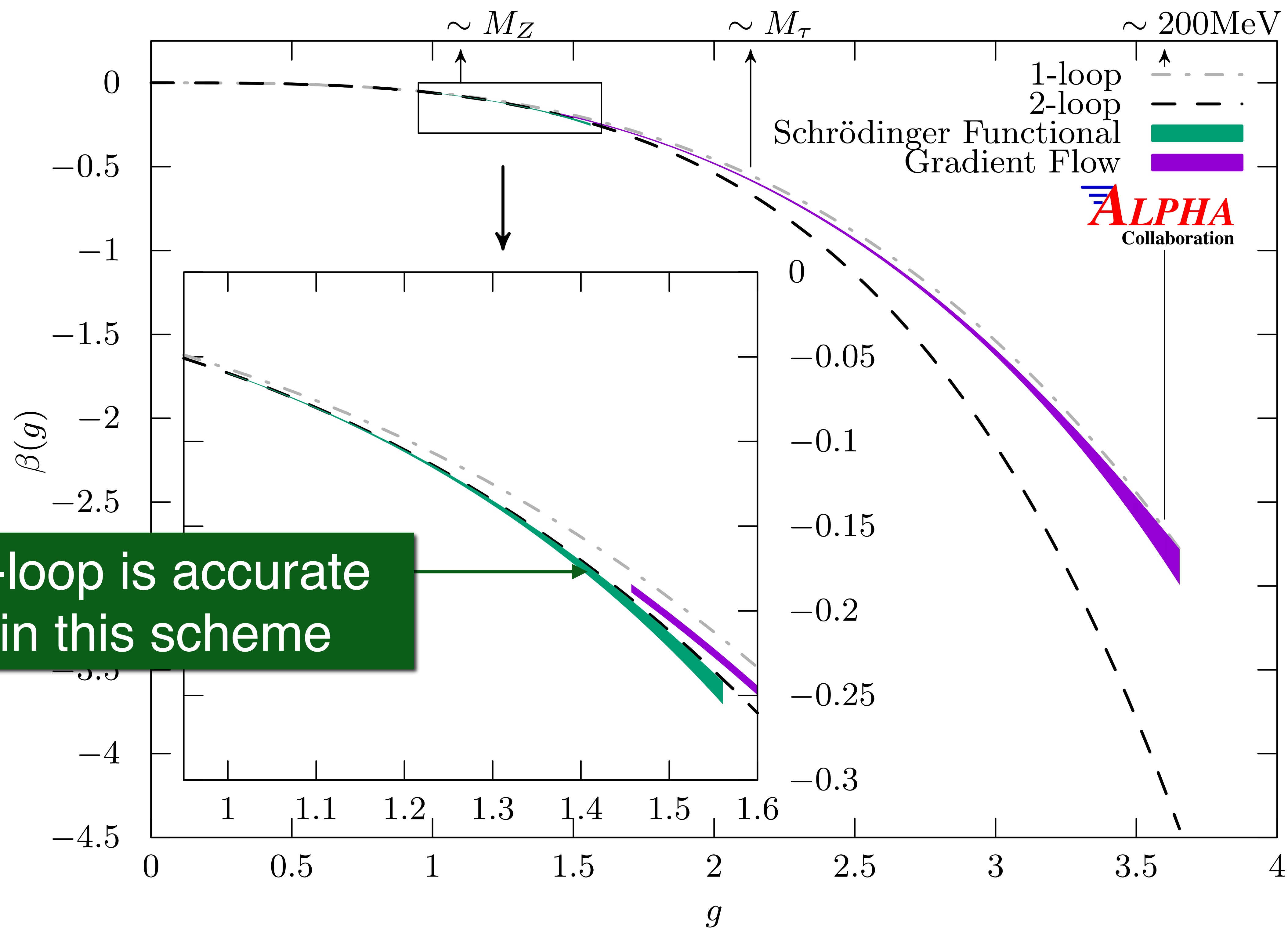
The non-perturbative β-functions

loop = order in g^2



The non-perturbative β-functions

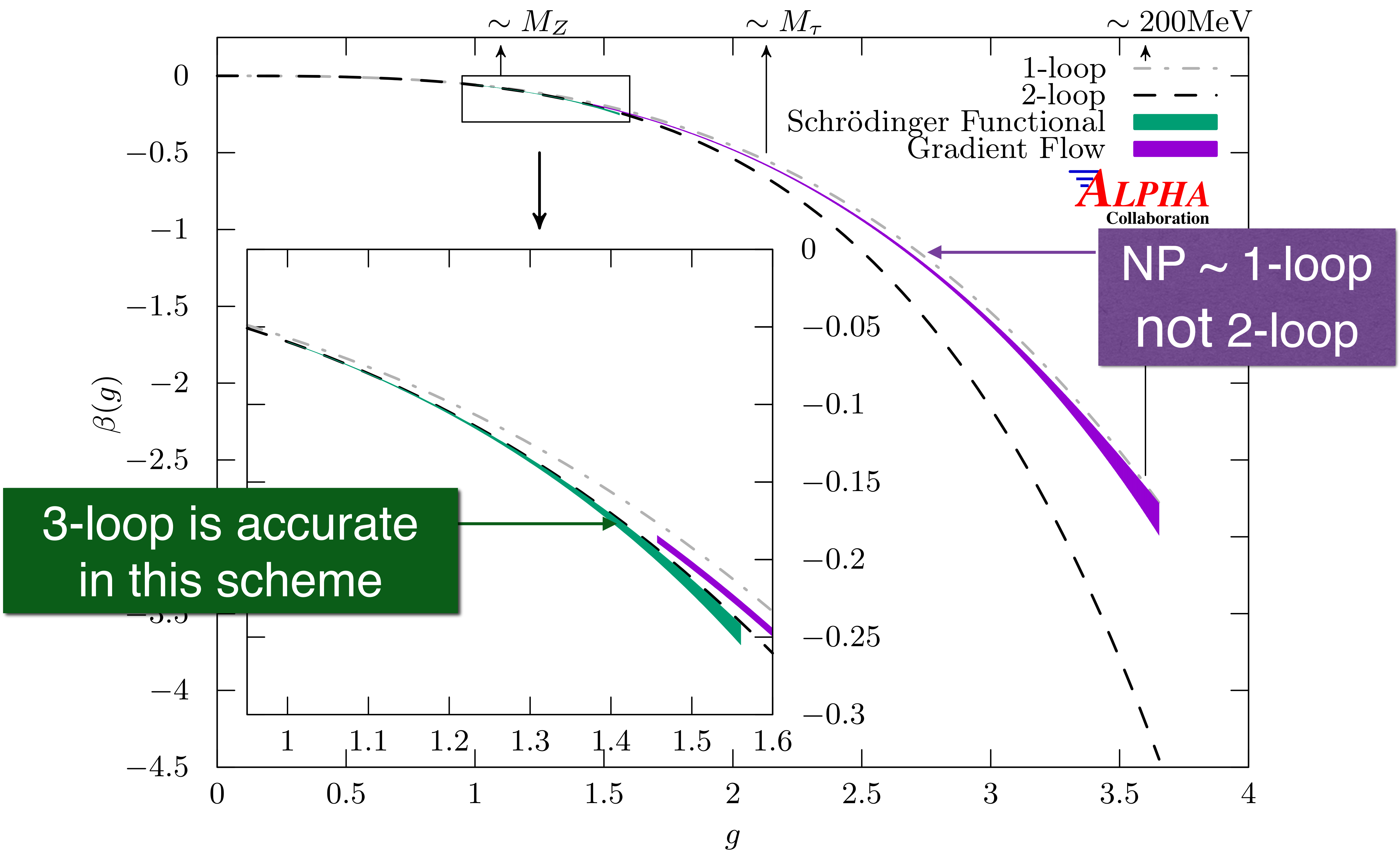
loop = order in g^2



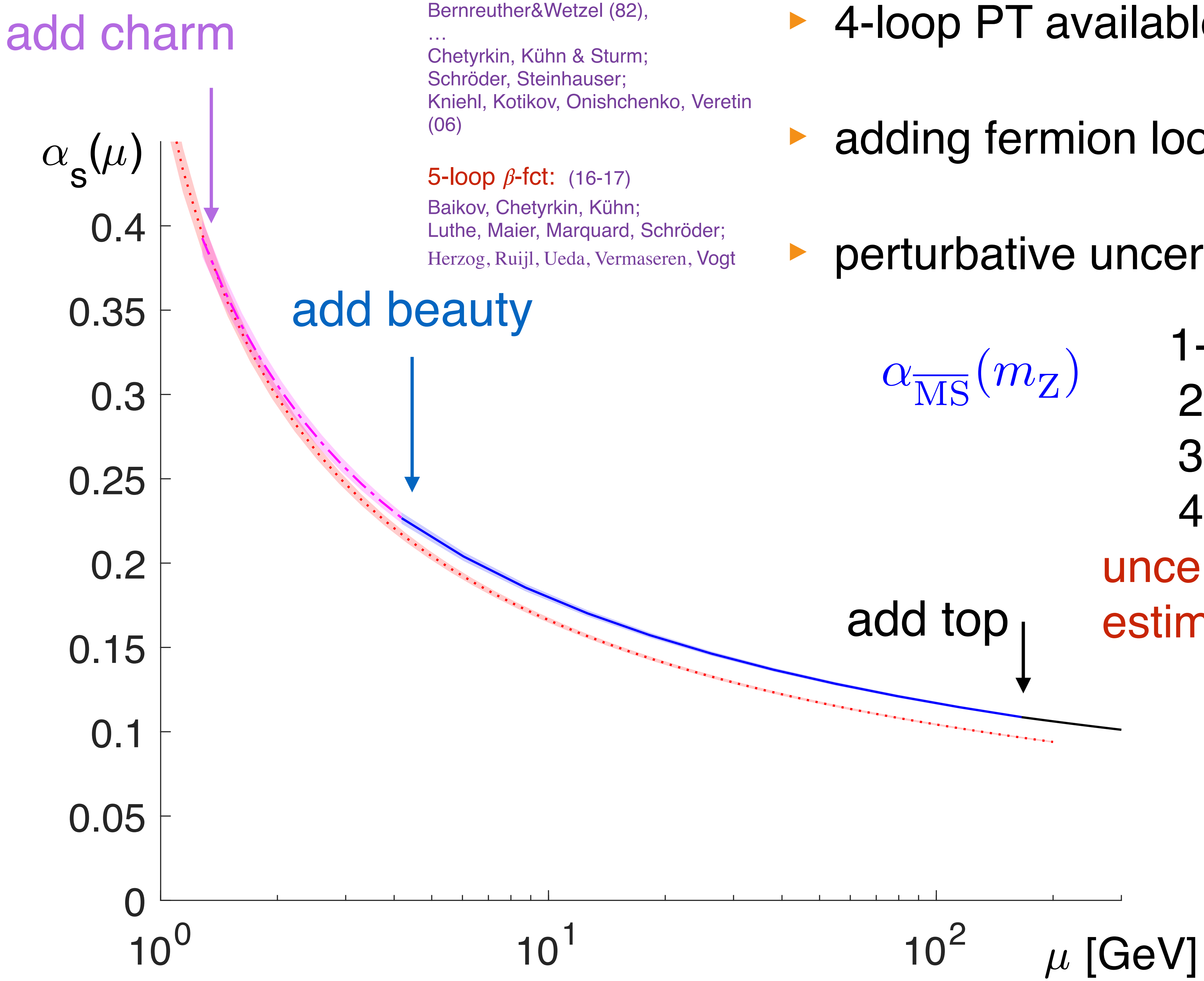
3-loop is accurate in this scheme

The non-perturbative β-functions

loop = order in g^2



Adding in c, b, t - quarks by perturbation theory



Weinberg (80),
Bernreuther&Wetzel (82),
…
Chetyrkin, Kühn & Sturm;
Schröder, Steinhauser;
Kniehl, Kotikov, Onishchenko, Veretin (06)

5-loop β-fct: (16-17)
Baikov, Chetyrkin, Kühn;
Luthe, Maier, Marquard, Schröder;
Herzog, Ruijl, Ueda, Vermaseren, Vogt

- 4-loop PT available
- adding fermion loops, “only”
- perturbative uncertainties are tiny

$\alpha_{\overline{\mathrm{MS}}}(m_Z)$

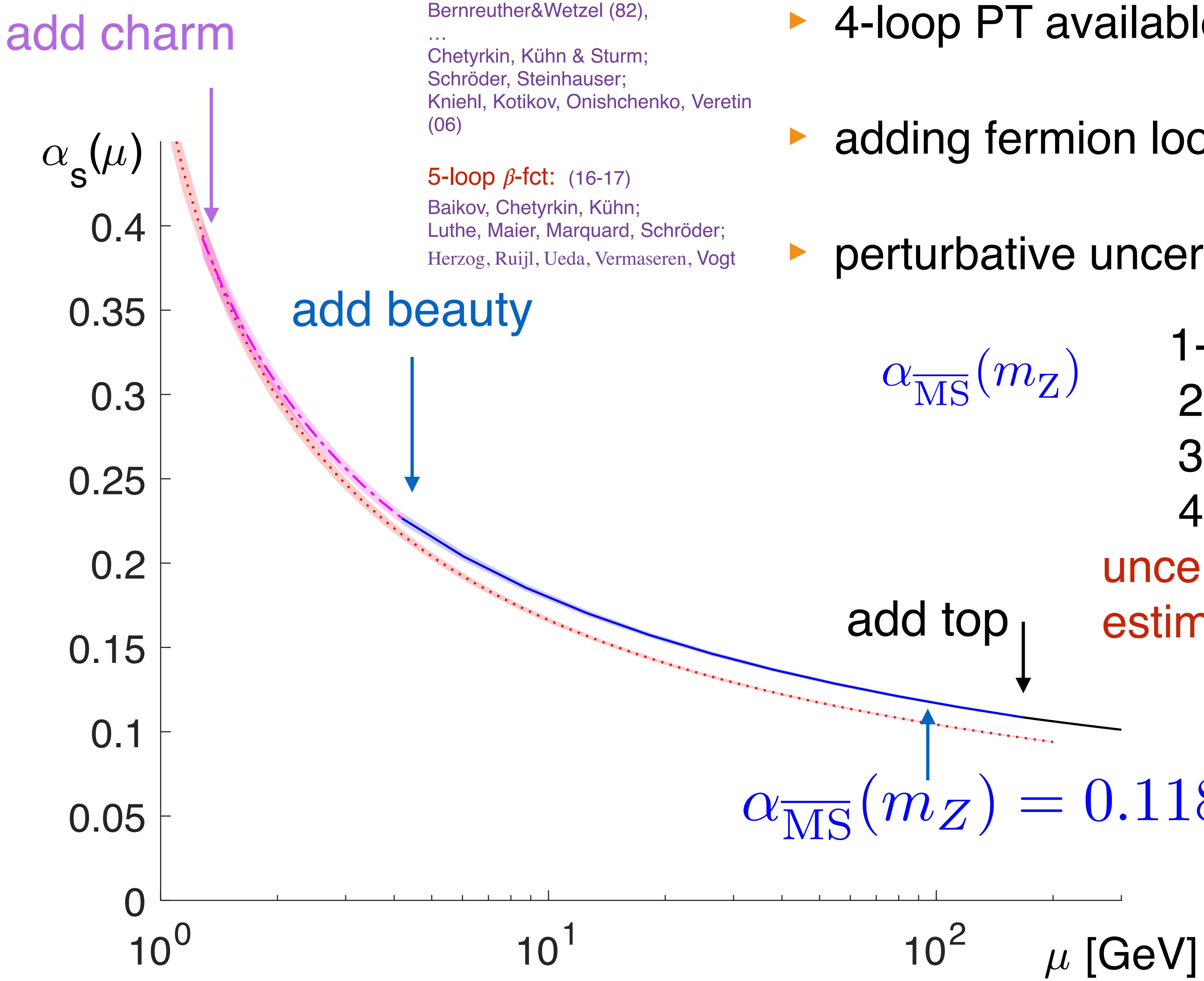
1-loop:	0.11701
2	0.00128
3	0.00019
4	0.00006

uncertainty estimate= 0.00025

Adding in c, b, t - quarks by perturbation theory

Weinberg (80),
Bernreuther&Wetzel (82),
…
Chetyrkin, Kühn & Sturm;
Schröder, Steinhauser;
Kniehl, Kotikov, Onishchenko, Veretin (06)

5-loop β-fct: (16-17)
Baikov, Chetyrkin, Kühn;
Luthe, Maier, Marquard, Schröder;
Herzog, Ruijl, Ueda, Vermaseren, Vogt



- 4-loop PT available
- adding fermion loops, "only"
- perturbative uncertainties are tiny

$\alpha_{\overline{\mathrm{MS}}}(m_Z)$

1-loop:	0.11701
2	0.00128
3	0.00019
4	0.00006

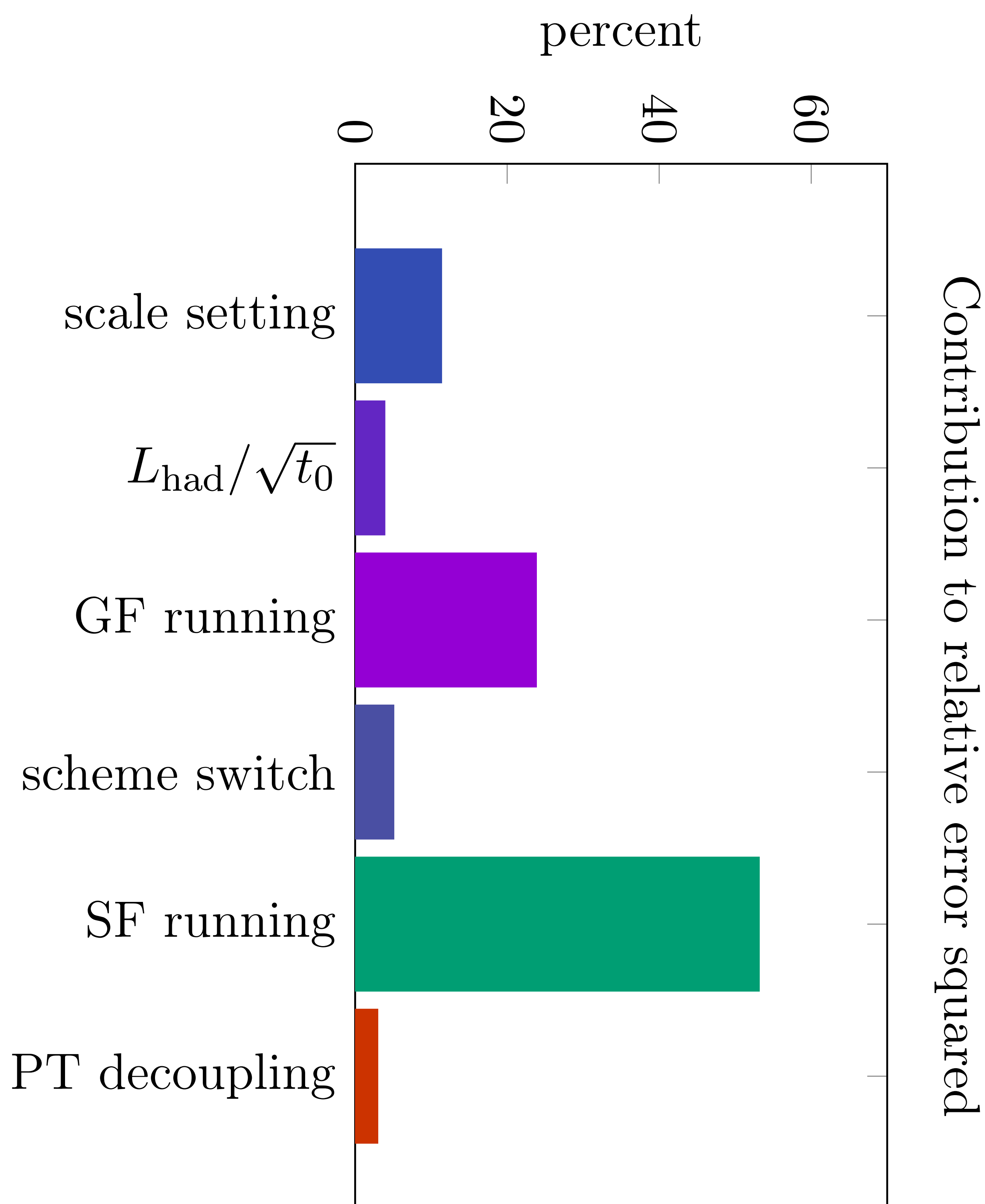
uncertainty estimate= 0.00025

$$\alpha_{\overline{\mathrm{MS}}}(m_Z) = 0.1185(8)(3)$$

Error budget

Scales	MeV
$1/L_\infty$	50
$f_\pi, f_K, m_\pi \ldots \sqrt{8t_0}$	150 -500
$\mu_{had} = 1/L_{had}$	200
μ_{swi}	4000
μ_{PT}	70000

error budget of our computation
(contribution to err^2)



dominated by non-perturbative running

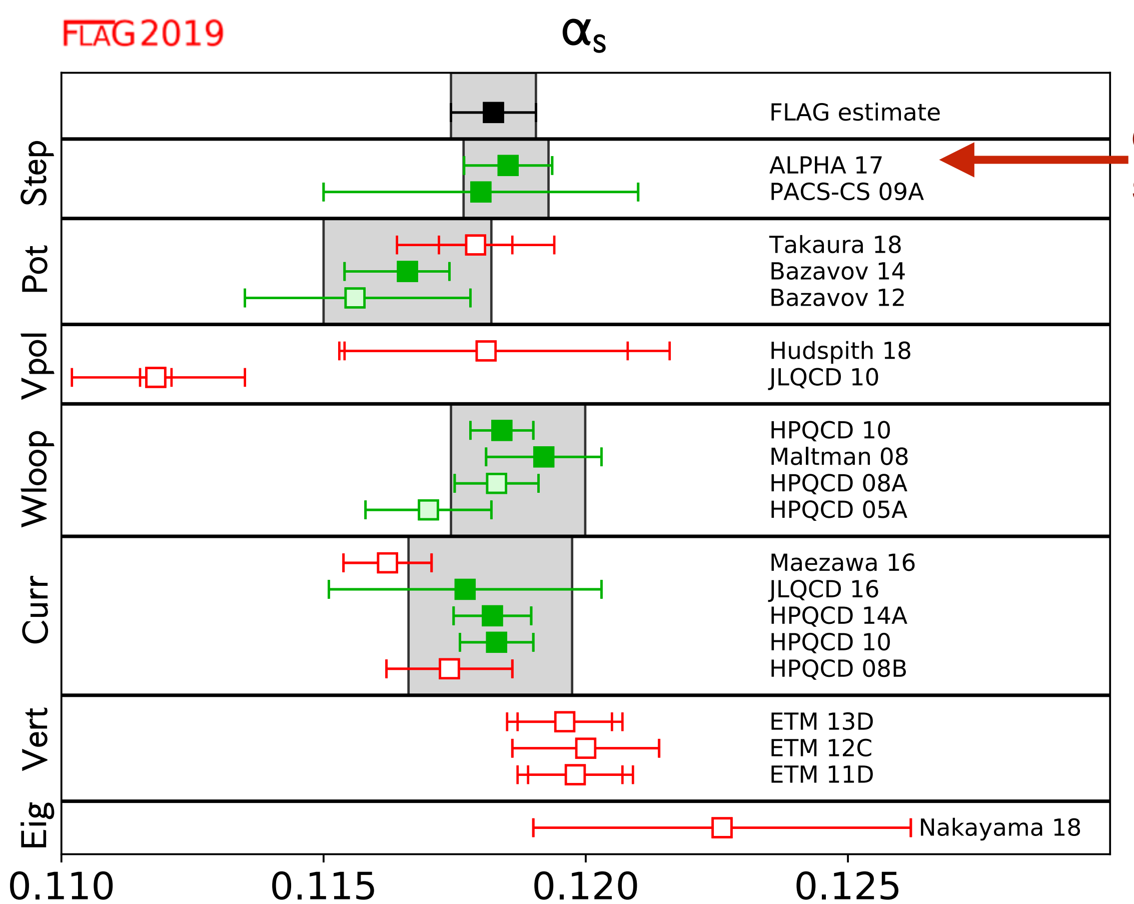
The result in comparison

FLAG Review 2019

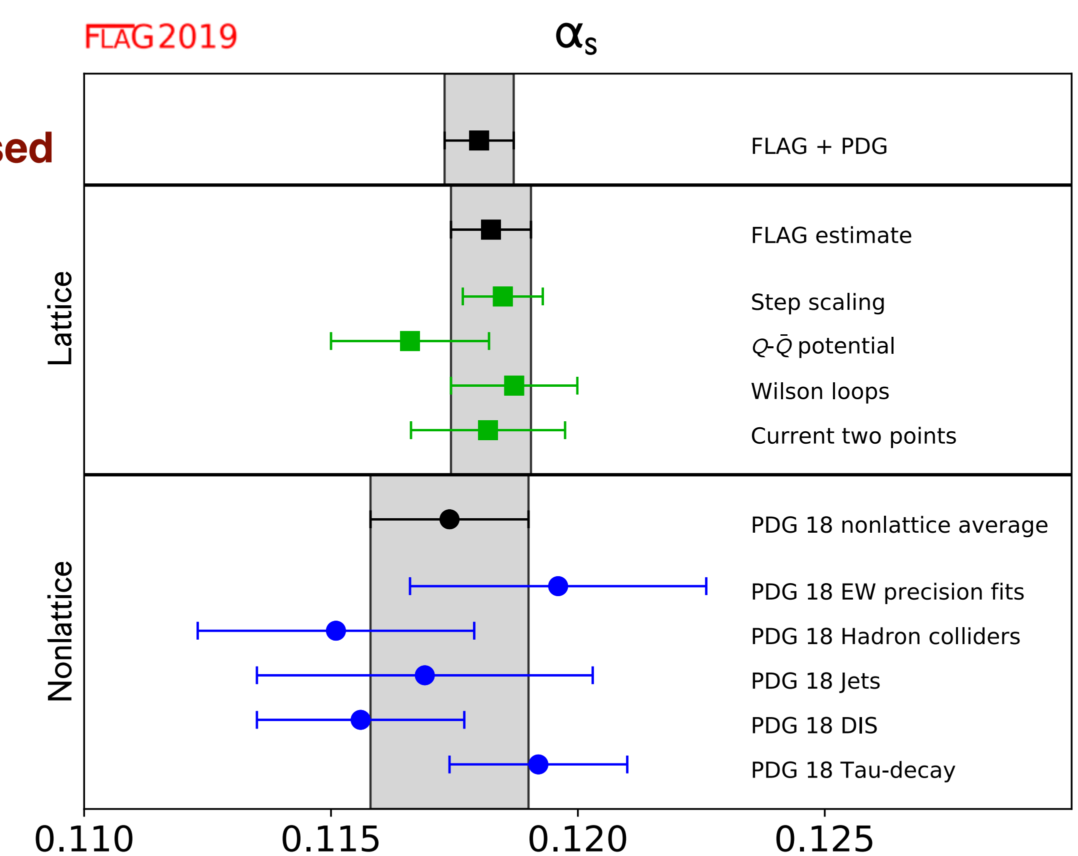
March 5, 2019

Flavour Lattice Averaging Group (FLAG)

S. Aoki,[1] Y. Aoki,[2,3] * D. Bečirević,[4] T. Blum,[5,3] G. Colangelo,[6] S. Collins,[7] M. Della Morte,[8]
P. Dimopoulos,[9] S. Dürr,[10] H. Fukaya,[11] M. Golterman,[12] Steven Gottlieb,[13] R. Gupta,[14]
S. Hashimoto,[2,15] U. M. Heller,[16] G. Herdoiza,[17] R. Horsley,[18] A. Jüttner,[19] T. Kaneko,[2,15]
C.-J. D. Lin,[20,21] E. Lunghi,[13] R. Mawhinney,[22] A. Nicholson,[23] T. Onogi,[11] C. Pena,[17] A. Portelli,[18]
A. Ramos,[24] S. R. Sharpe,[25] J. N. Simone,[26] S. Simula,[27] R. Sommer,[28,29] R. Van De Water,[26]
A. Vladikas,[30] U. Wenger,[6] H. Wittig[31]



discussed so far



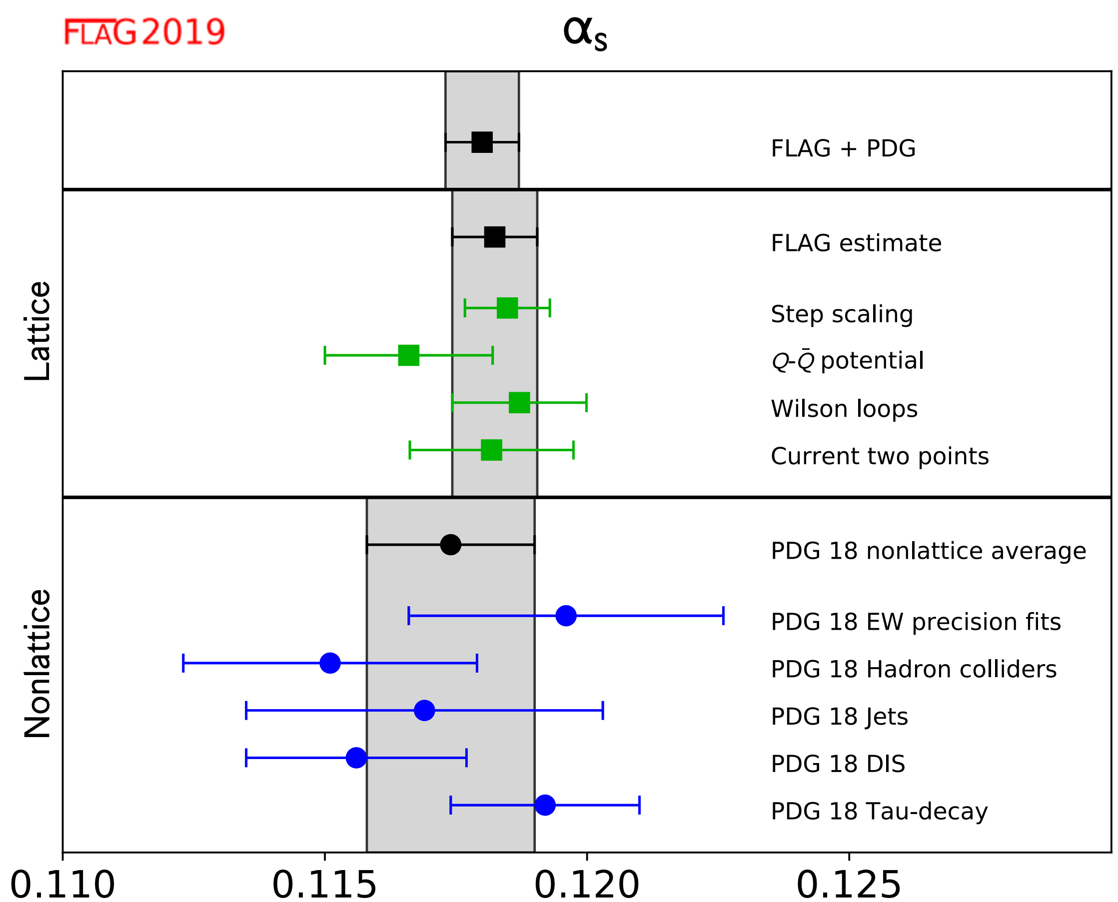
The result in comparison

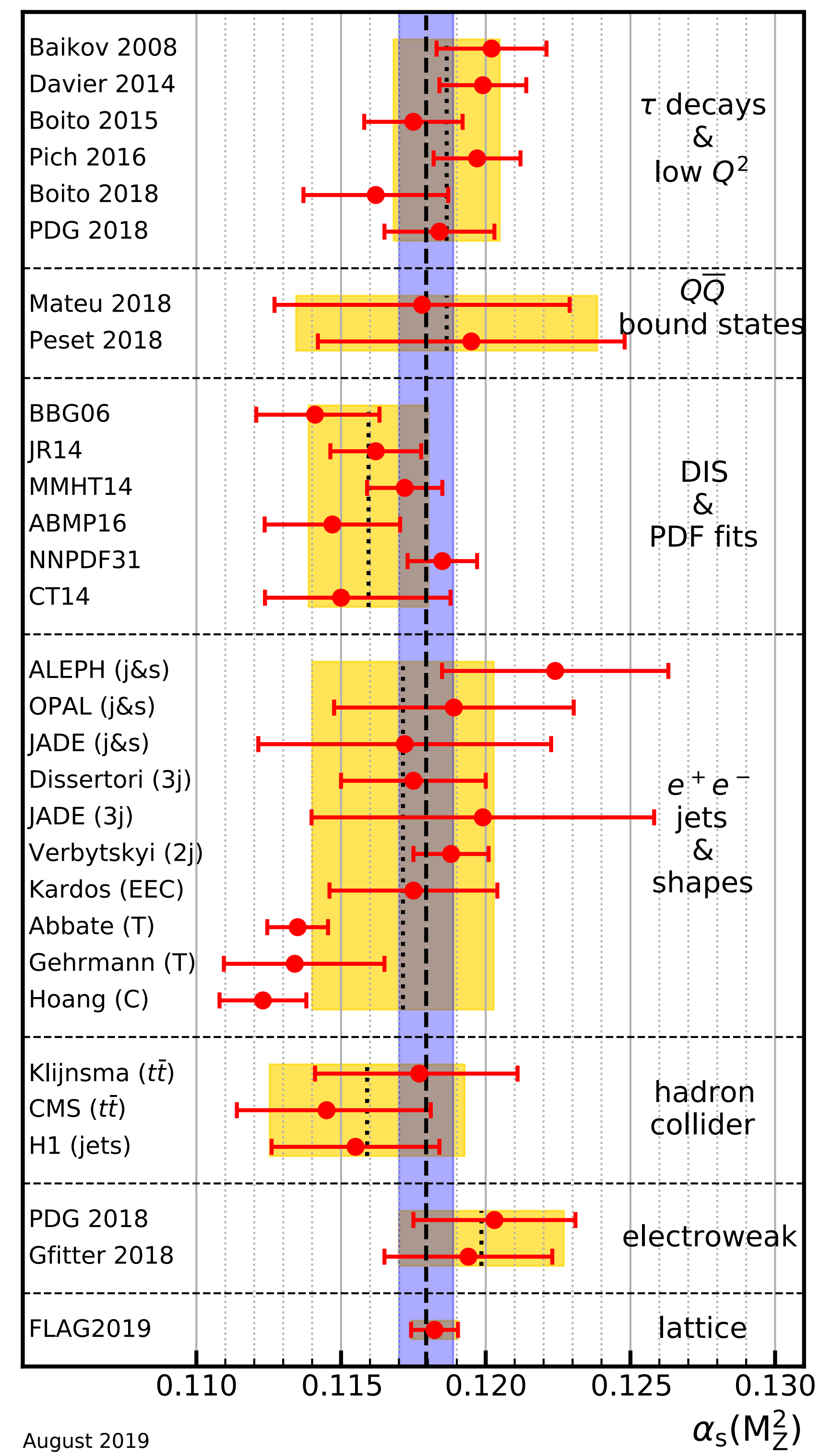
FLAG Review 2019

March 5, 2019

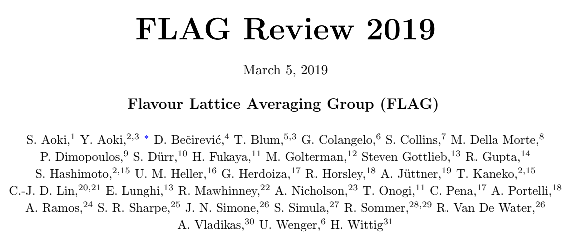
Flavour Lattice Averaging Group (FLAG)

S. Aoki,[1] Y. Aoki,[2,3] * D. Bečirević,[4] T. Blum,[5,3] G. Colangelo,[6] S. Collins,[7] M. Della Morte,[8] P. Dimopoulos,[9] S. Dürr,[10] H. Fukaya,[11] M. Golterman,[12] Steven Gottlieb,[13] R. Gupta,[14] S. Hashimoto,[2,15] U. M. Heller,[16] G. Herdoiza,[17] R. Horsley,[18] A. Jüttner,[19] T. Kaneko,[2,15] C.-J. D. Lin,[20,21] E. Lunghi,[13] R. Mawhinney,[22] A. Nicholson,[23] T. Onogi,[11] C. Pena,[17] A. Portelli,[18] A. Ramos,[24] S. R. Sharpe,[25] J. N. Simone,[26] S. Simula,[27] R. Sommer,[28,29] R. Van De Water,[26] A. Vladikas,[30] U. Wenger,[6] H. Wittig[31]





The result in comparison

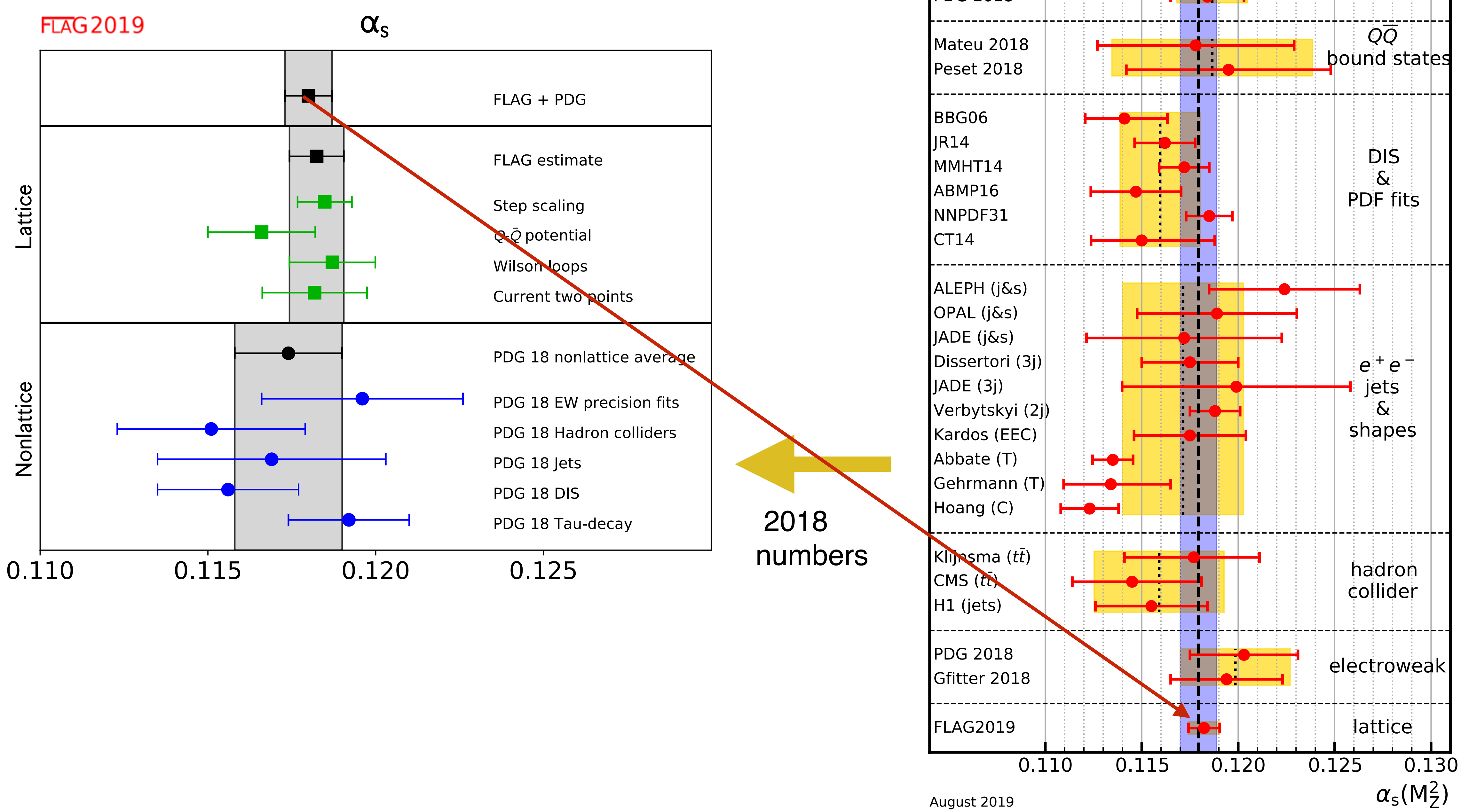


FLAG Review 2019

March 5, 2019

Flavour Lattice Averaging Group (FLAG)

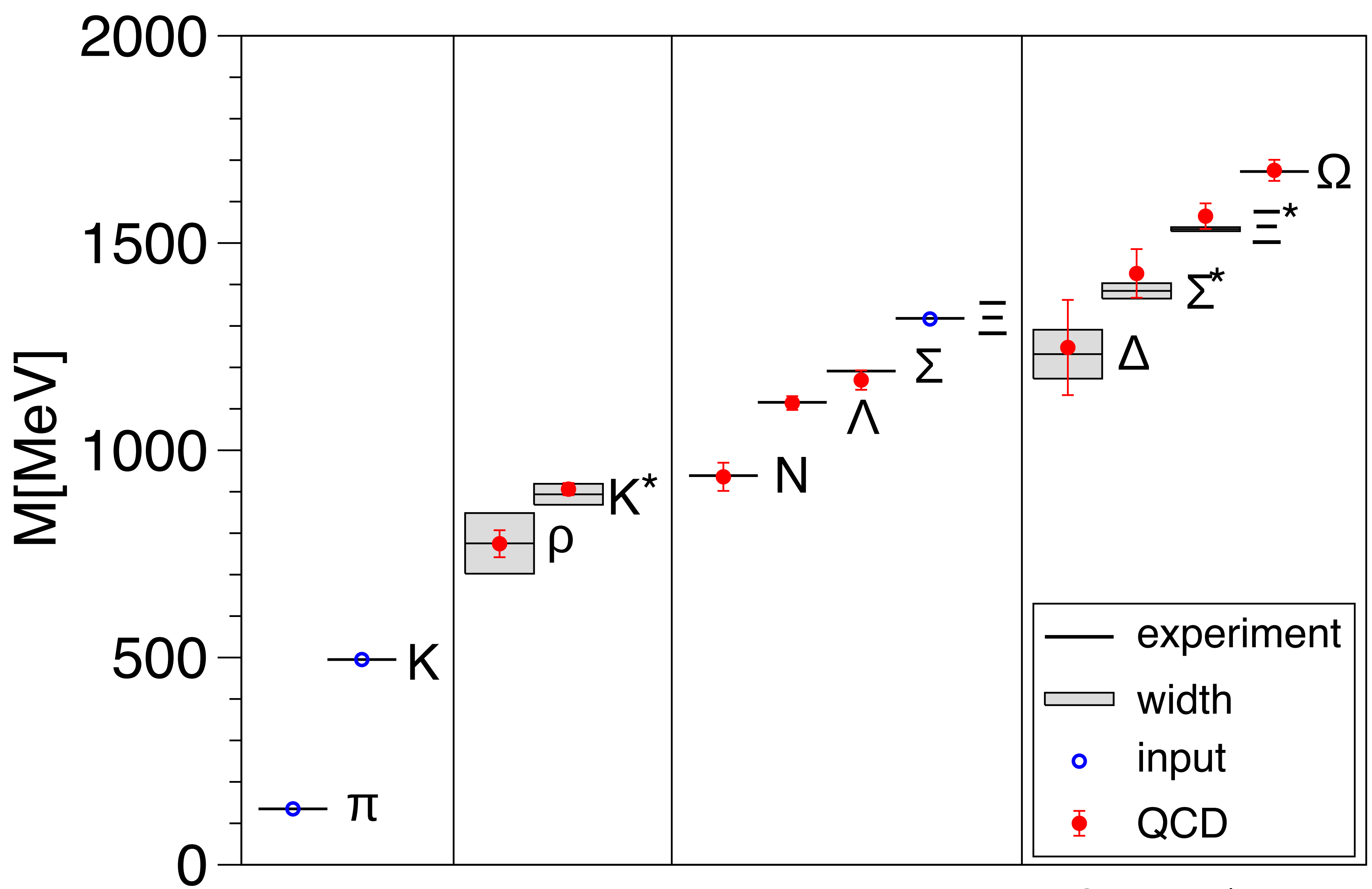
S. Aoki,[1] Y. Aoki,[2,3] * D. Bečirević,[4] T. Blum,[5,3] G. Colangelo,[6] S. Collins,[7] M. Della Morte,[8] P. Dimopoulos,[9] S. Dürr,[10] H. Fukaya,[11] M. Golterman,[12] Steven Gottlieb,[13] R. Gupta,[14] S. Hashimoto,[2,15] U. M. Heller,[16] G. Herdoiza,[17] R. Horsley,[18] A. Jüttner,[19] T. Kaneko,[2,15] C.-J. D. Lin,[20,21] E. Lunghi,[13] R. Mawhinney,[22] A. Nicholson,[23] T. Onogi,[11] C. Pena,[17] A. Portelli,[18] A. Ramos,[24] S. R. Sharpe,[25] J. N. Simone,[26] S. Simula,[27] R. Sommer,[28,29] R. Van De Water,[26] A. Vladikas,[30] U. Wenger,[6] H. Wittig[31]



Conclusions

- Lattice QCD, finite size techniques & high order PT
  → Control over strong interactions from lowest to highest energies

- Agreement with experiment → QCD valid at all energies



GF11
CP-PACS

BMW collaboration

Conclusions

- Lattice QCD, finite size techniques & high order PT
  → Control over strong interactions from lowest to highest energies

- Agreement with experiment → QCD valid at all energies

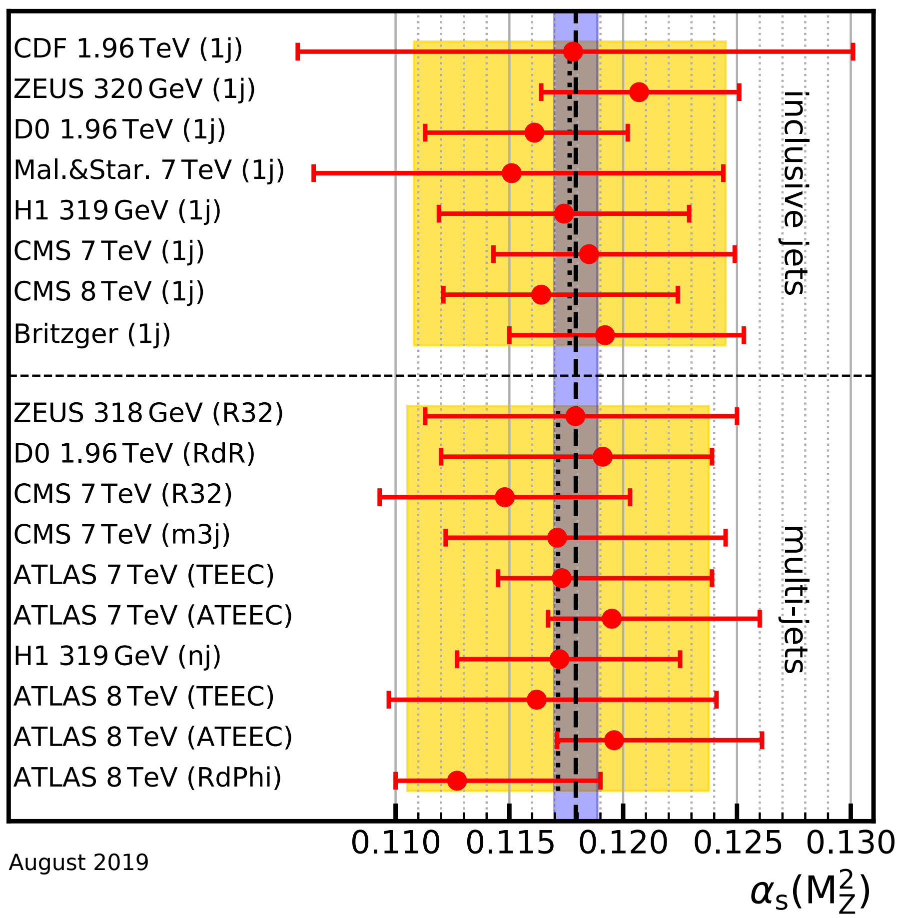


Figure 9.2: Summary of determinations of $\alpha_s(M_Z^2)$ at NLO from inclusive and multi-jet measurements at hadron colliders. The uncertainty is dominated by estimates of the impact of missing higher orders. The yellow (light shaded) bands and dotted lines indicate average values for the two sub-fields. The dashed line and blue (dark shaded) band represent the final world average value of $\alpha_s(M_Z^2)$.

Conclusions

- Lattice QCD, finite size techniques & high order PT
  → Control over strong interactions from lowest to highest energies
- Agreement with experiment → QCD valid at all energies
- Below 1% accuracy for $\alpha(m_Z)$
  → precision input for LHC, vacuum stability, BSM searches
- at α=0.1: PT is accurate
- at α=0.2: examples where PT is not accurate (not discussed here)

The Λ-parameter

- The $\boldsymbol{\Lambda}$-parameter

$$\Lambda = \mu \times \left(b_0 \bar{g}^2(\mu)\right)^{-b_1/(2b_0^2)} e^{-1/(2b_0\bar{g}^2(\mu))}$$
$$\times \exp\left\{-\int_0^{\bar{g}(\mu)} dx \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$

- is a renormalization group invariant (constant)

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\Lambda = 0$$

- With known, β-function, it is equivalent to $\boldsymbol{\alpha}(\mu)$

New strategy based on decoupling of heavy quarks

sketch of history

- Weinberg, …, Bernreuther+Wetzel, … Chetyrkin et al.
  N_f - dependent effective theory / effective coupling
  in mass-independent renormalization schemes
  4-loop relations
  note:
  $\Lambda_3 \ll M_{charm} \ll M_{bottom} \ll M_{top}$
  $\sim$ 0.3GeV 1 GeV 6 GeV 170 GeV

- Wuppertal+NIC group (2014 - 2019)
  charm-quark-mass dependence of low energy mass scales (e.g. nucleon mass can quantitatively be predicted
  by (above) pert. theory.

- now: turn the tables: predict Λ_3 from Λ_0 and low energy scale

New strategy based on decoupling

Physics Letters B 807 (2020) 135571

Contents lists available at ScienceDirect

Physics Letters B

ELSEVIER

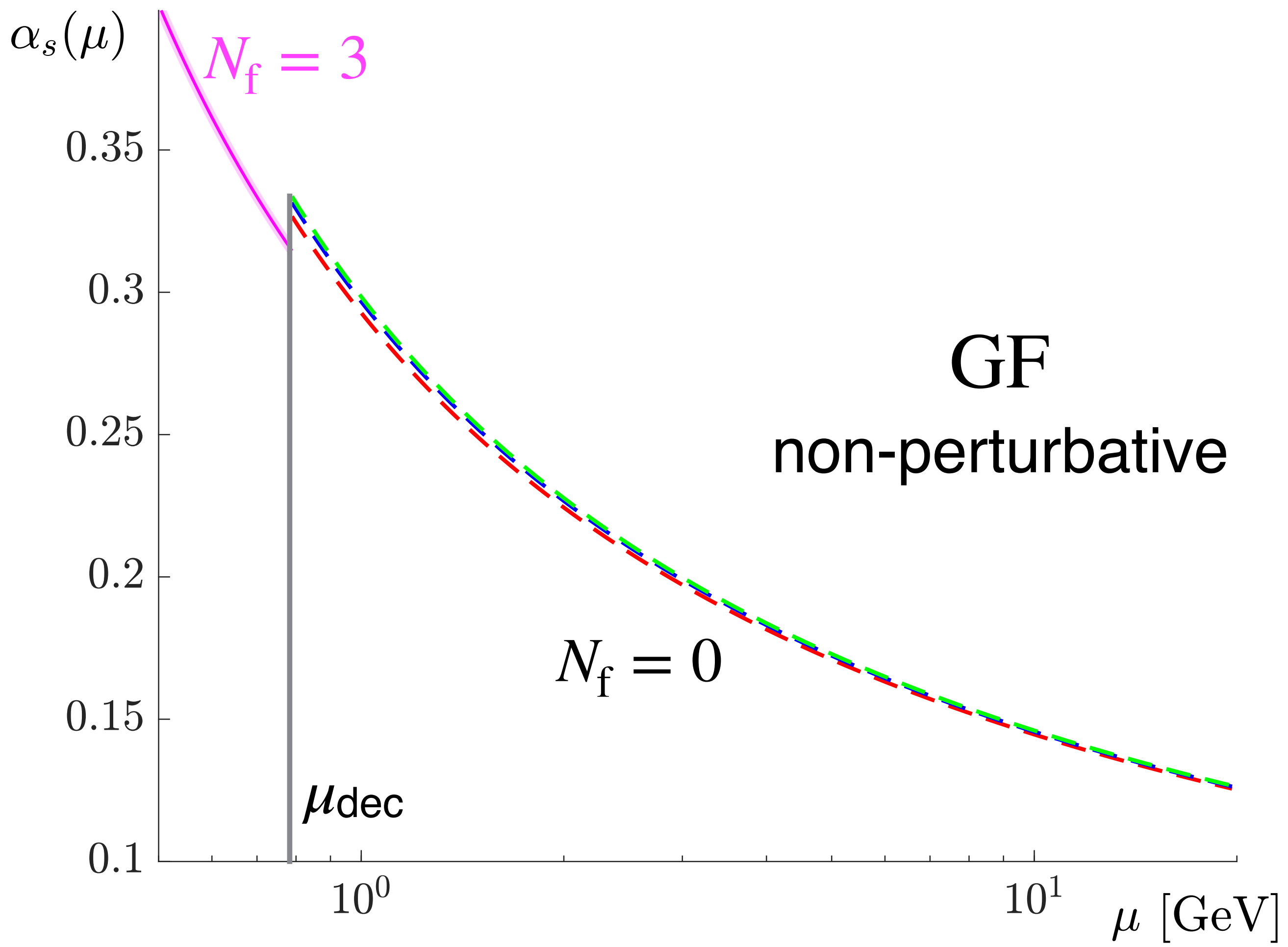
www.elsevier.com/locate/physletb

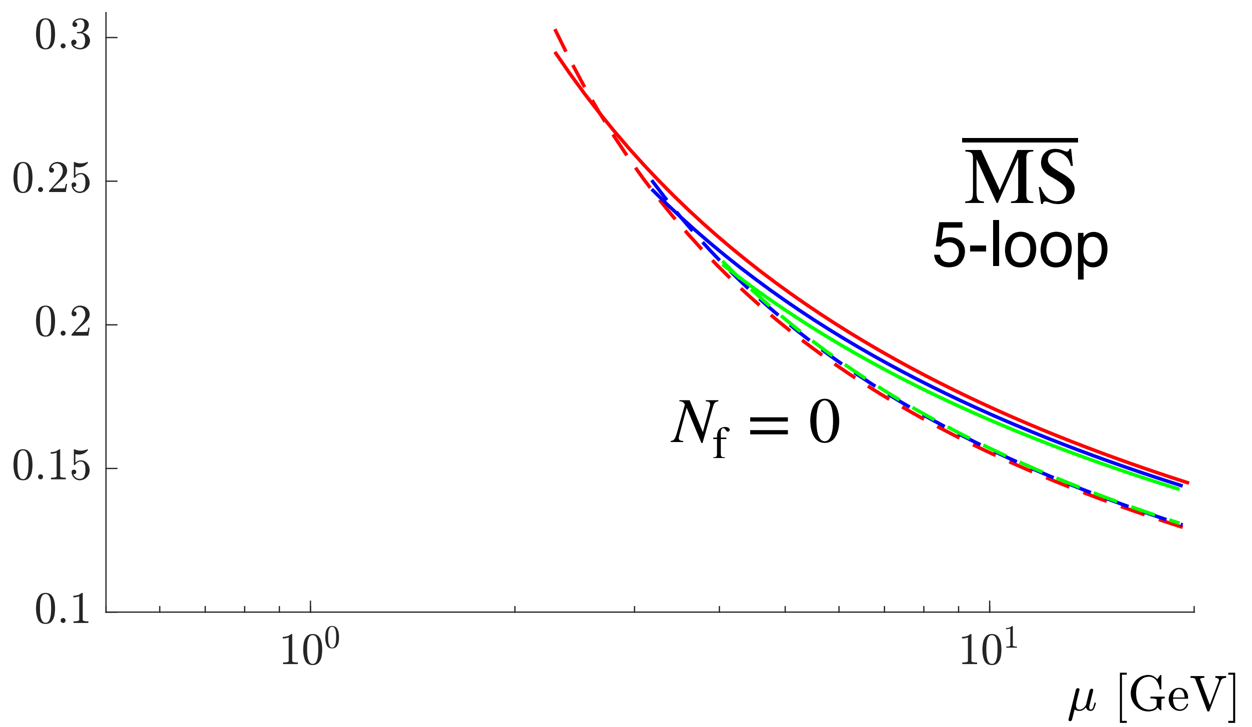
Non-perturbative renormalization by decoupling

ALPHA Collaboration

Mattia Dalla Brida [a], Roman Höllwieser [b], Francesco Knechtli [b], Tomasz Korzec [b], Alberto Ramos [c], Rainer Sommer [d,e,*]

New strategy based on decoupling in a nut shell



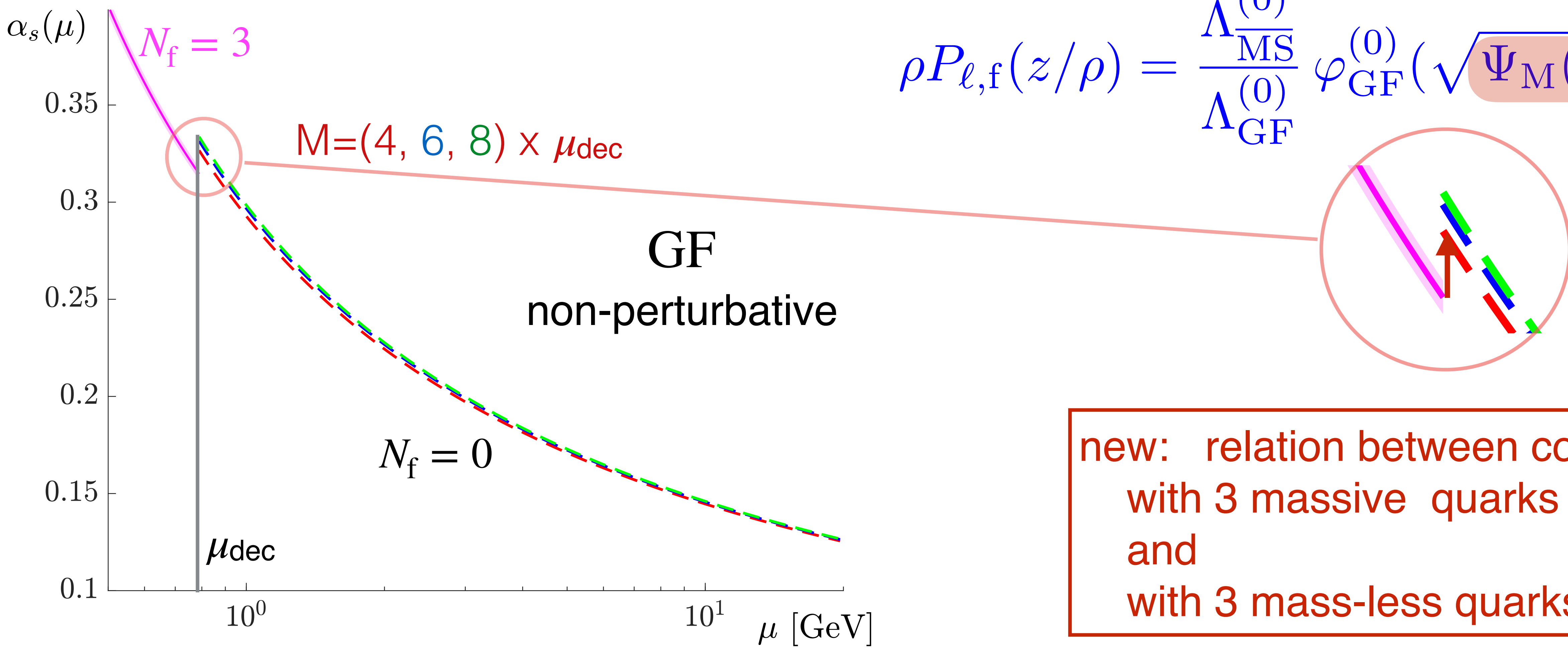


$$\rho P_{\ell,\mathrm{f}}(z/\rho) = \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\Lambda^{(0)}_{\mathrm{GF}}}\,\varphi^{(0)}_{\mathrm{GF}}\big(\sqrt{\Psi_{\mathrm{M}}(u_0, z)}\big)$$

$$\rho = \frac{\Lambda^{(3)}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}}\,.$$

$z = M/\mu_{\mathrm{dec}}$

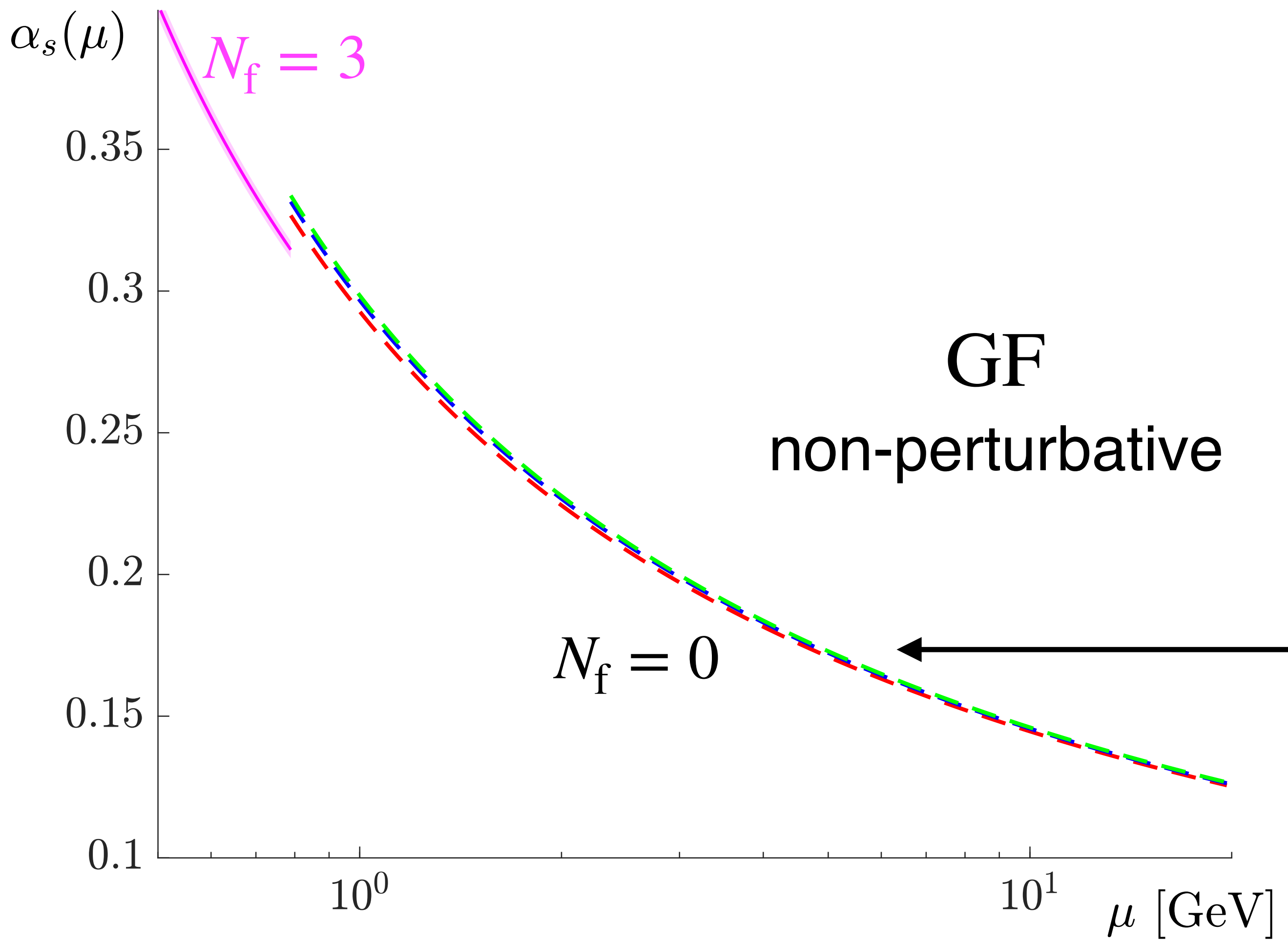
New strategy based on decoupling



$$\rho P_{\ell,\mathrm{f}}(z/\rho) = \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\Lambda^{(0)}_{\mathrm{GF}}}\,\varphi^{(0)}_{\mathrm{GF}}\left(\sqrt{\Psi_\mathrm{M}(u_0, z)}\right)$$

new: relation between couplings
with 3 massive quarks
and
with 3 mass-less quarks

New strategy based on decoupling

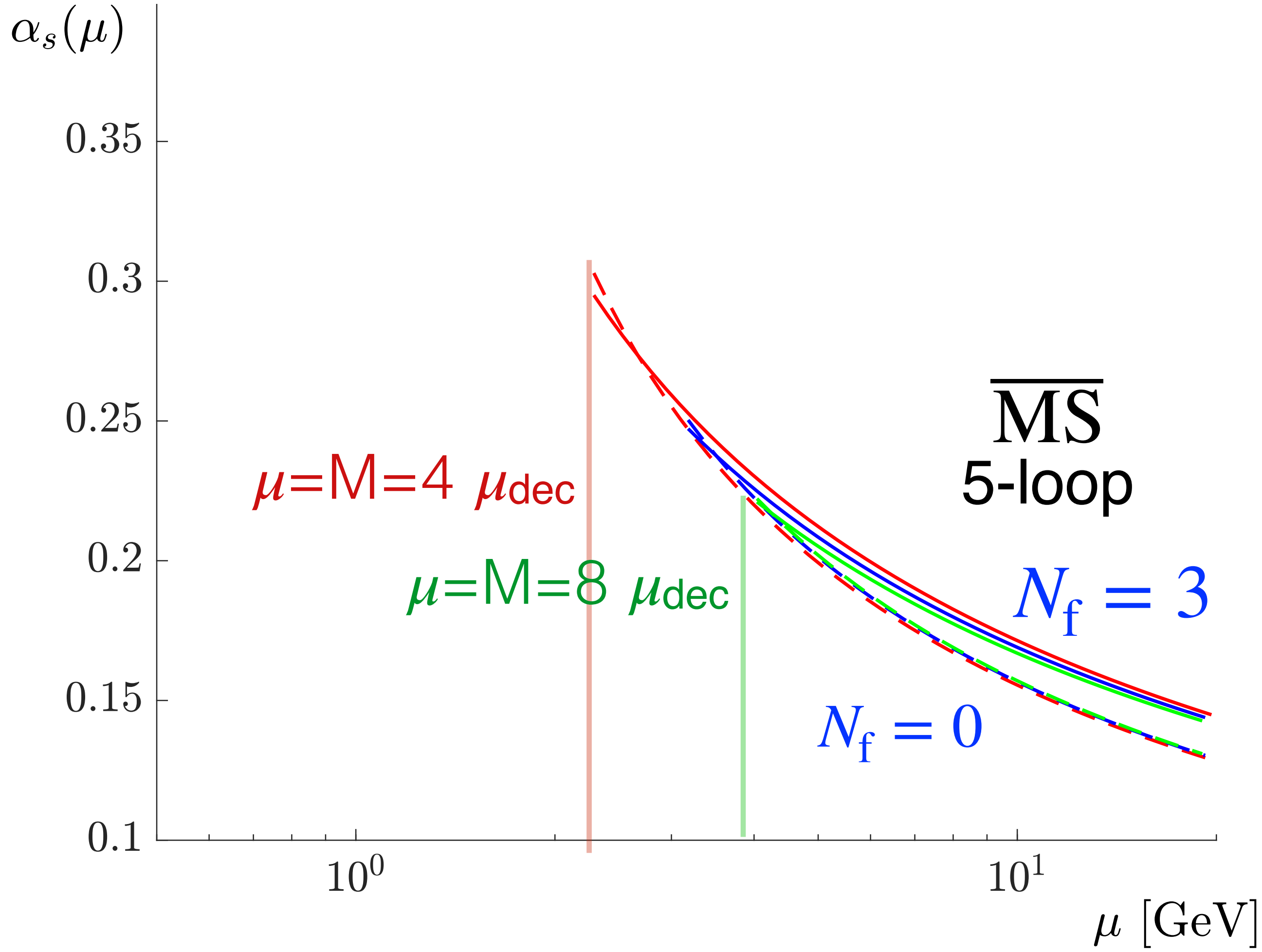


$$\rho P_{\ell,\mathrm{f}}(z/\rho) = \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\Lambda^{(0)}_{\mathrm{GF}}} \, \varphi^{(0)}_{\mathrm{GF}}\left(\sqrt{\Psi_{\mathrm{M}}(u_0, z)}\right)$$

non-perturbative but well-known [Dalla Brida & Ramos] function given by $\boldsymbol{\beta}$-function in finite volume GF scheme

New strategy based on decoupling

$$\rho P_{\ell,\mathrm{f}}(z/\rho) = \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\Lambda^{(0)}_{\mathrm{GF}}}\,\varphi^{(0)}_{\mathrm{GF}}\big(\sqrt{\Psi_{\mathrm{M}}(u_0,z)}\big)$$

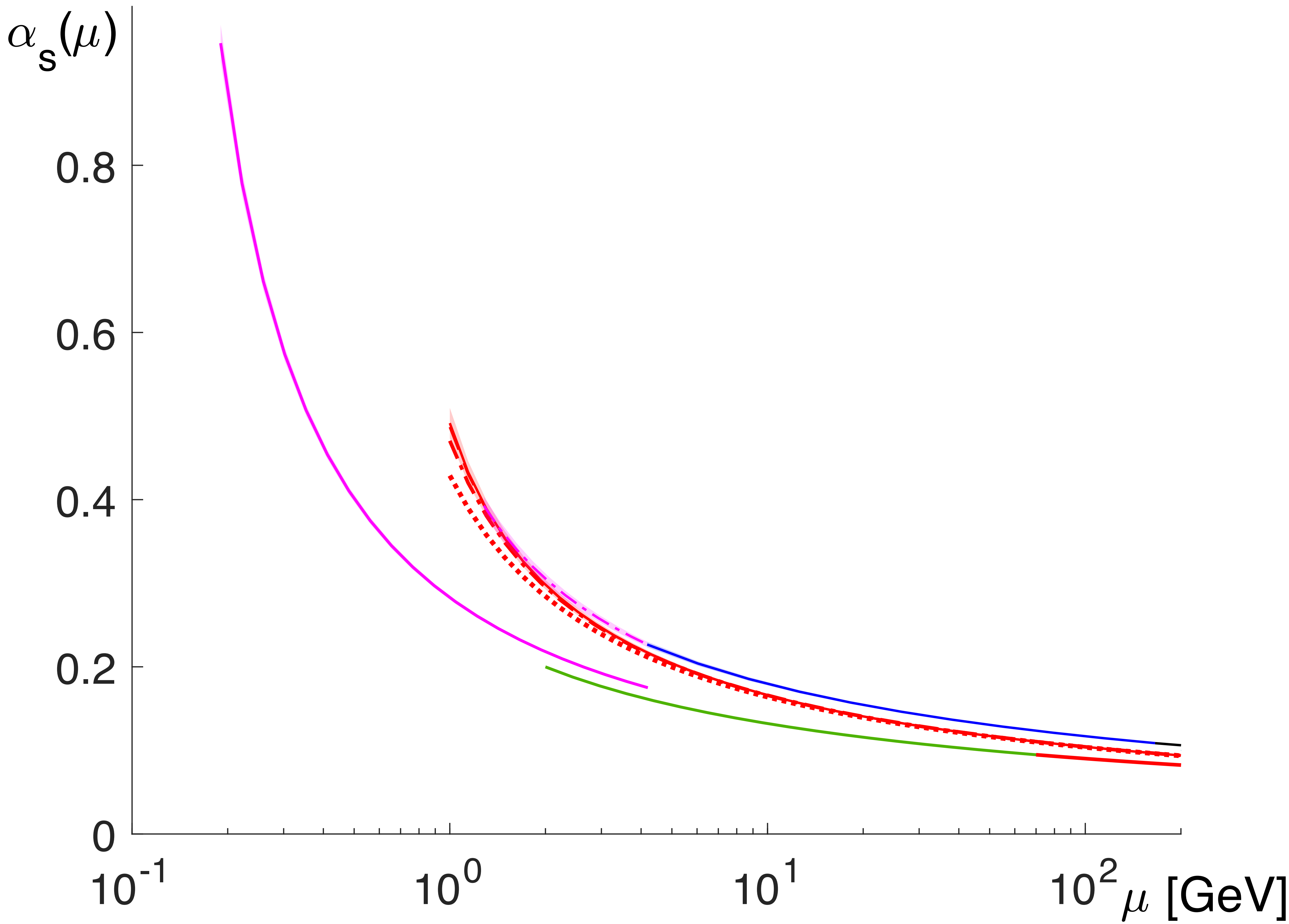


known perturbative function

New strategy based on decoupling

- very promising
- reduction of error by factor 0.5 seems reachable
- that will be good enough for a while to come
  (until there is a linear collider)

Thank you



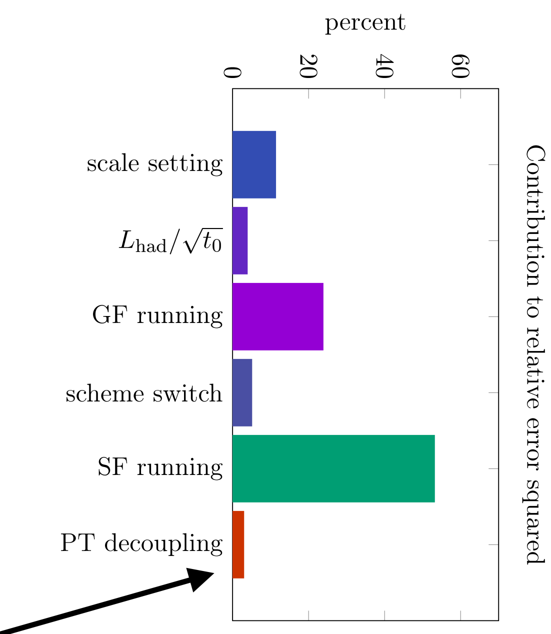
Additional

$N_f < 5$

- Only N_f=3 (or 4) are reached by direct computation
- Threshold matching QCD(N_f) and QCD(N_{f+1}) by perturbation theory
- Corrections are small in perturbation theory
- Exploratory NP investigation exists
  [M. Bruno, J. Finkenrath, F. Knechtli, B. Leder, R. S., 2015]

Be aware

- f_π, f_K depend on V_{ud}, V_{us}, and the SM



- perturbation theory for decoupling, $N_f=3 \rightarrow N_f=5$ looks great.

  can it be entirely misleading?
  then 0.0003 error would be wrong.

  non-perturbative tests have confirmed
  perturbation theory for decoupling with precision

A small warning about PT

A small warning about PT

Test where Λ is constant