SIGNAL FORMATION AND SIGNAL PROCESSING IN DETECTORS

LECTURES AT THE UNIVERSITY OF FREIBURG

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LECTURE 2

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Outline overview

1. What is a detector "signal"?

2. Charge transport in gases and solids

3. Induced signals on electrodes
   - Schottky-Ramo Theorem
   - Current, charge or voltage?
   - Applying Ramo to detectors
   - Structured electrodes
   - (calculation of E_W by "conformal mapping")

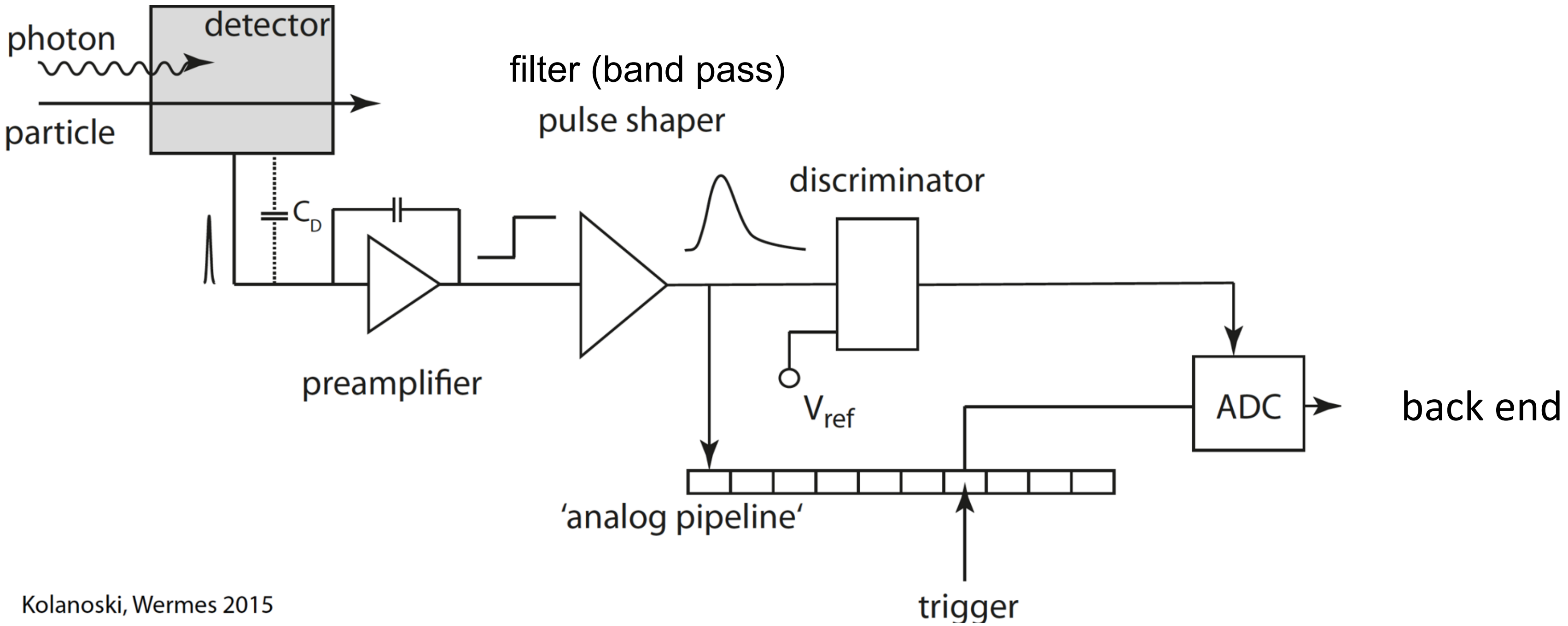
4. Signal fluctuations and (electronic) noise
   - Why bother?
   - Signal fluctuations (Fano noise)
   - Electronic noise

5. Readout of signals
   - Amplification
   - (Excursion: Laplace transform)
   - Filtering
   - Discrimination
   - Digitisation
   - (Example: a readout chip)

6. Signal transmission off detector

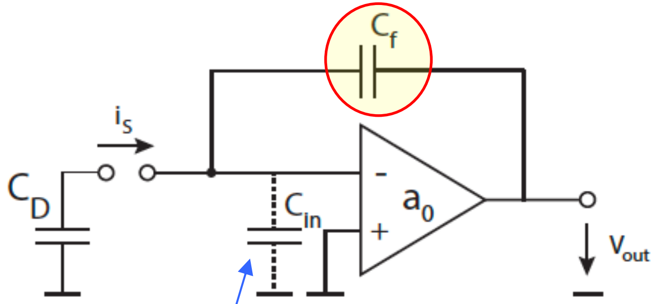
7. Noise of a readout system
   - Explicit calculation of noise
     - ATLAS pixel detector
     - ATLAS strip detector
     - ATLAS Liq. Argon calorimeter

Our typical system



Kolanoski, Wermes 2015

Remember: Charge Sensitive Amplifier (CSA)



(b) Capacitive feedback.

$$C_{in} = C_f\,(a_0 + 1)$$

dynamic input capacitance
(should be very large, else C_D incompletely discharged => unwanted x-talk possible)

charge (sensitive) amplifier (CSA)
(= current integrator)

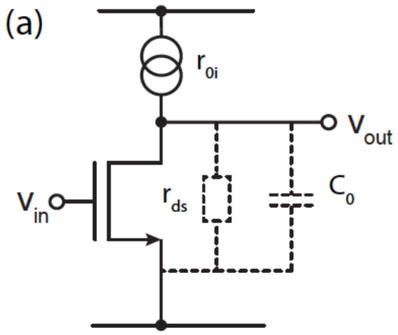
The signal current is integrated on C_f

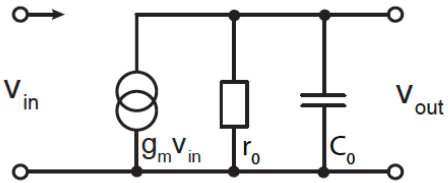
$$v_{out}(t) = -a_0\,v_{in}(t) = -\frac{1}{C_f}\int_0^t i_S\,dt' = -\frac{Q_S(t)}{C_f}$$

$$A_Q = \left|\frac{v_{out}}{Q_S}\right| \approx \frac{1}{C_f}$$

gain

Frequency behaviour





equivalent circuit

$$\boxed{a(\omega)} = -\frac{v_{out}}{v_{in}} = -\frac{i_{out}}{v_{in}} \cdot (r_0 \| C_0)$$

$$= -g_m \frac{r_0}{1 + i\omega r_0 C_0} \boxed{= \frac{a_0}{1 + i\frac{\omega}{\omega_c}}}$$

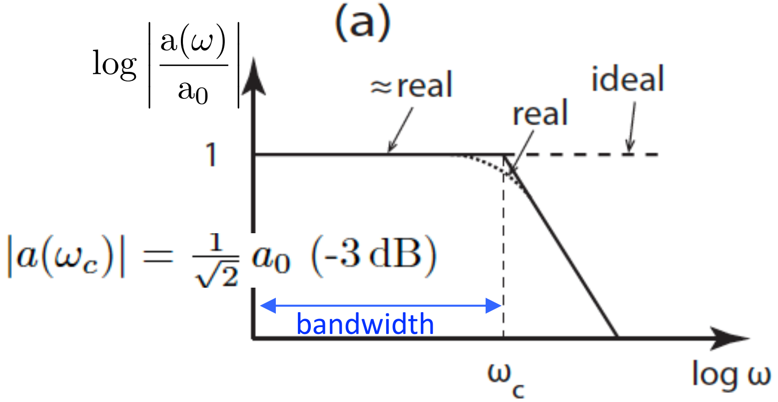
$$g_m = i_{out}/v_{in}$$

transfer function

$$\omega_c = \frac{1}{r_0 C_0}$$

$$a_0 = -\frac{v_{out}}{v_{in}}(\omega \to 0) = -\frac{i_{out}}{v_{in}} r_0 = -g_m r_0$$

Bode Plot

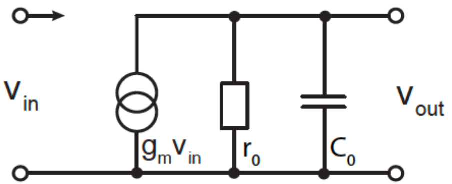


$$v(t) = v_0 (1 - e^{-t/\tau})$$

frequency domain

time domain

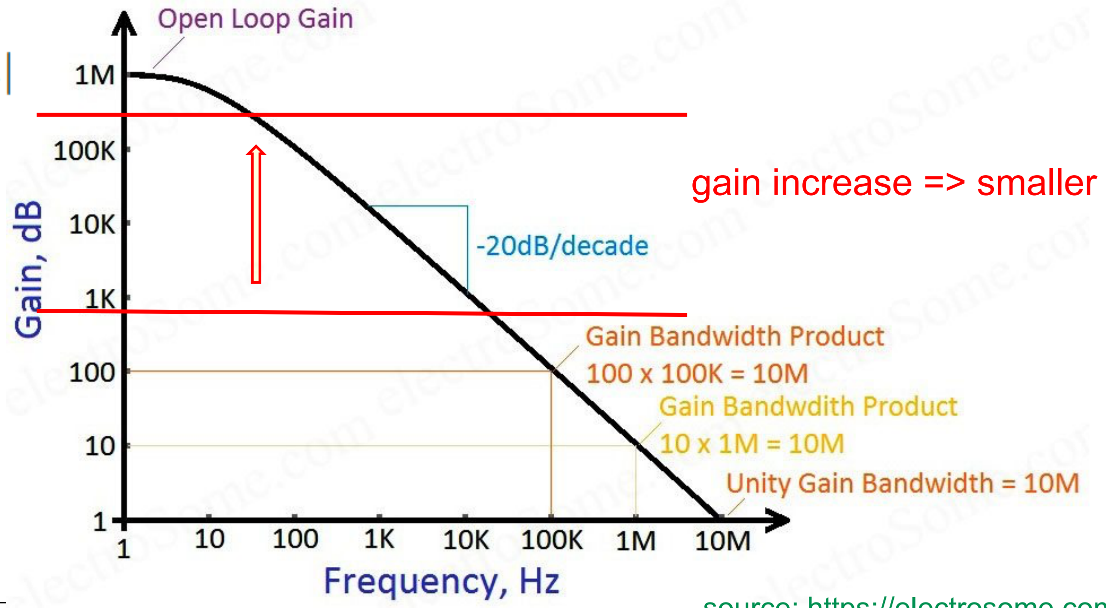
Gain × Bandwidth product



$$\mathrm{GBP} = |a(\omega) \times \omega| \approx g_m r_0 \cdot \frac{1}{r_0 C_0} = \frac{g_m}{C_0}$$

≈ CONST ! (for a single stage, single pole amplifier)

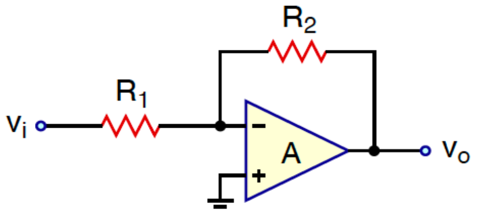
$$a_{dB} = 20 \log_{10} |a_v|$$



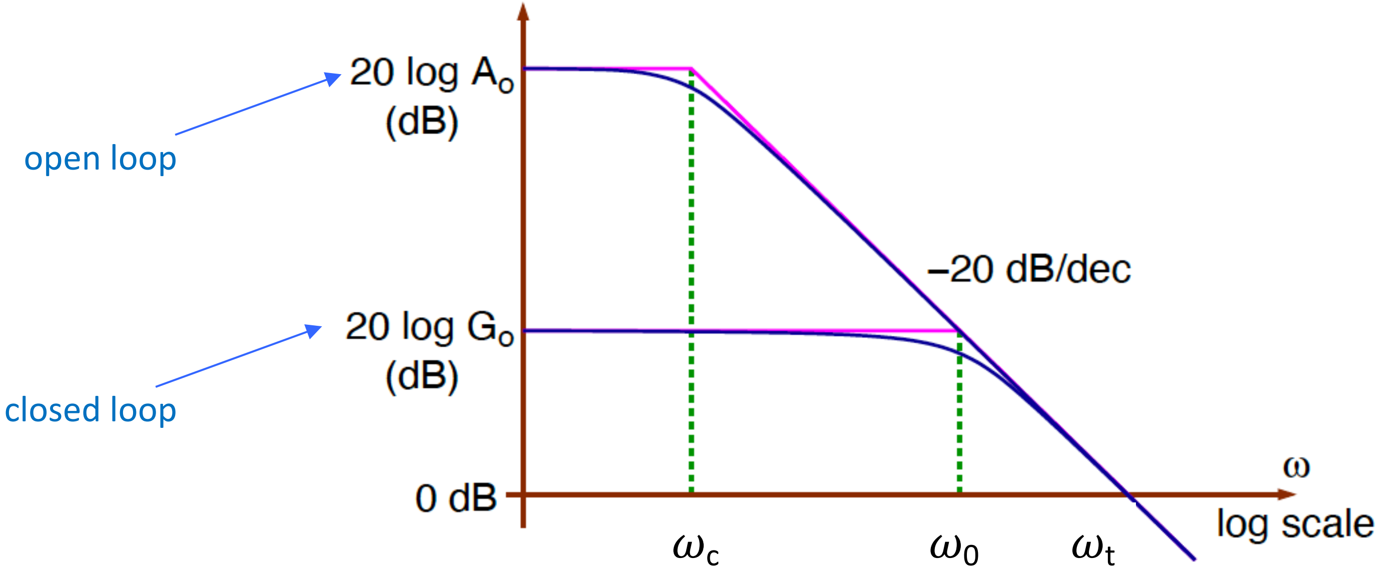
gain increase => smaller BW => slower response

source: https://electrosome.com/opamp/

Reducing the gain => increasing the BW



e.g. by (inverted) feedback



CSA input impedance and "timing"

remember dyn. C_{in}

$$C_{in} = \frac{Q_S}{v_{in}} = C_f\,(a_0 + 1)$$

$$Z_{in}(f) = \frac{1}{i\omega C_{in}} = \frac{1}{(a(\omega)+1)\cdot i\omega C_f} \approx \frac{1}{ia(\omega)\,\omega C_f}$$

$$a(\omega) = \frac{a_0}{1 + i\frac{\omega}{\omega_c}}$$

at low frequencies

$$a(\omega) \to a_0 \quad \Rightarrow \quad Z_{in}(\omega) \to \frac{1}{ia_0\omega C_f}$$

behaves as a capacitor

$$a(\omega) \to -ia_0\frac{\omega_c}{\omega} \quad \Rightarrow \quad Z_{in}(\omega) \to \frac{1}{\left(-ia_0\frac{\omega_c}{\omega}\right)\cdot i\omega C_f}$$

at high frequencies

$$= \frac{1}{a_0\omega_c C_f} = \frac{1}{\mathrm{GBP}\cdot C_f} = \frac{C_0}{C_f}\,\frac{1}{g_m} = R_{in}$$

C_0 = cap. of amplifier

behaves as a resistor, since $g_m \triangleq 1/R$

=> "timing"

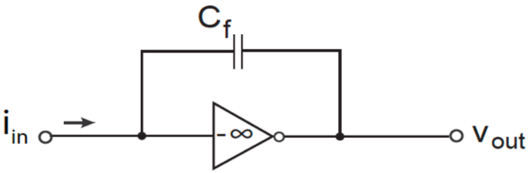
$$\boxed{\tau_{CSA} = C_D\,R_{in} = \frac{C_D}{C_f}\,\frac{1}{a_0\omega_c} = \frac{C_D}{C_f}\,\frac{C_0}{g_m}}$$

det. capacitance

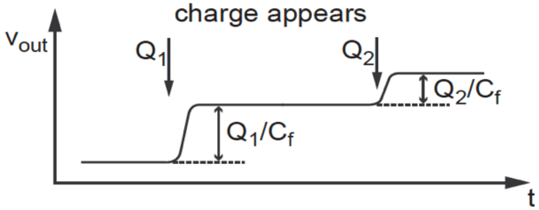
power

CSA: discharging C_f

ideal situation w/o reset



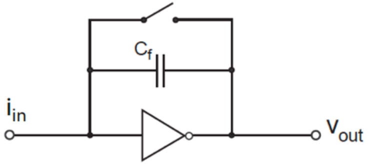
(a) Circuit diagram.



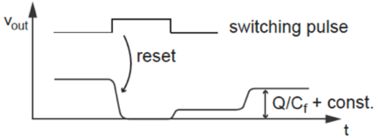
(b) Output voltage.

=> “pile-up” => saturation

discharging options

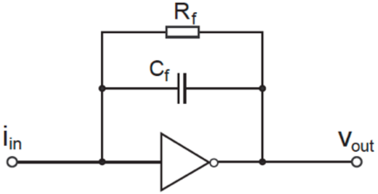


(a) Switch reset: circuit.

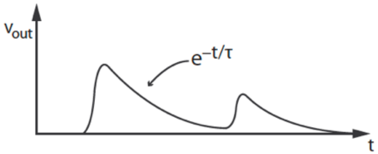


(b) Switch reset: output voltage.

“reset” switch

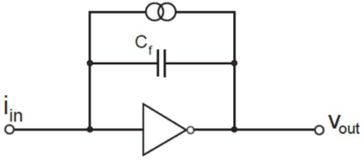


(c) Reset via resistor: circuit.

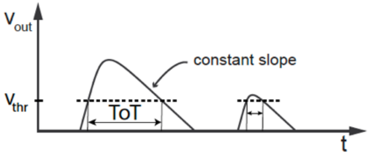


(d) Reset via resistor: output voltage.

resistor R_f



(e) Reset via current source: circuit diagram.



(f) Reset via current source: output voltage.

a current source

Short excursion: Laplace Transform

Laplace transform

- Convenient method to switch from time to frequency domain and back.
- Very similar to Fourier transform (Laplace encompasses Fourier)

$$F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)\, e^{-st} dt\,,$$

$$\boxed{s} = \sigma + \boxed{i\omega}$$

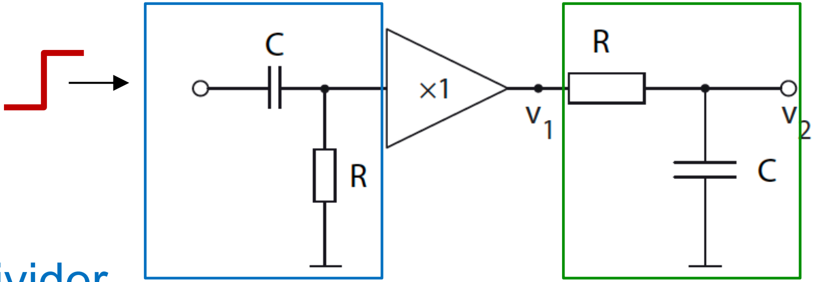
σ = a constant generating convergence for time-wise constraint functions

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} F(s) e^{st} ds = \begin{cases} f(t) & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0 \end{cases}$$

Laplace transform

Operation or function	Time domain $f(t) = \mathcal{L}^{-1}[F(s)]$	Frequency domain $F(s) = \mathcal{L}[f(t)]$
linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
convolution	$\int_0^\infty f(t-t')g(t')dt'$	$F(s)G(s)$
nth derivative	$\frac{d^n}{dt^n} f(t)$	$s^n F(s)$
time integration	$\int_0^t f(t)dt$	$\frac{1}{s} F(s)$
scaling of t	$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
time shift	$f(t-t_0)$	$e^{-st_0} F(s)$
damping	$e^{-s_0 t} f(t)$	$F(s+s_0)$
multiplication	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
δ function	$\delta(t)$	1
derivative of the δ function	$\frac{d^n}{dt^n} \delta(t)$	s^n
step function	$\Theta(t)$	$\frac{1}{s}$
falling exponential	e^{-at}	$\frac{1}{s+a}$
rising exponential	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
power function	t^n	$\frac{n!}{s^{n+1}}$

Laplace Transform: example CR-RC filter (shaper)



transfer function:

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)}$$

voltage divider

$$\frac{v_{out}(t)}{v_{in}(t)} = \frac{i(t)R}{\frac{1}{C}\int i(t)dt + i(t)R}$$

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{i(s)R}{\frac{1}{sC}i(s) + i(s)R}$$

$$\boxed{H_1(s) = \frac{sRC}{1 + sRC}}$$

$$\boxed{H_2(s) = \frac{1}{1 + sRC}}$$

=> CR-RC: $H(s) = H_1(s) \times H_2(s)$

$$v_1(s) = H_1(s)\, v(s) = \frac{sRC}{1 + sRC} v(s) = \frac{s\tau}{1 + s\tau} v(s)\,,$$

$$v_2(s) = H_2(s)\, v_1(s) = \frac{1}{1 + sRC} v_1(s) = \frac{s\tau}{(1 + s\tau)^2} v(s)$$

step function input

$$v(t) = V_0 \Theta(t)$$

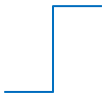
$$\Rightarrow v(s) = V_0 \frac{1}{s}$$

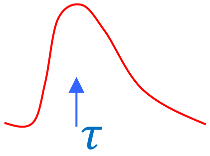
$$\Rightarrow \quad v_2(s) = \frac{V_0 \tau}{(1 + s\tau)^2} \quad \Rightarrow \quad \boxed{v_2(t) = V_0 \frac{t}{\tau} e^{-t/\tau}}$$

Pulse shaping

necessary (i) to avoid pile-up of signals
(ii) to reduce the bandwidth => reduce noise (see later)

in time domain



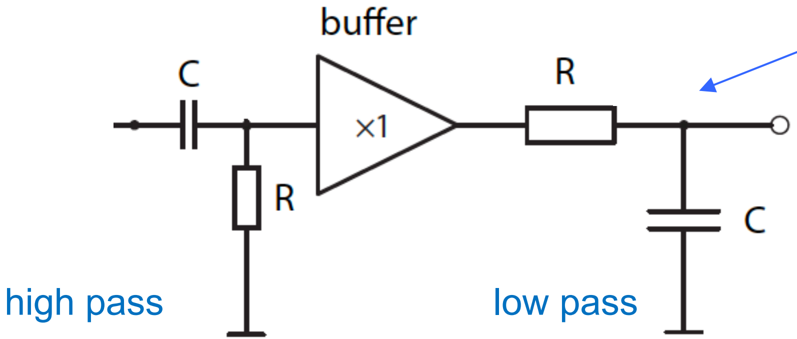


$$v_{sh}(t) = A\frac{t}{\tau}\,\mathrm{e}^{-\frac{t}{\tau}}$$

peaking at τ with $v(\tau) = A / 2.71$

in frequency domain $1/s$

$$H(s) = A\,\frac{s\tau}{(1+s\tau)^2}$$



usually realised as
CR – RC shaper

$$A(t) = \frac{t}{\tau}\,e^{-\frac{t}{\tau}}$$

or as

$(CR)^N$ – $(RC)^M$ shaper $$A(t) = \frac{1}{M!}\left(\frac{t}{\tau}\right)^M e^{-\frac{t}{\tau}}$$

for N = 1

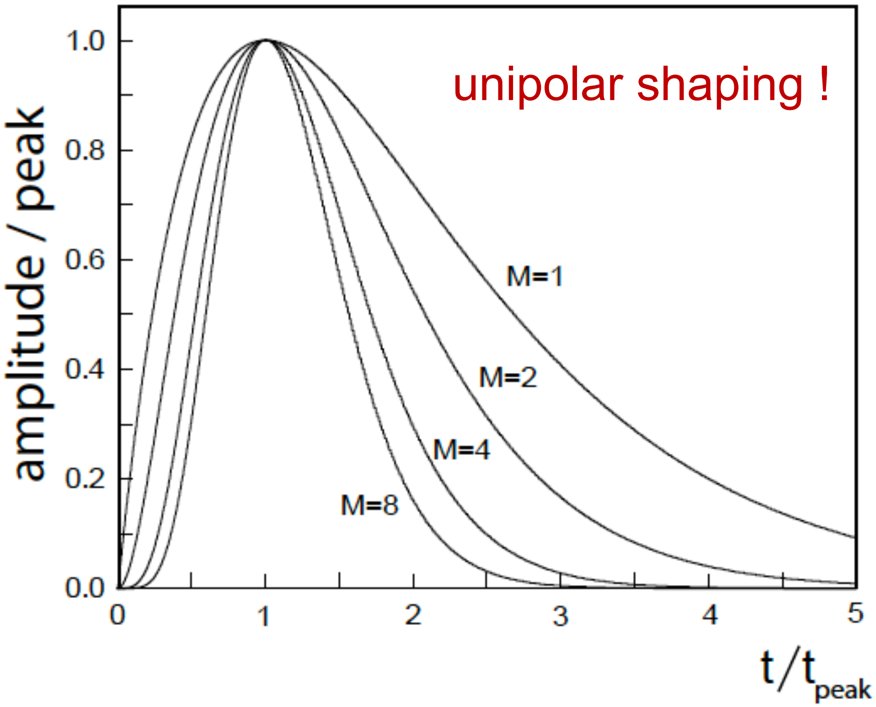


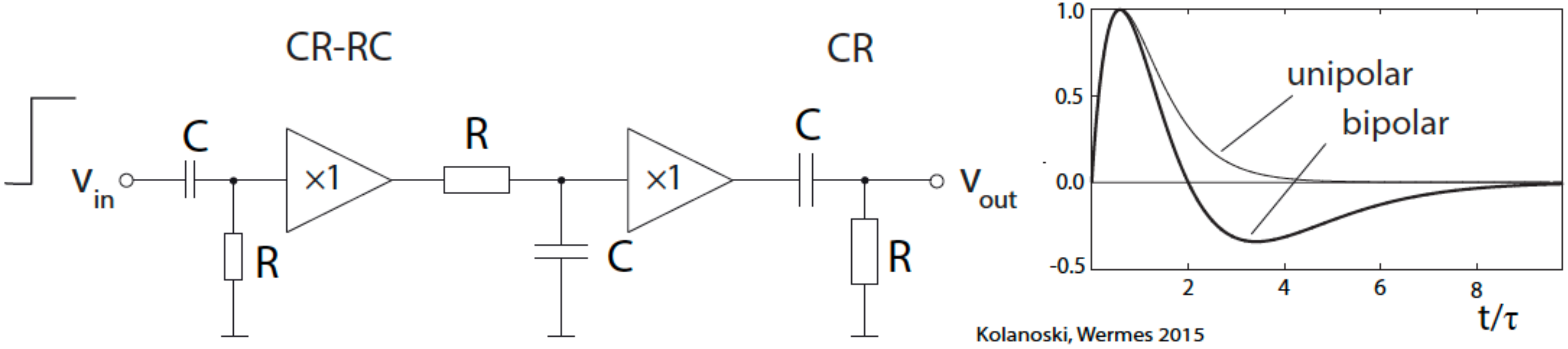
Fig. 17.12 Step response output of CR–(RC)^M shaper stages of different order M. Amplitudes and times are normalised to their respective peak amplitude.

With increasing M
=> shape more Gaussian and, if one chooses a shorter peaking time, also narrower in absolute terms.
=> better double-pulse resolution

However, the electronic effort is substantially larger than for CR–RC shapers.

Bipolar shaping

By adding a further high-pass filter to any unipolar shaper a bipolar pulse shape is obtained with equal area in both wings. This can be done by adding a CR high-pass filter after M low-pass filters.



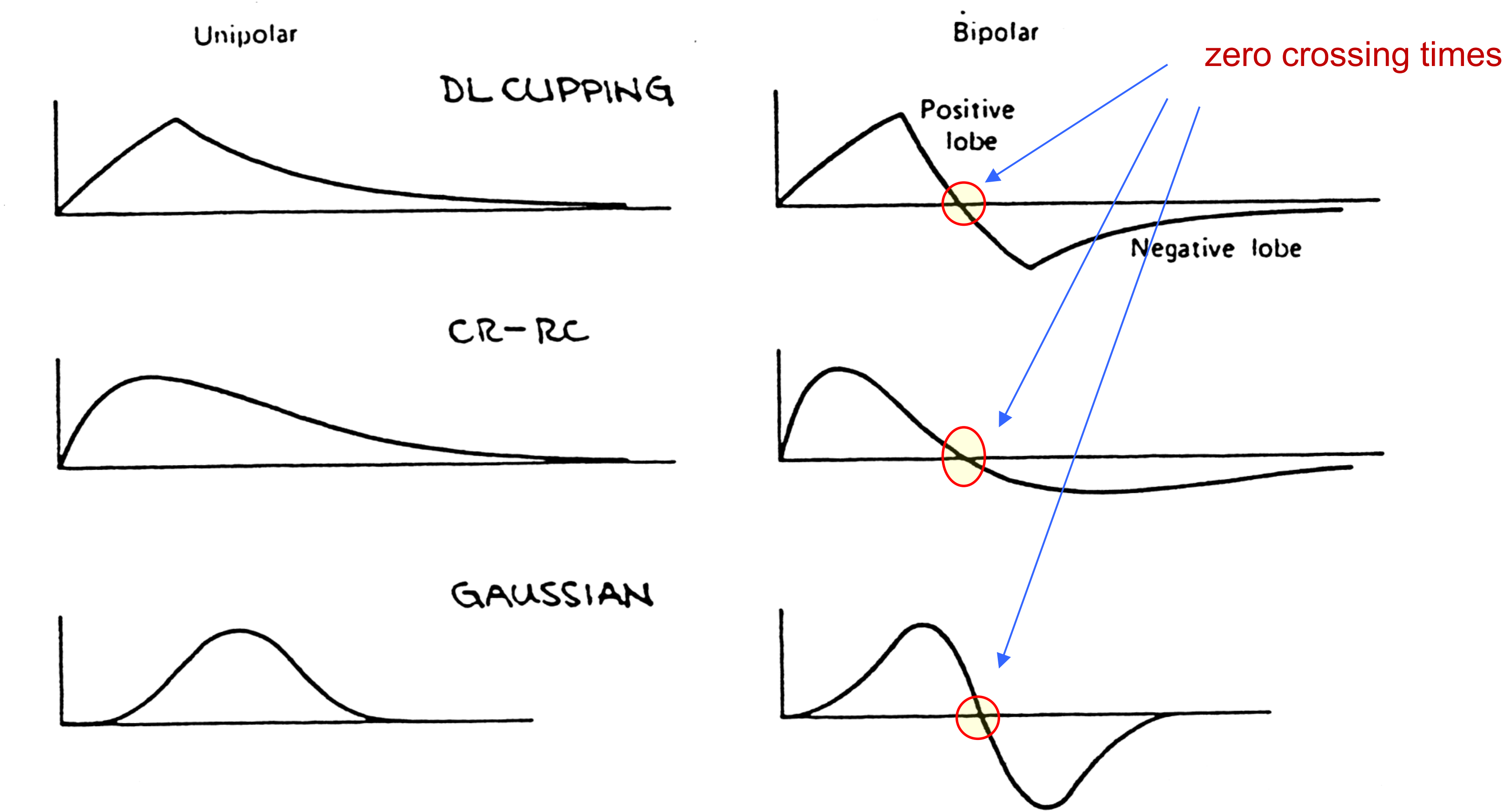
advantages

- digests high count rates
- is simple and robust against baseline fluct.
- is good for time-critical applications (use zero-crossing time as observable)

less good

- somewhat more power consumption
- somewhat worse S/N
- needs larger chip area

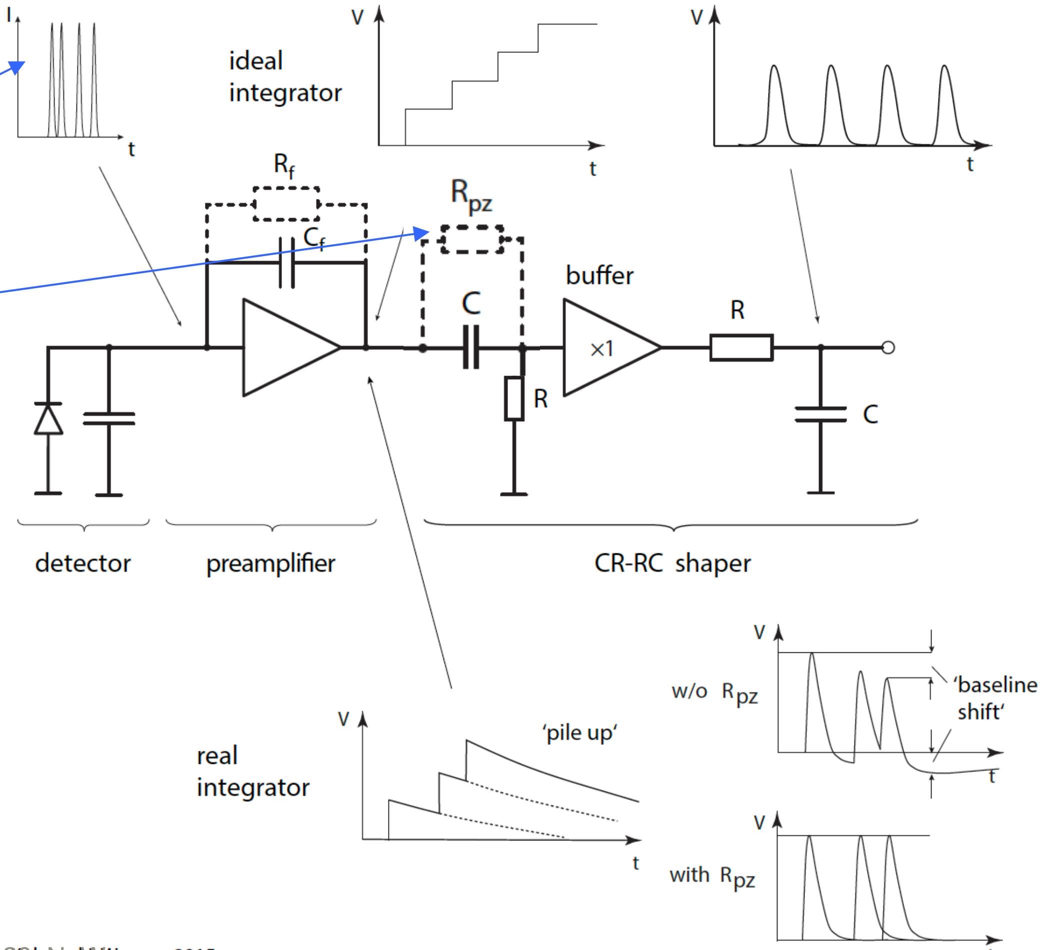
Unipolar and bipolar filtering



The amplifier system = preamp + shaper (= filter)

input signals
= δ fctns.

pole-zero
cancellation



ideal

real

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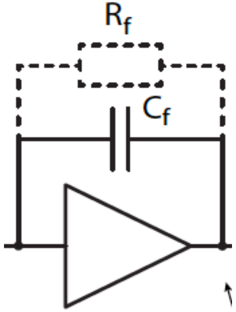
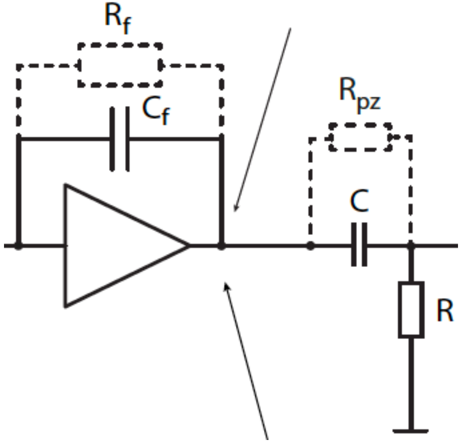
Undershoot and pole-zero cancellation

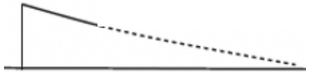
Without further corrections unipolar shaping also features unwanted undershoots in realistic applications. Origin is the (slow) discharge of the feedback capacitor C_f .



$$v_{out}(s) = \frac{R_f}{1 + s\tau_f} i_{in}(s)$$

pole on real axis for $s=\pm 1/\tau_f$
(resp. for Bode plot and position of ω_c)



works fine for a  input (1/s), but not for a  pulse, because the CR of the shaper no longer cancels the 1/s in the step function pulse, which leads to the undershoot

but with R_{pz}

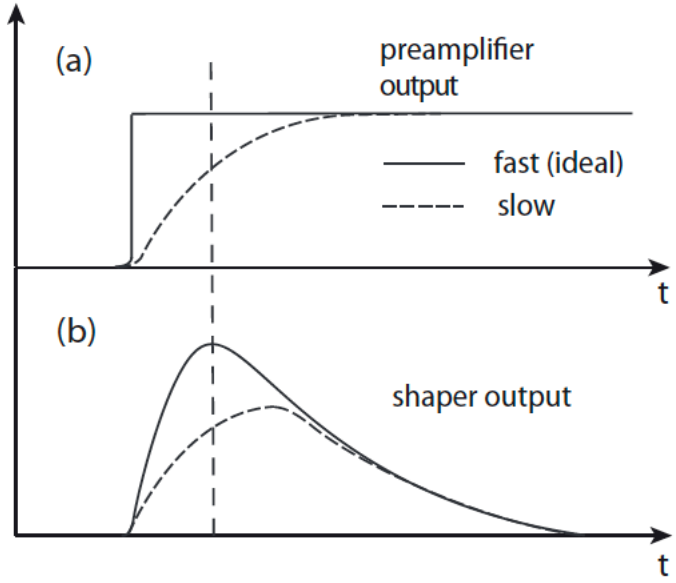
$$H(s) = H_{CSA}(s) \cdot H_{pz}(s) \quad = \quad \frac{R_f}{1+s\tau_f} \cdot \frac{1+s\tau_{pz}}{1+\frac{R_{pz}}{R}+s\tau_{pz}}$$

$$\underset{\tau_{pz}=\tau_f}{=} \quad \frac{R_f}{1+\frac{R_{pz}}{R}+s\tau_f} \quad \underset{R_{pz}\ll R}{\longrightarrow} \quad \frac{R_f}{1+s\tau_f}$$

original H(s) restored

Some more things …

“ballistic deficit” or “shaping loss”



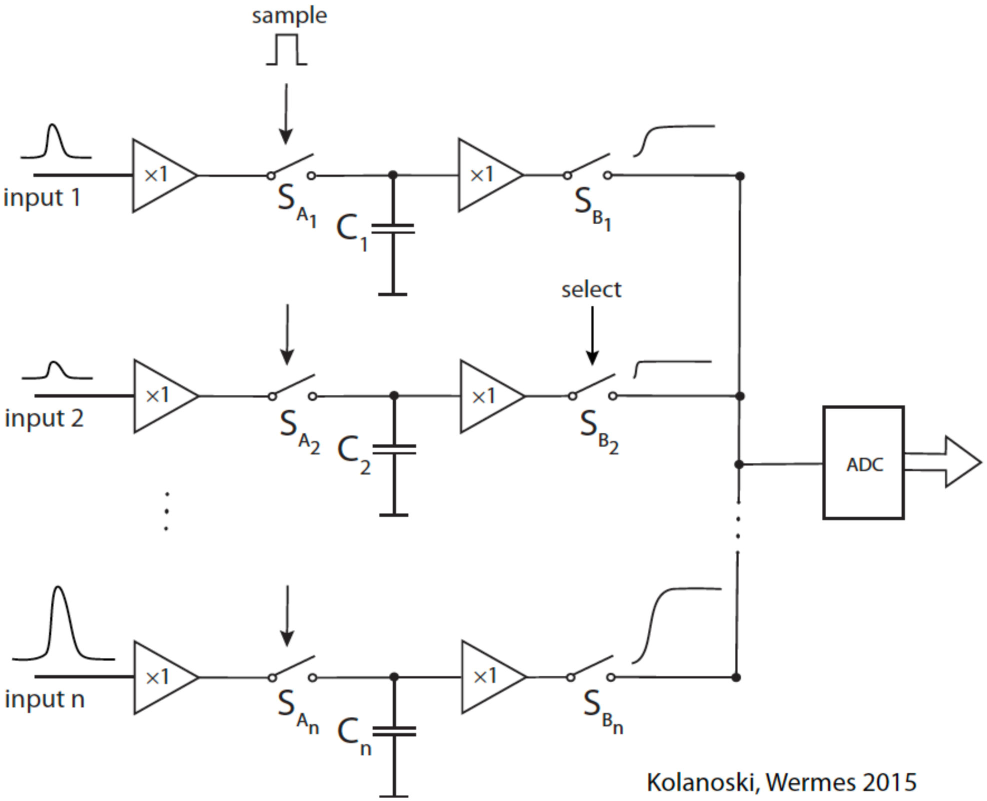
- If pulse evolution at the shaper input takes much longer than the shaper τ_{CR}
- caused e.g.
  - by a large charge collection time
  - by a large input capacitance
  - by an intrinsically slow preamplifier.

=> The v_{out} (shaper) is trimmed by the slow rise of the preamplifier output pulse.
The falling edge of the shaper output already sets in before the preamplifier output has reached its maximum value.

skip S&H?

The "sample and hold" technique

- If signal shape and arrival time are known
- => sample v_{in} at a fixed time, hold them for Δt, and process later
- Sample pulse (derived from trigger and delayed coincident with the peak) opens the switches S_A for sampling on the C_i .
- readout the switches S_B are successively closed, hence sequentially transferring the stored voltages serially onto the ADC
- By fast sampling the pulse in successive time intervals and storing the voltages in a series of sample-hold cells, called analog memory, one can memorise the whole pulse shape (wave form sampling).



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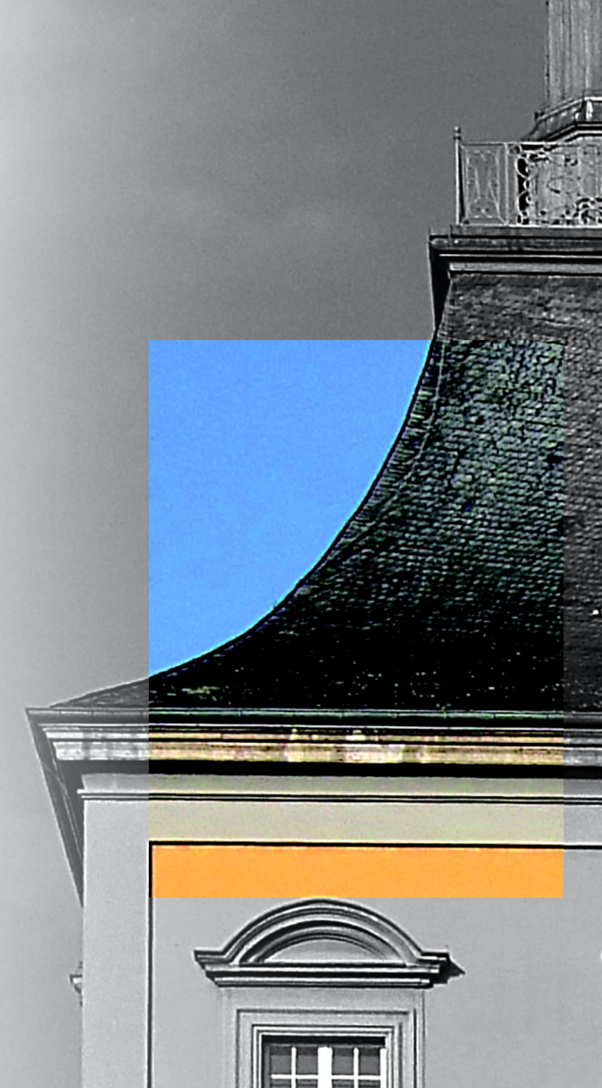
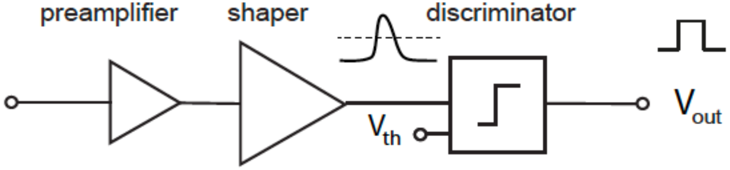
The discriminator



Discrimination and "time walk"

= voltage comparator
needed to filter out
noise hits

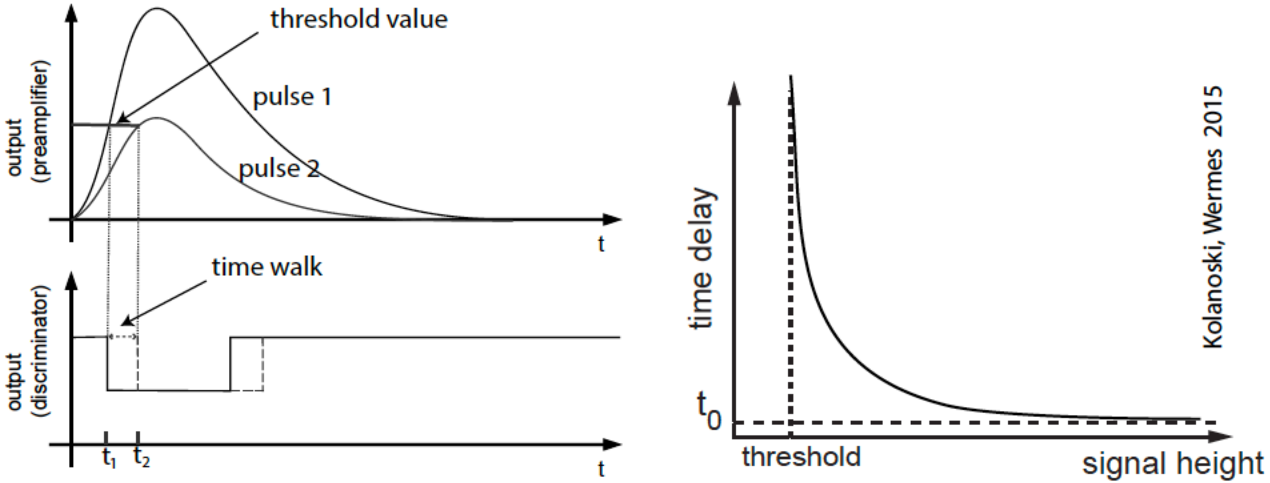


(a) Readout system with amplifier, shaper, and discriminator.

standard = leading edge discrimination

time
walk

typical: best timing performance if threshold at ~10-20% of peak value (depending on noise)



(b) Time walk.

(c) Dependence of time walk on the signal pulse height (schematic).

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when stable timing is an issue

- goal: discriminate at the same fraction, constant in time.
- An inverted and attenuated pulse is superimposed on the delayed pulse.
- Note time-stable zero-crossing point.

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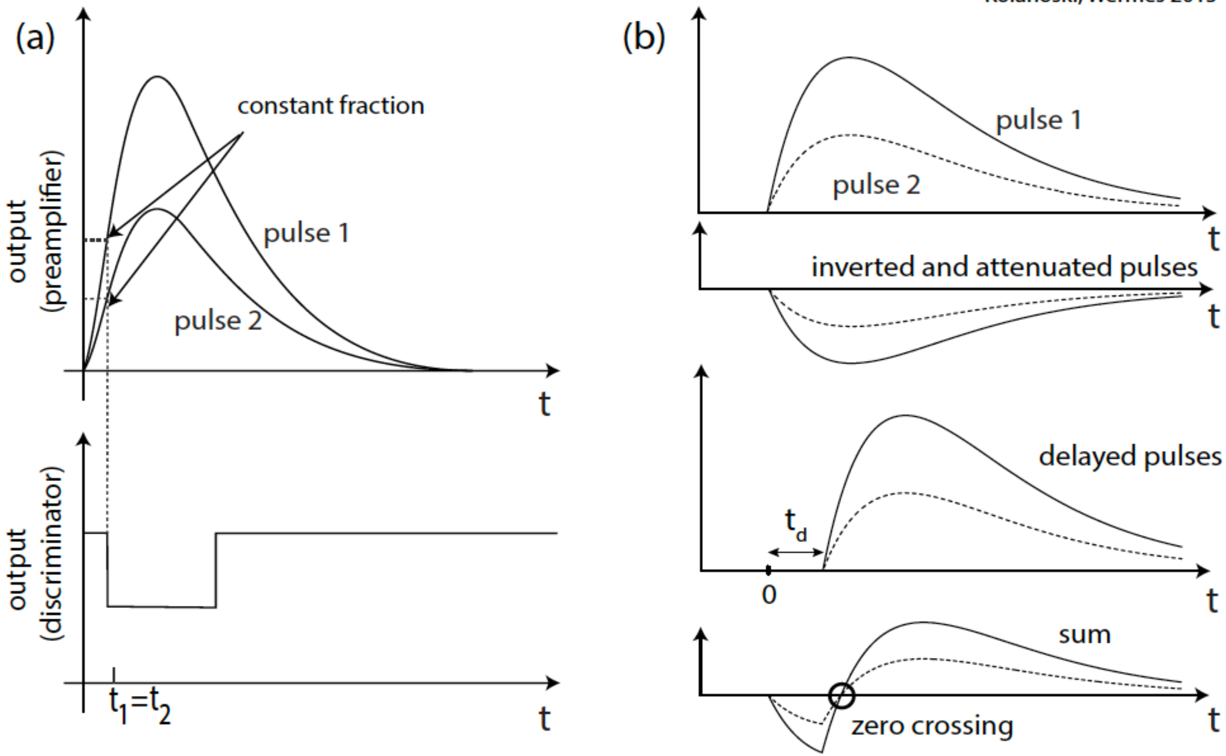
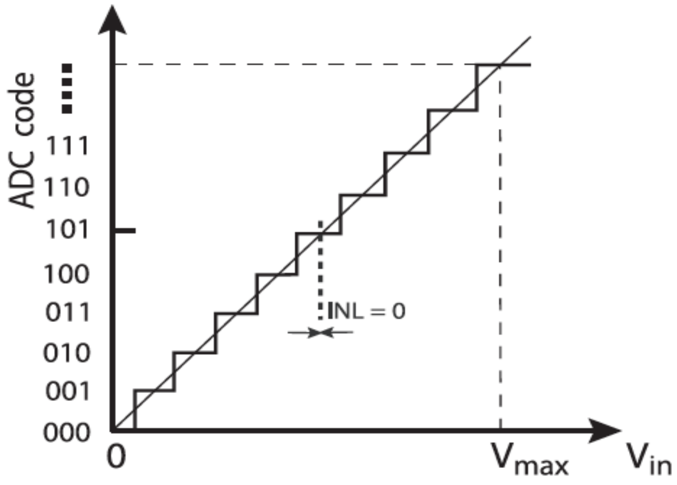


$$v_{sum}(t) = v_{in}(t - t_d) - k v_{in}(t)$$

(Elements of ...) Digitisation

Analog - to – Digital conversion (ADC)



$$1\,\text{LSB} \mathrel{\widehat{=}} \frac{V_{max} - V_{min}}{2^n - 1}$$

quantization error $\delta = \frac{\text{LSB}}{\sqrt{12}}$

SNR = full input range / δ

$$= 20 \log_{10} \left(2^n \sqrt{12}\right) \approx (6.02\,n + 10.8)\,\text{dB}$$

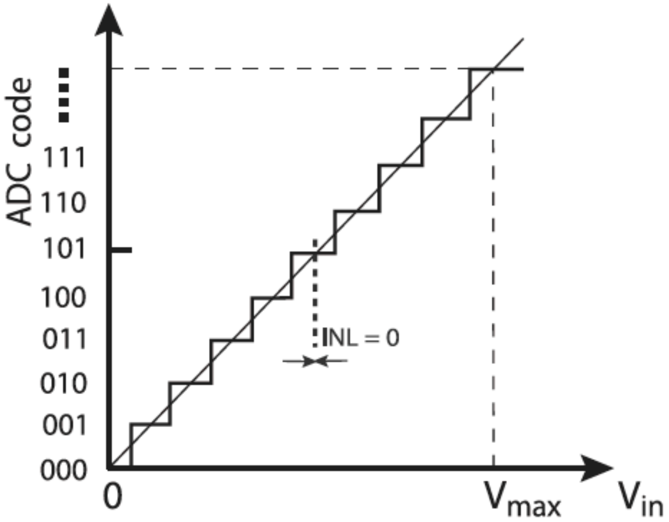
(n = no. of bits)

What matters?

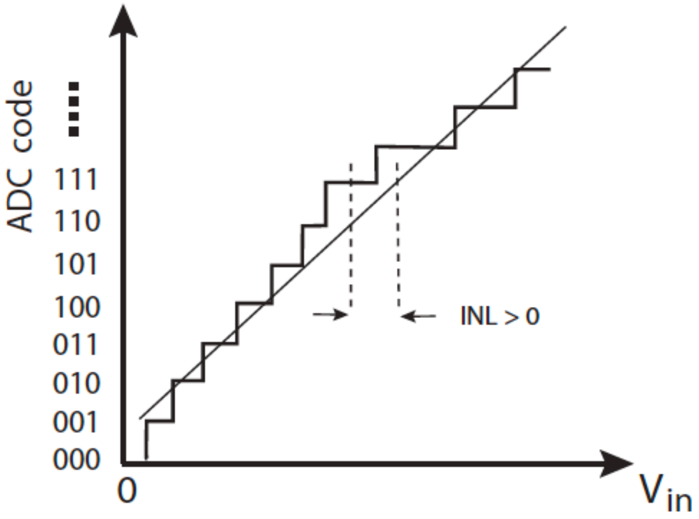
- Resolution: precision of the code;
- INL (integral non-linearity): proportionality of output to input;
- DNL (differential non-linearity): homogeneity of digitisation steps;
- Conversion speed
- Rate capability: how fast successive signals can still can be correctly digitised
- Stability: sensitivity of conversion quality with wrt. to time, temperature, other params

ADC errors

quantization error
zero-point error
slope error
linearity errors



(a) Perfectly linear ADC.



(b) Integral non-linearity.

INL describes the total deviation from the expected linear behaviour.
INL = maximum deviation of the measured midpoints from ideal line
given as fraction of V_{max} or (usually) in units of the voltage step which corresponds to the lowest-valued bit (LSB units).

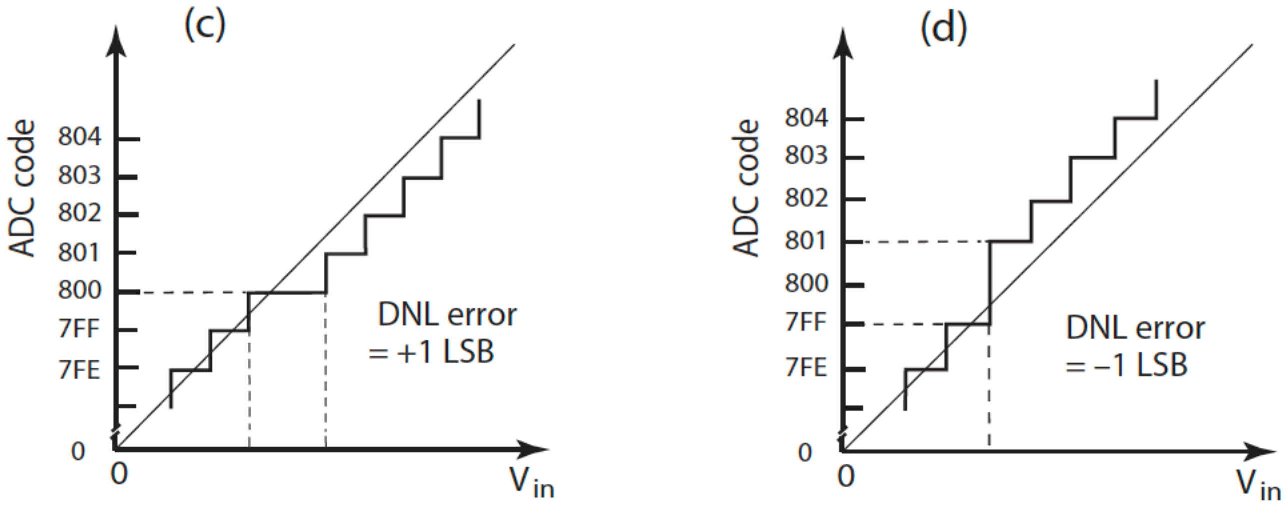
ADC errors … Differential Non-Linearity

DNL = measure of the homogeneity of subsequent digitisations

The midpoints of the voltages of subsequent codes should ideally be separated by exactly 1 LSB. The DNL describes local relative deviations of ADC codings from the (ideal) step width (1 LSB):

$$\mathrm{DNL_i} = \frac{V_{i+1} - V_i}{\mathrm{LSB}} - 1 \quad \text{for} \quad i = 0, 1, 2, \ldots, 2^n - 2$$

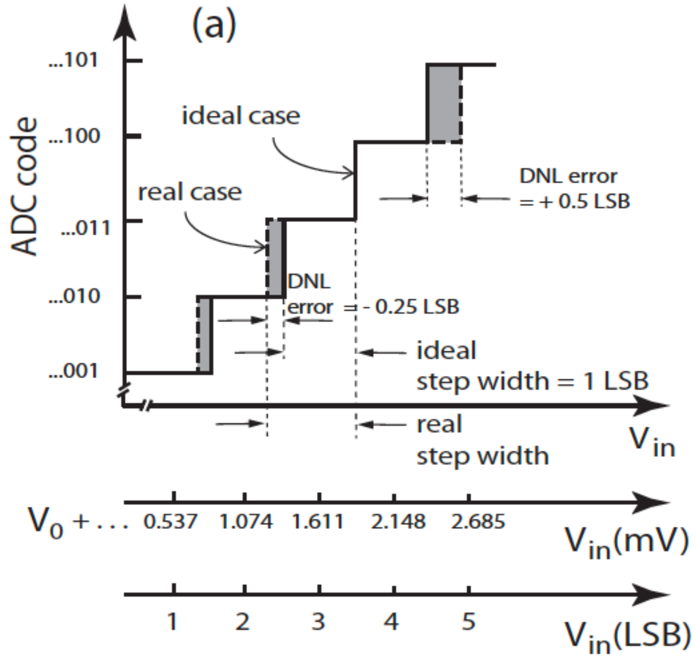
specified as a fraction or a multiple of ±1 LSB.

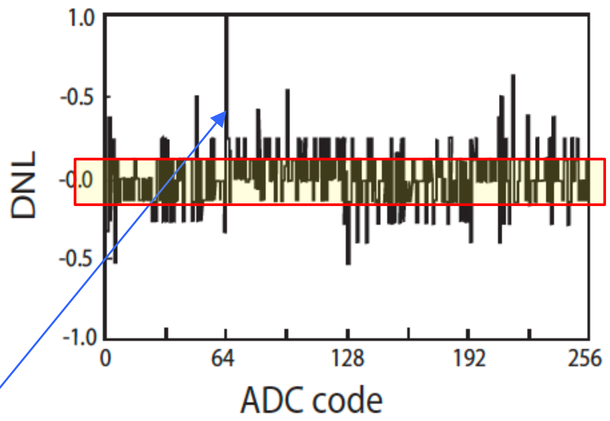


DNL errors larger than −1 LSB are required to guarantee no ‘missing codes’.

Kolanoski, Wermes 2015

DNL





measurement: sweep V_{in} linearly and plot DNL_i as a function of the code.

for overall DNL either $\mathrm{DNL} := \max_i(|\mathrm{DNL}_i|)$ or $\mathrm{DNL}_{rms} := \left(\frac{1}{2^n - 2} \sum_{i=0}^{2^N-2} (DNL_i)^2 \right)^{1/2}$

ADC concepts

SAR = Successive Approximation Register

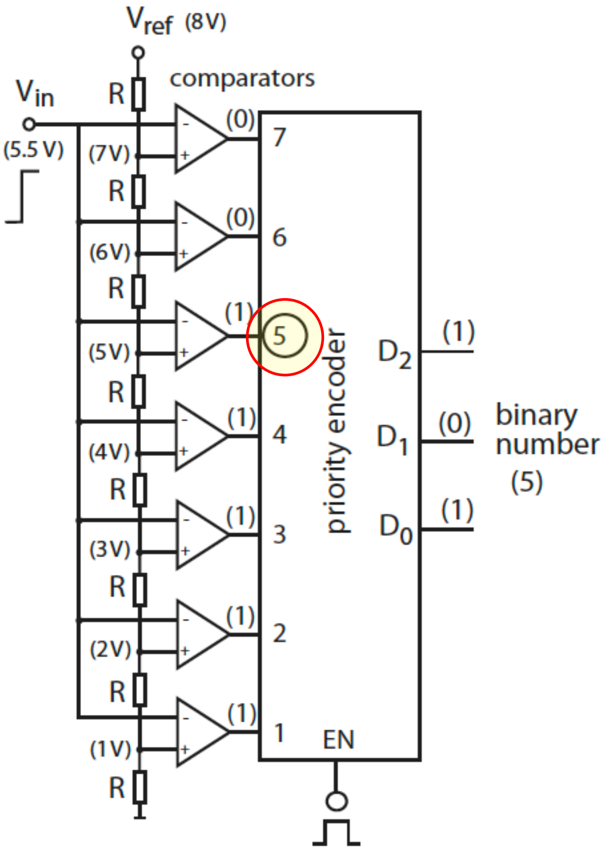
Flash ADC

Wilkinson ($\rightarrow$ Dual Slope) ADC

Pipeline ADC

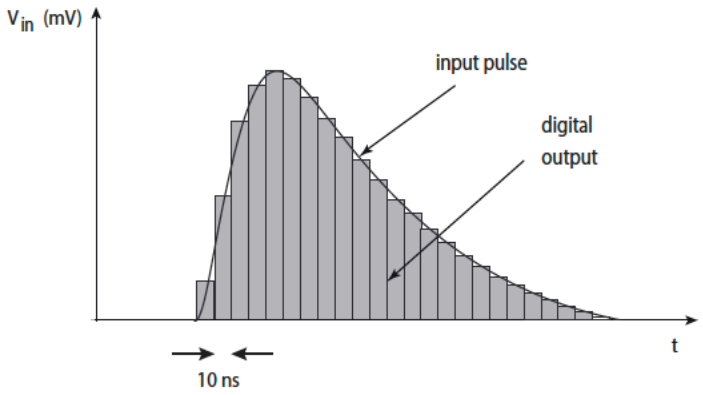
… more

here
3-bit



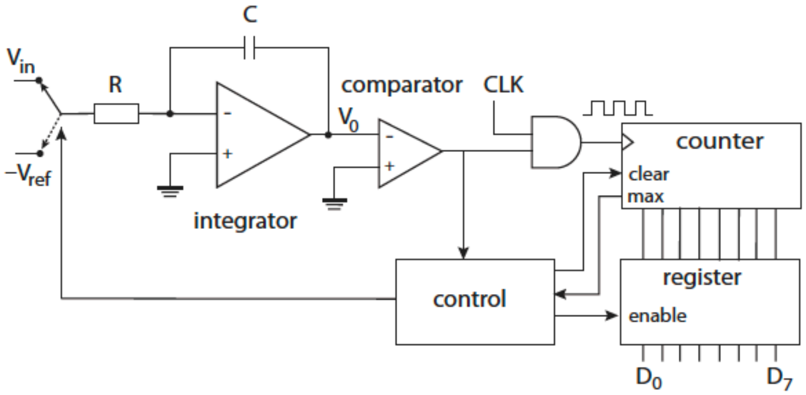
(a) Flash ADC block diagram.

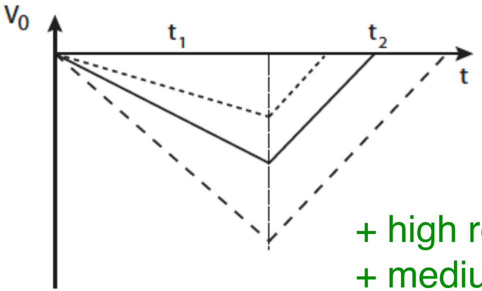
+ fast
– high power cons.
large chip area needed



(b) Digitisation of a pulse.

Dual Slope (Wilkinson) ADC





+ high resolution
+ medium circuit effort
+ chip area ~independent of resolution
+ independent of fabrication variations
+ good noise immunity
− comparison to other ADCs

- Feedback capacitor is linearly charged and discharged, controlled by a counter.
- Charging for a time t_1, usually until the counter reaches its maximum value determined by V_{in} .
- Then the counter is reset and the input is switched to $-V_{ref}$ (discharging starts).
- Slope (and time) of discharge depends on $-V_{ref}$
- Slope (and time) of charge depends on V_{in} .
- Discharging proceeds with constant slope (set by $-V_{ref}$) but with variable duration t_2 until V_0 reaches zero.
- The duration (t_2) depends on the height of V_0 reached after the end of the charging process.
- => The counter value at t_2 encodes V_{in} and is transferred into the register.

ADCs

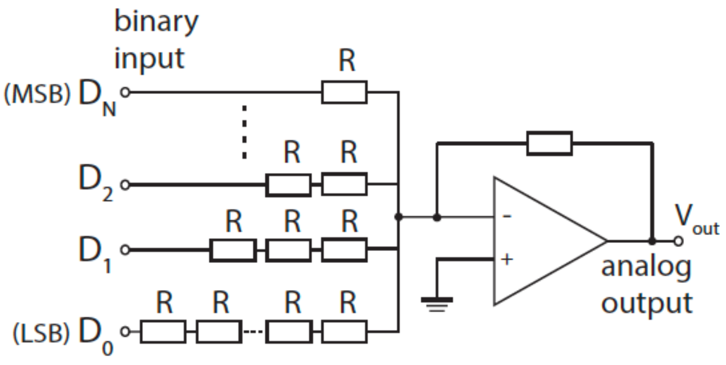
ADC type	Rel. speed (samples per s)	Resolution (bits)	Chip area	Power consumption
SAR	slow–medium ($< 2\,\mathrm{Ms/s}$)	8–16	small	low
Dual-slope	slow ($< 100\,\mathrm{ks/s}$)	12–20	medium	low
Flash	very fast ($< 5\,\mathrm{Gs/s}$)	4–12	large	high
Pipeline	fast ($< 500\,\mathrm{Ms/s}$)	8–16	medium	medium

skip DAC, TDC

DAC, TDC, etc.

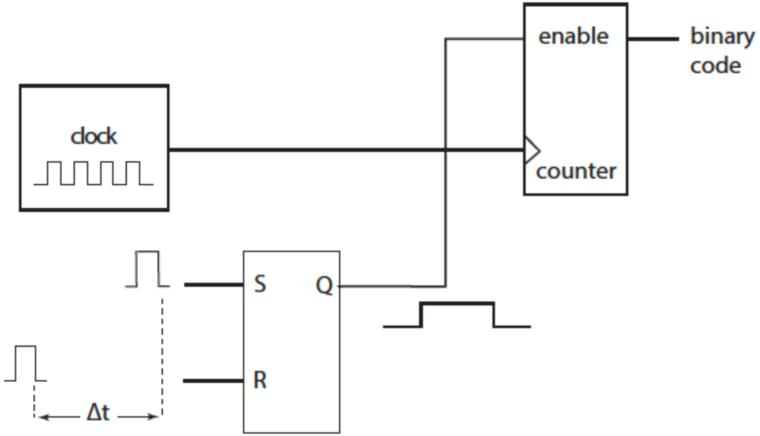
DACs are characterised similarly to ADCs by resolution, linearity, and conversion speed. Integral (INL) and differential (DNL) nonlinearity are also defined as for ADCs

DAC



(a) Resistive ladder.

TDC

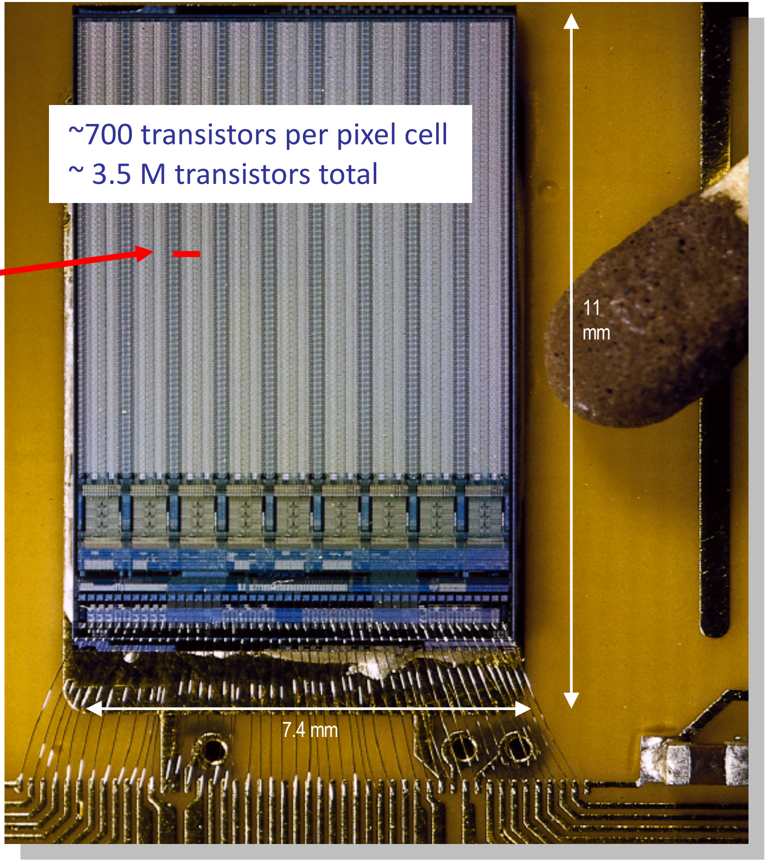


AN EXAMPLE FOR WHAT WE DISCUSSED SO FAR

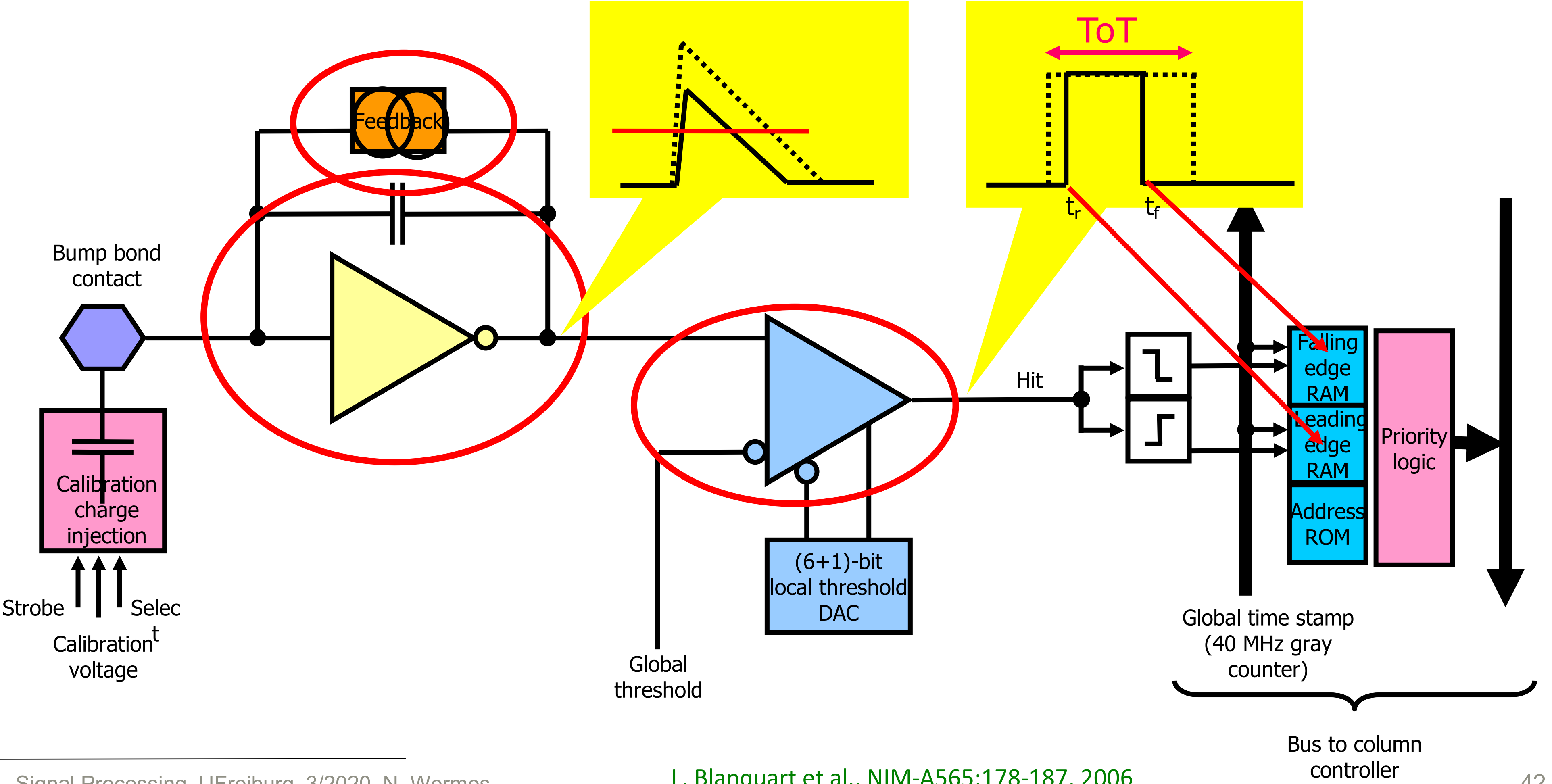
(ATLAS) PIXEL READOUT CHIP

Pixel Frontend Chip

- ATLAS FE chips (FE-I3 and FE-I4)
  - 250 nm (IBL 130 nm) CMOS technology
  - pixel cell size: 50 x 400 µm^2
  - 18 columns x 160 rows = 2880 cells
  - parallel processing in all cells
    - - amplification
    - - zero suppression



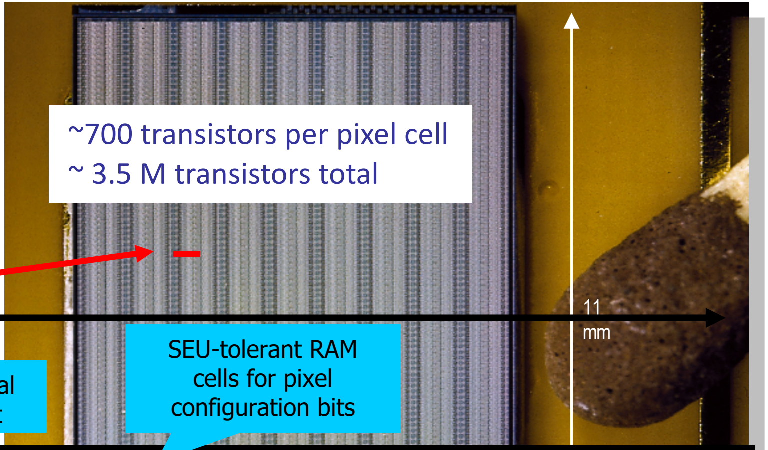
Pixel cell: amplifier + quasi shaper + discriminator

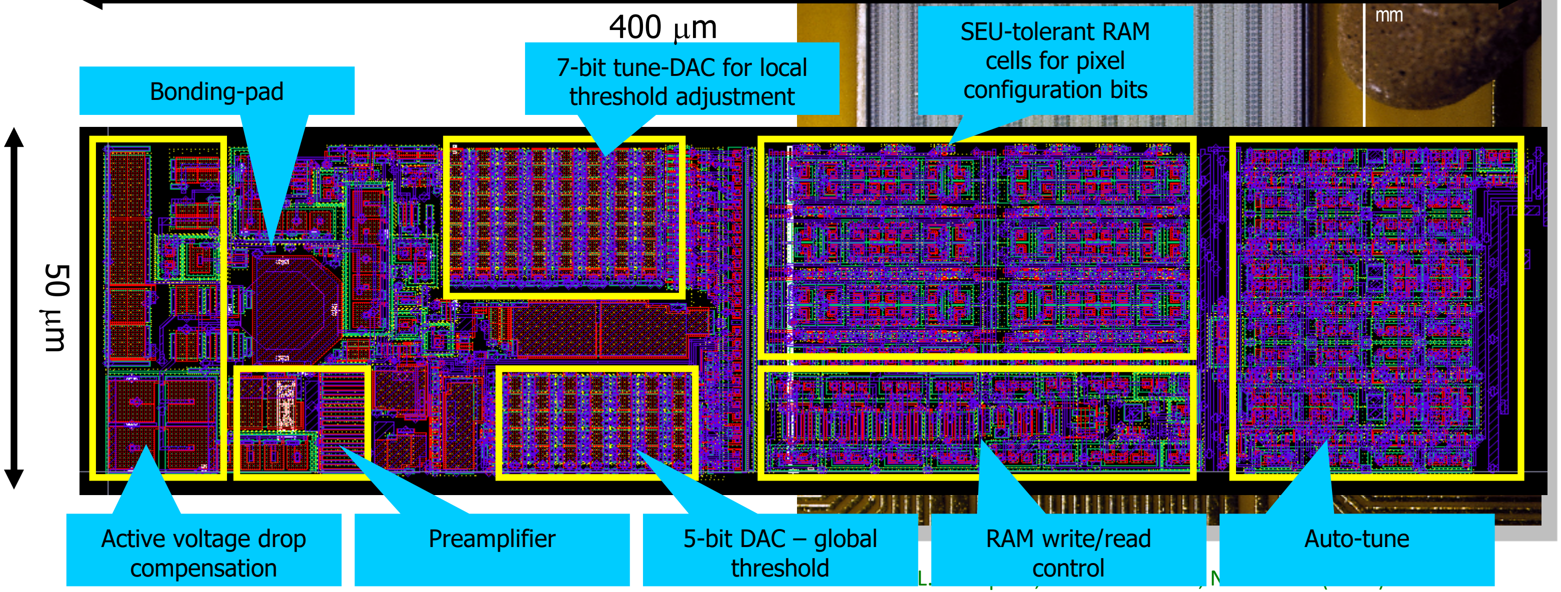


L. Blanquart et al., NIM-A565:178-187, 2006

Pixel Frontend Chip

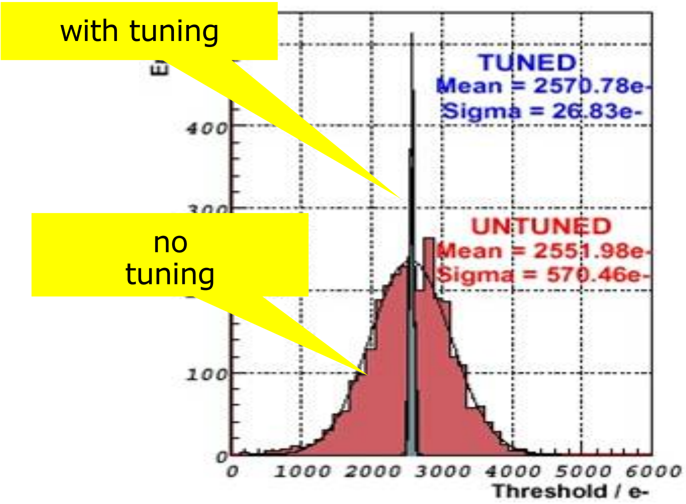
- ATLAS FE-I3
  - 250 nm CMOS technology
  - pixel cell size: 50 x 400 μm^2
  - 18 columns x 160 rows = 2880 cells
  - parallel processing in all cells
    - - amplification
    - - zero suppression



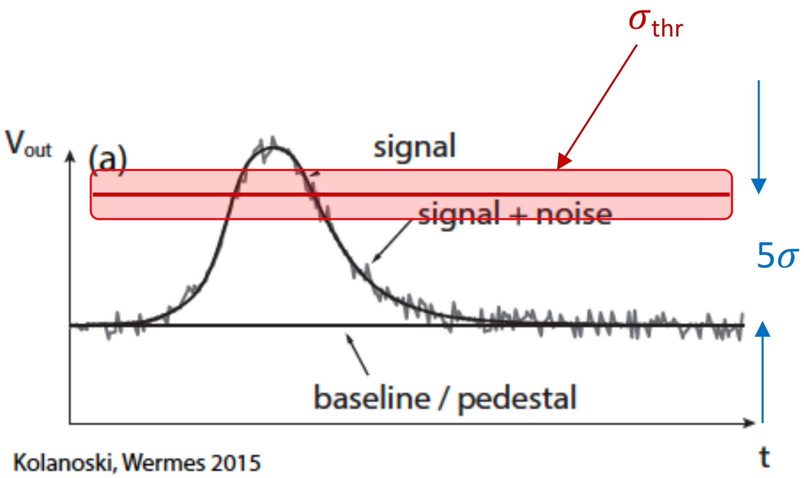


Requirements on the electronics performance

- small noise hit rate → low noise and small threshold dispersion
- $\sigma_{noise} \oplus \sigma_{threshold}$ < 500 e⁻ @ a threshold of 2500 e⁻
- time stamping < 20 ns after BX for all signal heights



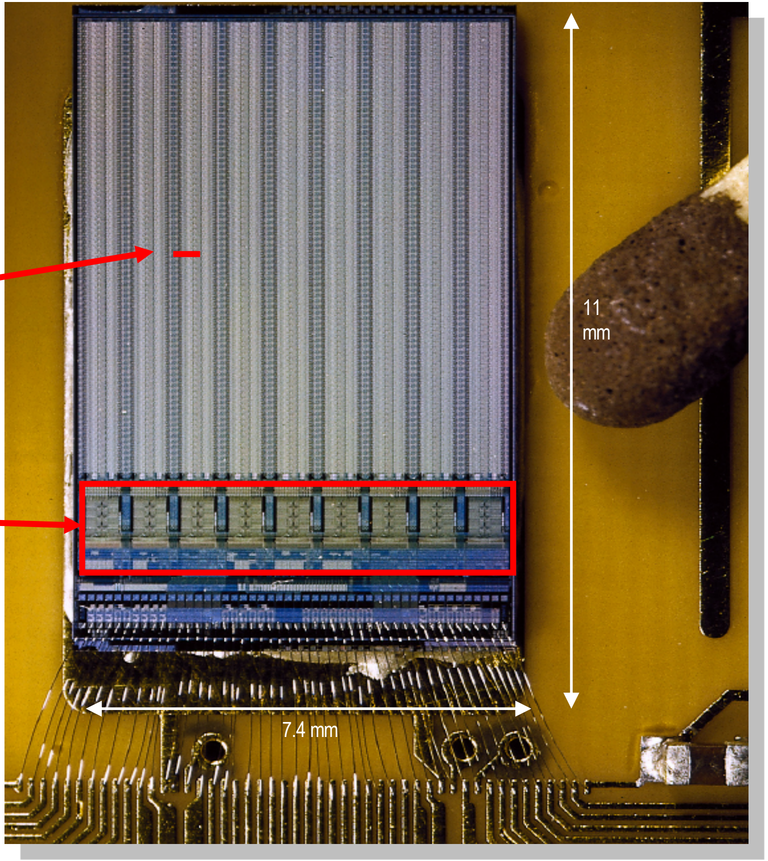
Distribution of pixel cell thresholds



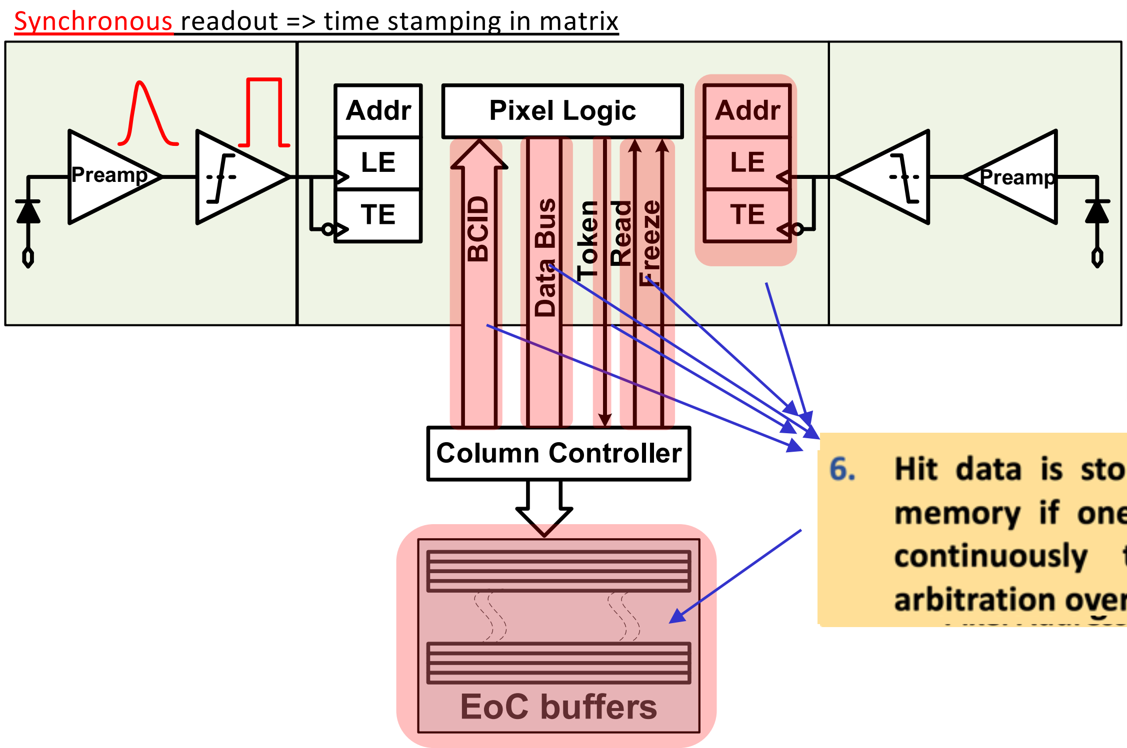
Pixel Frontend Chip

- ATLAS FE-Chip
  - 250 nm CMOS technology
  - pixel cell size: 50 x 400 µm^2
  - 18 columns x 160 rows = 2880 cells
  - parallel processing in all cells
    - amplification
    - zero suppression

- end of column logic
  - storage of hit information during trigger latency (2.5 µs)
  - hit selection upon L1 trigger



A readout architecture (“column drain”)



- BC ID (40 MHz) distributed in the column

- Hit timing stamped in pixel
  - LE: leading edge
  - TE: trailing edge

  => Time of arrival: LE

  => Ana. info. from ToT

- Hits read out sequentially, following a token passing scheme on a shared column bus

6. **Hit data is stored in the trigger memory if one exists or else is continuously transmitted after arbitration over different columns**

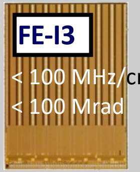
- Hits are removed if no trigger conicidence occurs.
- Hit information agreeing with L1 trigger time are read out.

skip upgrades

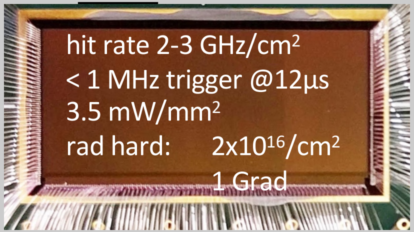
skip transmission lines

Pixel R/O-Chip for HL-LHC rates (and radiation)

- Effort and costs so large that joint approach (cross experiments) is needed -> **RD53** (20 Institutes)
- Higher hit rate (not smaller pixel size) requires higher logic density **-> 65nm TSMC**



RD53A/



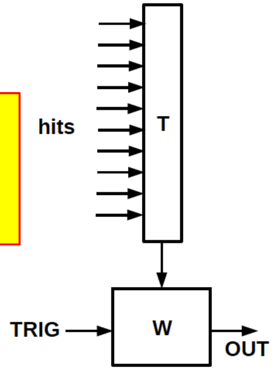
FE-I3	FE-I4	RD53A
250 nm technology	**130 nm technology**	**65 nm technology**
pixel size 400 × 50 µm²	**pixel size 250 × 50 µm²**	**pixel size 50 × 50 µm²**
3.5 M transistors	**70 M transistors**	**~ 1000 M transistors**

Pixel R/O philosophy changes -> better architectures

UNIVERSITÄT BONN

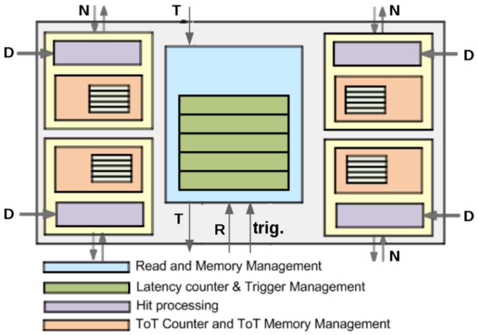
1st generation

- column drain R/O
- FE-I3 like



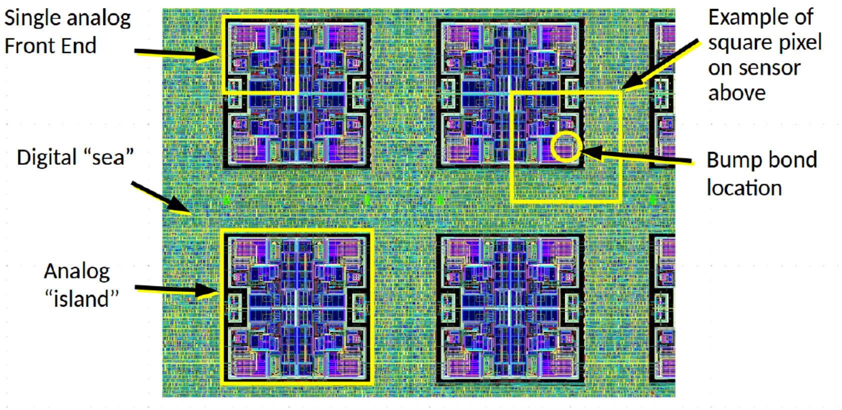
2nd generation

- 4-pixel region logic
- efficient for clusters
- FE-I4 like



3rd generation

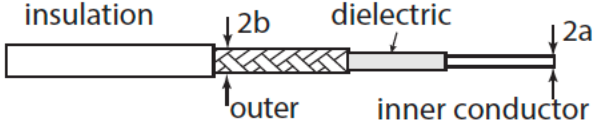
- region architectures with grouped logic -> regional hit draining
- surrounded by synthesized logic ("digital sea")
- RD53 like



"analog islands in digital sea"

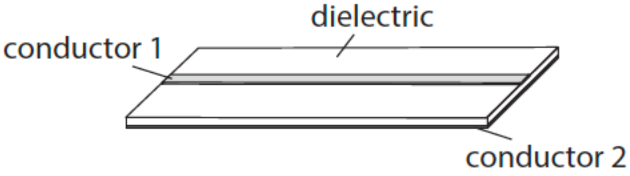
Signal transmission off the detector (transmission lines)

Coaxial cable and microstrip lines



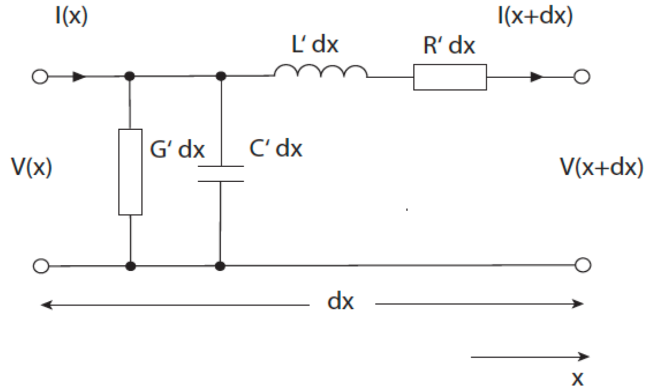
(a) Coaxial cable transmission line.

$$L' \approx \frac{\mu\mu_0}{2\pi} \ln\frac{b}{a} \qquad C' \approx \frac{2\pi\epsilon\epsilon_0}{\ln(b/a)}$$



(b) Microstrip line.

equivalent circuit



telegraph (wave) equations

$$\boxed{\begin{aligned} \frac{\partial^2 V}{\partial x^2} - L'C'\frac{\partial^2 V}{\partial t^2} &= (L'G' + R'C')\frac{\partial V}{\partial t} + R'G'V \\ \frac{\partial^2 I}{\partial x^2} - L'C'\frac{\partial^2 I}{\partial t^2} &= (L'G' + R'C')\frac{\partial I}{\partial t} + R'G'I\,, \end{aligned}}$$

solutions = forw. & backw. running waves

$$\boxed{V(x,t) = V_0\, e^{i\omega t - \gamma x} = V_0\, e^{\,i(\omega t \mp \beta x)}\, e^{\mp\alpha x}}$$

damping

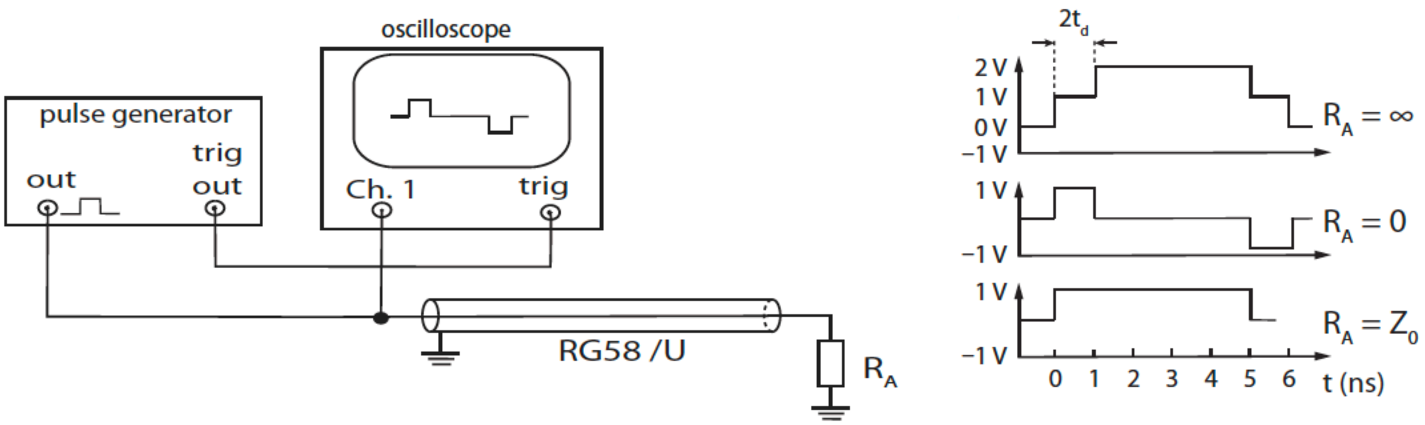
$$c_{ph} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\epsilon\epsilon_0\mu\mu_0}}$$

propagation velocity

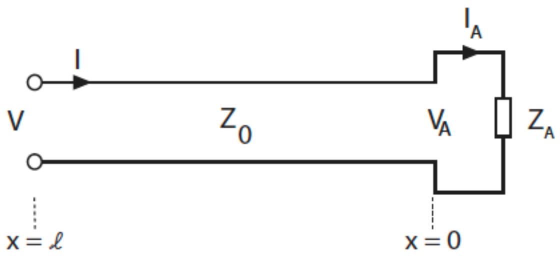
$$Z_0 = \frac{V}{I} = \sqrt{\frac{L'}{C'}}$$

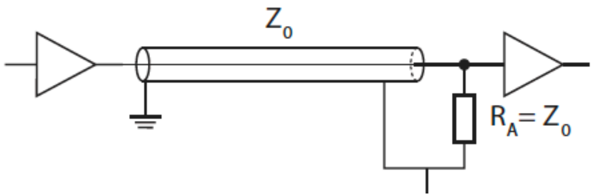
wave impedance of transmission line

Avoid reflections at unterminated ends …



Termination schemes





standard
(receiving-end termination)

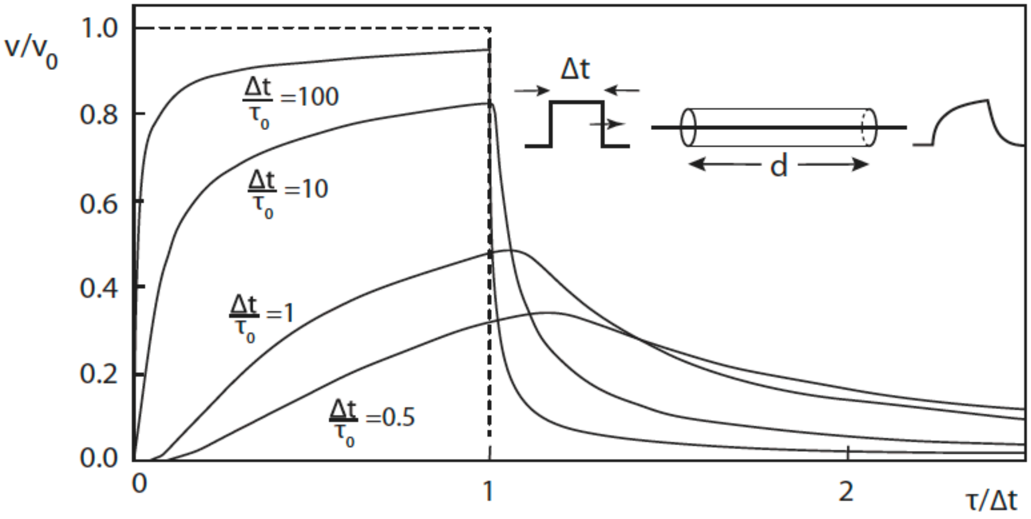
(additional) sending-end termination
however: 50% reduction of signal from left.

note: -▷- have Z_{in} = high, Z_{out} = low

Cable damping and eye diagram

$$V(x,t) = V_0\, e^{i\omega t - \gamma x} = V_0\, e^{\,i(\omega t \mp \beta x)}\, \boxed{e^{\mp \alpha x}} \quad \Longrightarrow \quad v(t) = v_0\, \mathrm{erfc}\left(\frac{1}{2\sqrt{\tau/\tau_0}}\right)$$

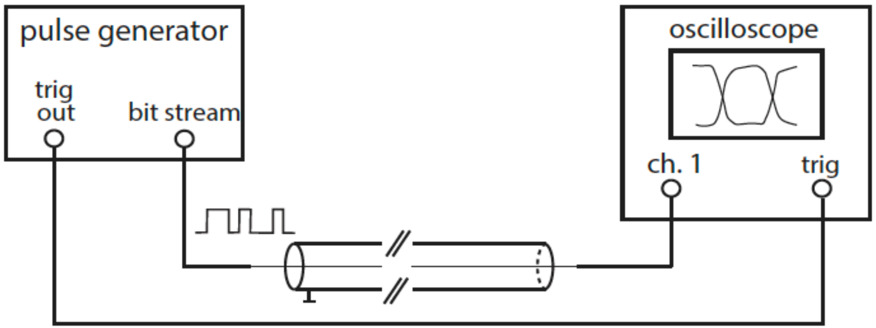
τ_0 = cable-characteristic, frequency-dependent rise time function



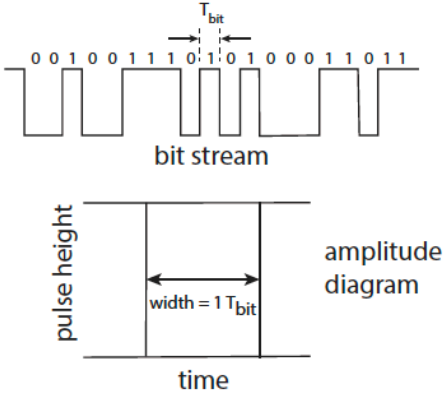
$\tau = t - d/c_{ph}$

pulse length Δt

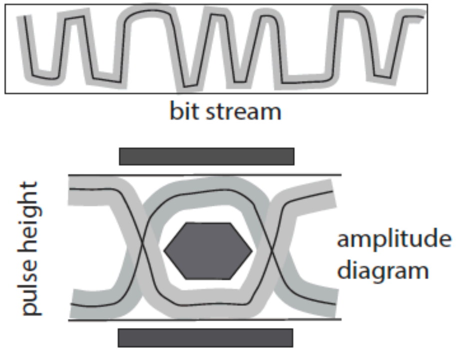
Cable damping and eye diagram



(a) Measurement set-up.



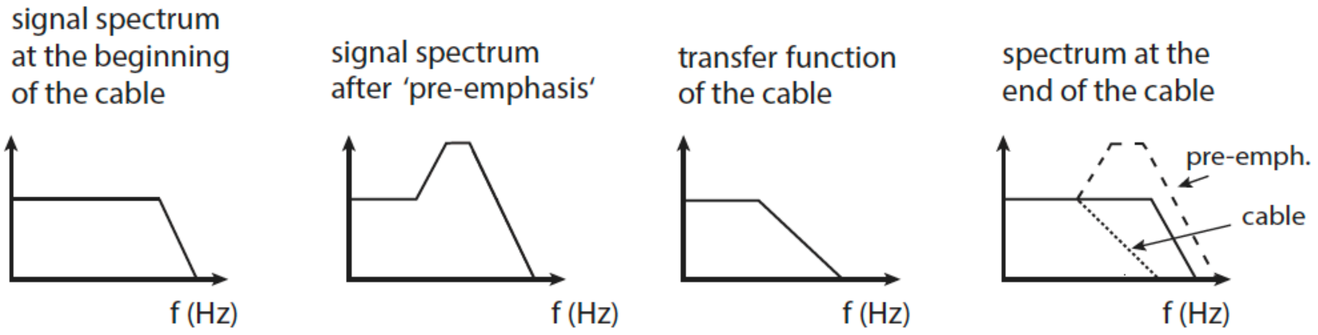
(b) Ideal transmission.



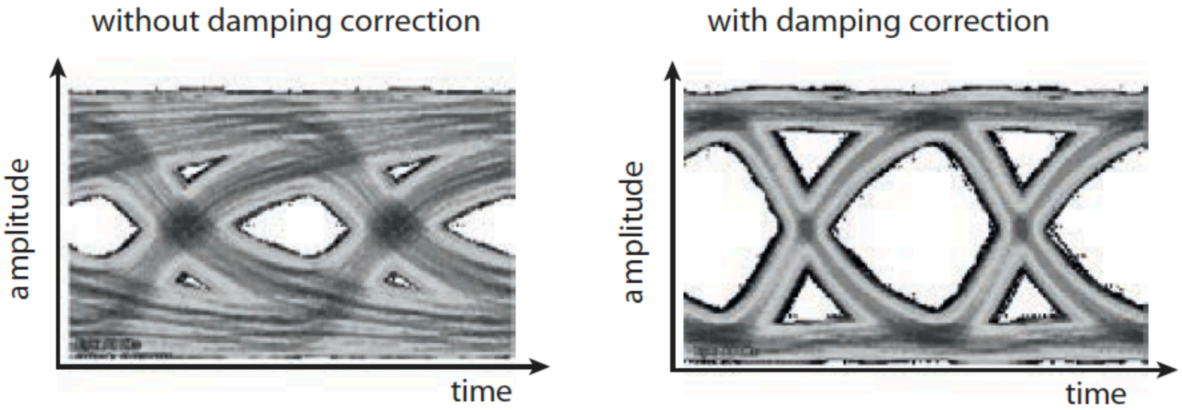
(c) Real transmission.

Kolanoski, Wermes 2015

Pre-emphasis

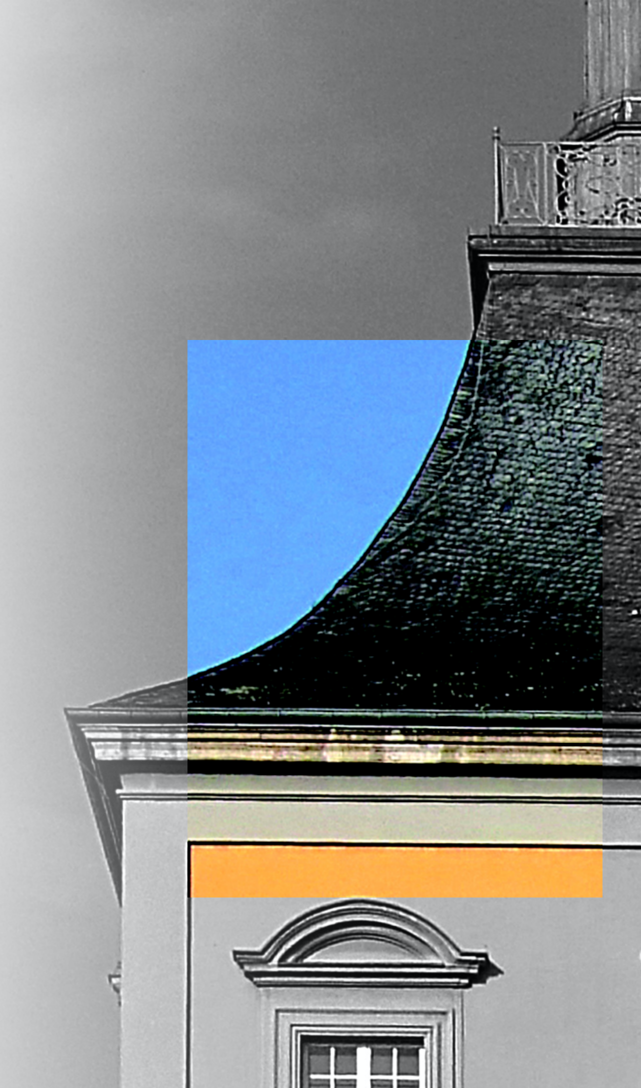


(a) Damping correction by means of *pre-emphasis*-filter (double logarithmic represesntation).

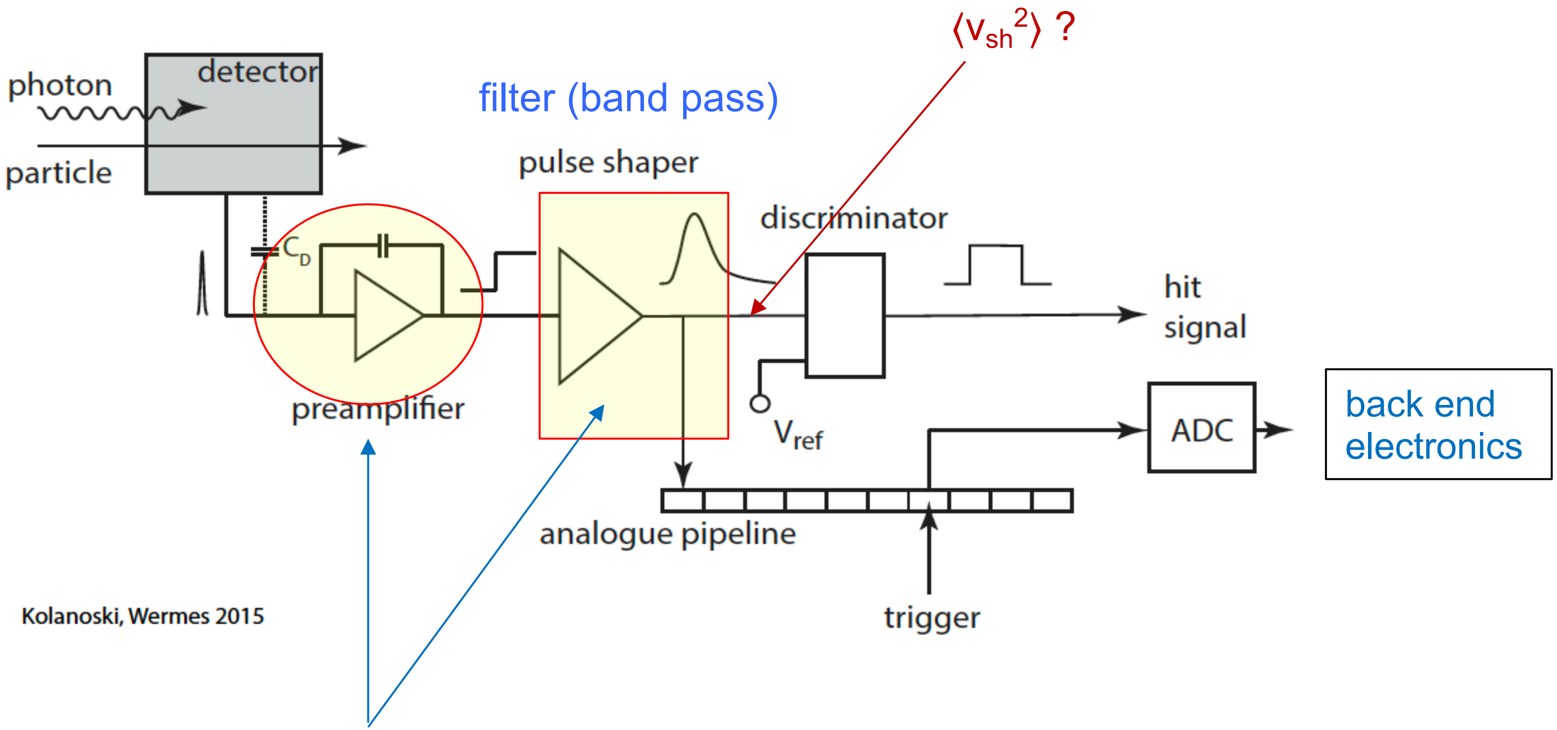


(b) Eye diagram without (left) and with (right) correction.

Noise
of a readout system

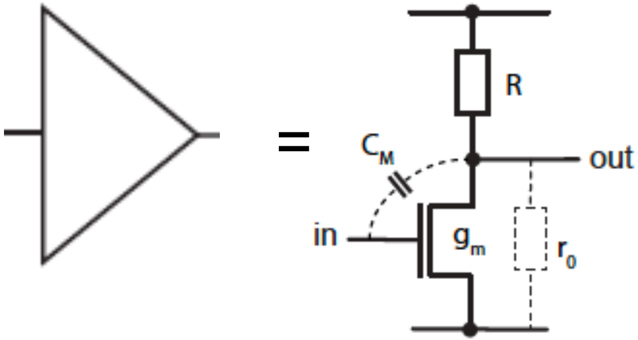


Generic R/O scheme: the dominant noise components

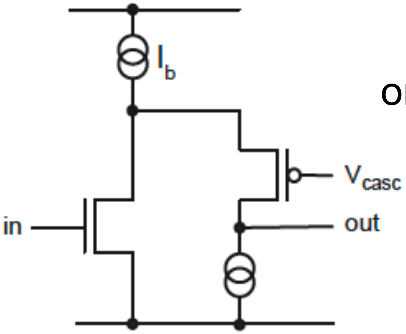


the dominant noise of a system is hidden in these parts

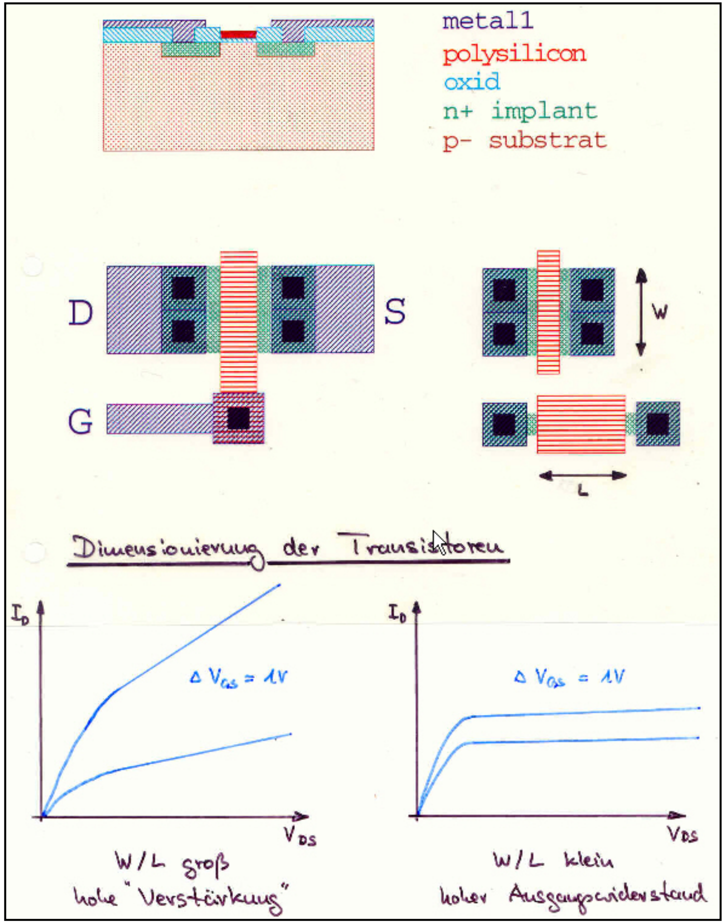
Most critical wrt noise: the (pre)amplifier



or

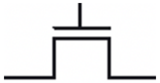


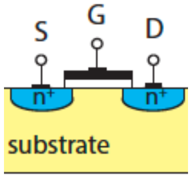
or ….

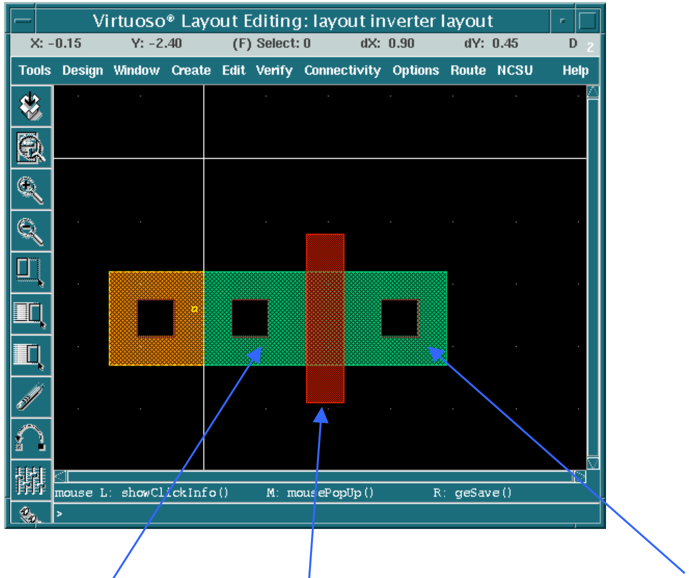


Layout: nMOS and pMOS

MOS transistor



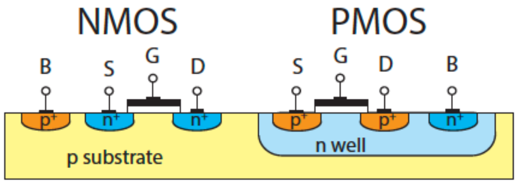


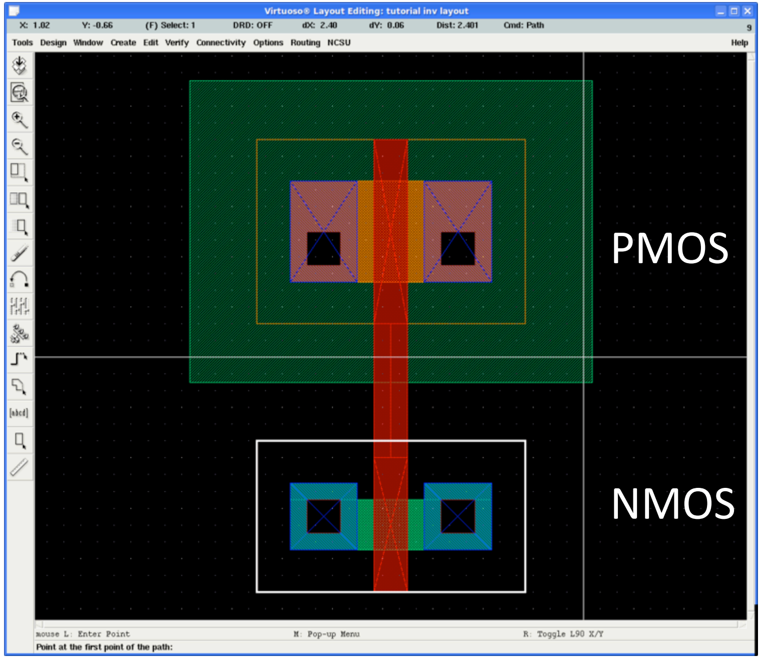


Source

Gate (area = W × L)

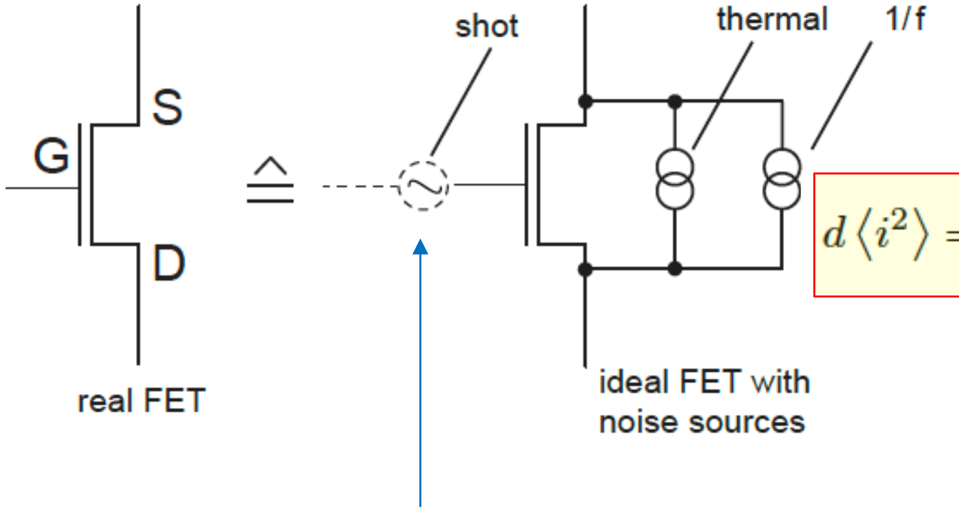
Drain





Noise in a MOSFET

$$\boxed{\frac{d\langle i^2\rangle_{\text{therm}}}{df}} = \frac{4kT}{R}df \rightarrow \frac{4kT}{\frac{1}{2/3g_m}} \boxed{= 4kT\frac{3}{2}g_m}$$

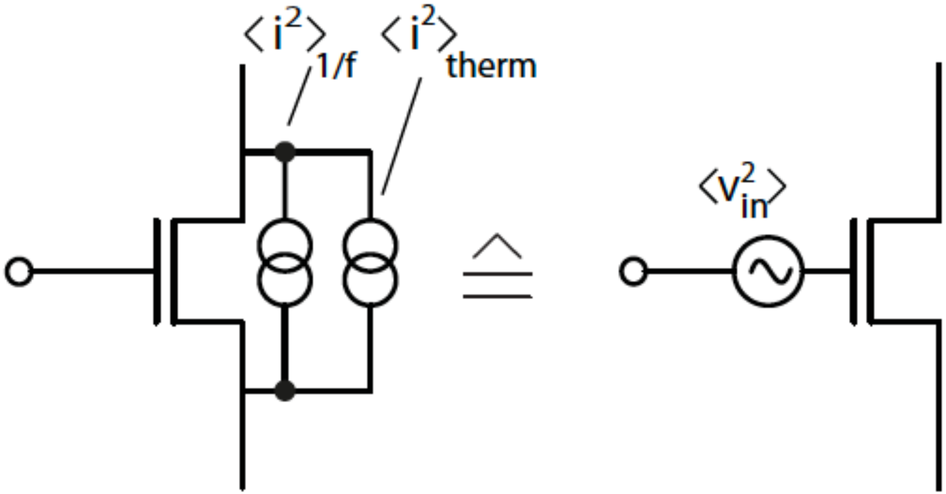


$$\boxed{d\left\langle i^2\right\rangle = K_\alpha \cdot \frac{1}{f^\alpha}df}$$

if shot noise input exists
(e.g. from leakage current of a Si detector)

Noise in a MOSFET (current or voltage?)

“parallel current noise can be described by serial voltage noise”



$$\langle i_D^2 \rangle = \langle (g_m v_{\rm in})^2 \rangle \qquad\qquad g_m = \partial I_D / \partial V_{GS} \propto \sqrt{I_D} \propto \sqrt{W/L}$$

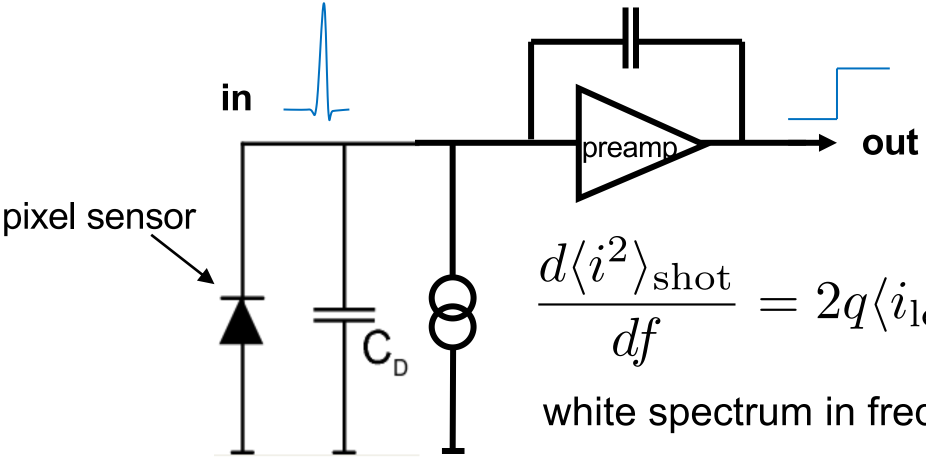
(in saturation)

Noise in an ionisation (typical here semiconductor) detector

three physical noise sources:

number fluctuations of quanta → 1. shot noise and 2. 1/f noise
velocity fluctuations of quanta → 3. thermal noise

where do they appear in a typical pixel detector readout chain ?



$$\frac{d\langle i^2\rangle_{\mathrm{shot}}}{df} = 2q\langle i_{\mathrm{leak}}\rangle$$

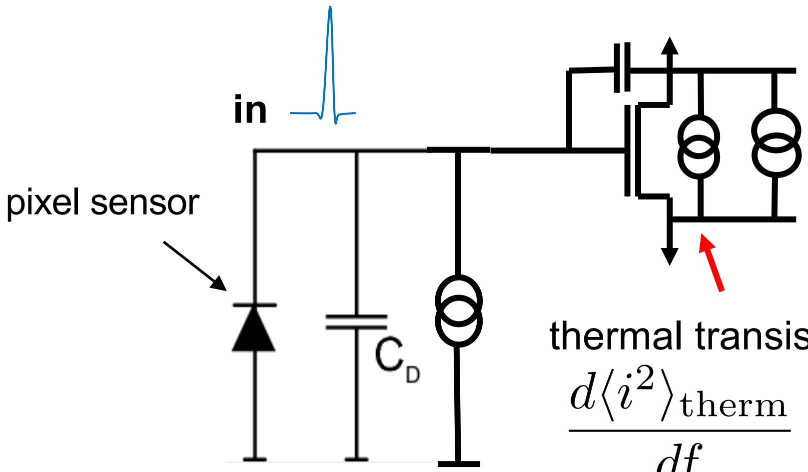
white spectrum in frequency f

Noise in a typical semiconductor detector

three physical noise sources:

number fluctuations of quanta → 1. shot noise and 2. 1/f noise
velocity fluctuations of quanta → 3. thermal noise

where do they appear in a typical pixel detector readout chain ?



transistor 1/f noise

$$d\left\langle i^2 \right\rangle = K_\alpha \cdot \frac{1}{f^\alpha}\, df$$

thermal transistor channel (white) noise

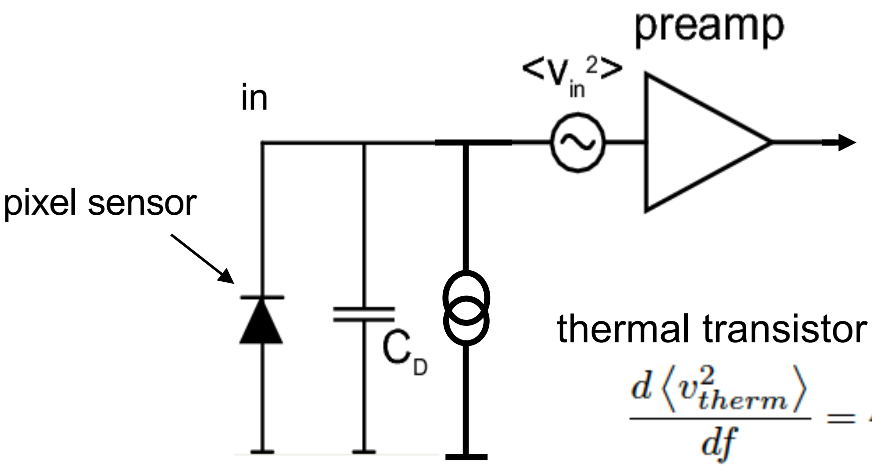
$$\frac{d\langle i^2 \rangle_{\rm therm}}{df} = 4kT\frac{3}{2}g_m$$

Noise in a typical semiconductor detector - III

three physical noise sources:

number fluctuations of quanta → 1. shot noise and 2. 1/f noise
velocity fluctuations of quanta → 3. thermal noise

where do they appear in a typical pixel detector readout chain ?



transistor 1/f noise

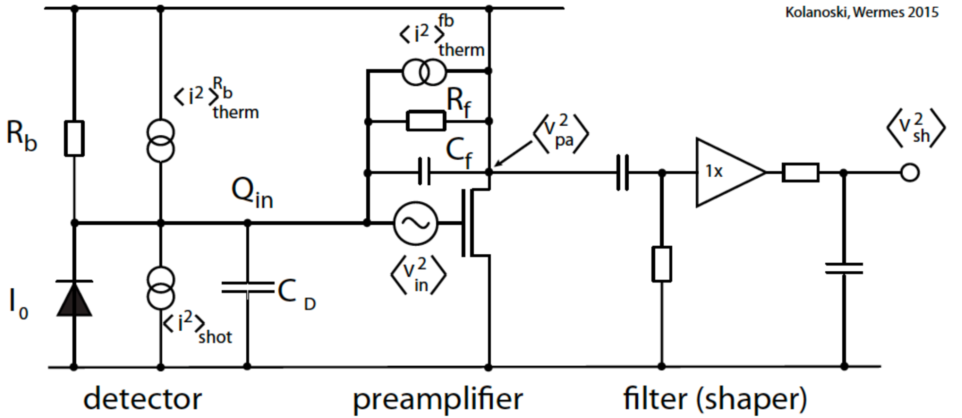
$$\frac{d\left\langle v_{1/f}^2\right\rangle}{df} = K_f \frac{1}{C'_{ox} W L} \frac{1}{f}$$

thermal transistor channel noise

$$\frac{d\left\langle v_{therm}^2\right\rangle}{df} = 4kT \frac{2}{3} \frac{1}{g_m}$$

CR – RC shaper (N=M=1)

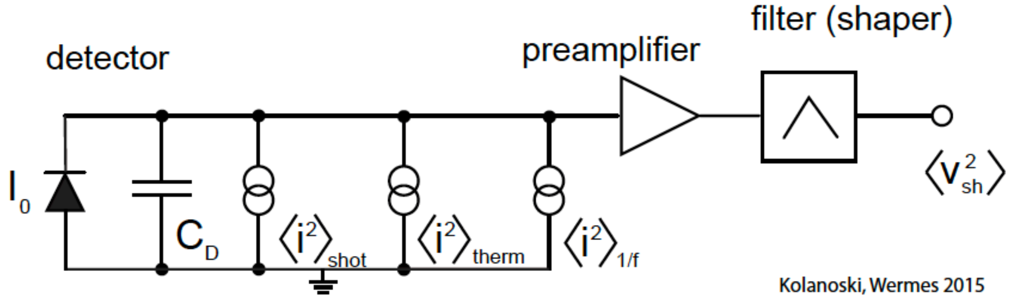
the situation discussed so far …



It is useful to treat the serial voltage noise sources (1/f and thermal noise) as equivalent parallel current noise via the capacitance C_D at the input of the preamplifier. This is possible via the relationship

$$\langle i_{in}^2 \rangle \approx \langle v_{\rm in}^2 \rangle \, (\omega \boxed{C_D})^2 \qquad (\text{with } \omega = 2\pi f)$$

equivalent circuit



We then get …

$$\frac{d\left\langle i_{\text{in}}^2\right\rangle}{d\omega} = \underbrace{2eI_0\frac{1}{2\pi}}_{\text{‘shot’}} + \underbrace{\frac{K_f}{C_{ox}WL}\frac{1}{f}(\omega C_D)^2}_{\text{‘1/f’}} + \underbrace{\frac{4kT}{g_m}\frac{2}{3}(\omega C_D)^2}_{\text{‘thermal’}}$$

This noise current, flowing through the feedback capacitance C_f , generates a noise voltage behind the preamplifier:

$$\left\langle v_{\text{pa}}^2\right\rangle = \left\langle i_{\text{in}}^2\right\rangle\left(\frac{1}{\omega C_f}\right)^2$$

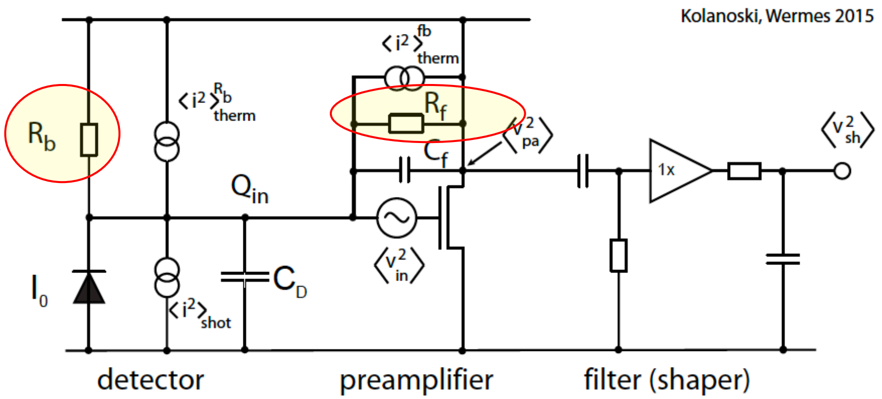
$$\begin{aligned}\frac{d\left\langle v_{\text{pa}}^2\right\rangle}{d\omega} &= \frac{eI_0}{\pi\omega^2 \boxed{C_f^2}} + K_f\frac{1}{C'_{ox}WL}\boxed{\frac{C_D^2}{C_f^2}}\frac{1}{\omega} + \frac{4}{3\pi}\frac{kT}{g_m}\boxed{\frac{C_D^2}{C_f^2}} \\ &= \sum_{k=-2}^{0} c_K\omega^k\end{aligned}$$

$$c_{-2} = \frac{e}{\pi}I_0\frac{1}{C_f^2}\,, \qquad c_{-1} = K_f\frac{1}{C_{\text{ox}}WL}\frac{C_D^2}{C_f^2}\,, \qquad c_0 = \frac{4}{3\pi}kT\frac{1}{g_m}\frac{C_D^2}{C_f^2}$$

… what about thermal noise in R-feedback ?

$$\frac{d\langle i^2\rangle_{\rm therm}}{df} = \frac{4kT}{R}df$$

=> choose R_f large enough!



Is/was argued to be small …
it acts on the preamplifier input in a very similar way as the leakage current shot noise contribution, i.e.

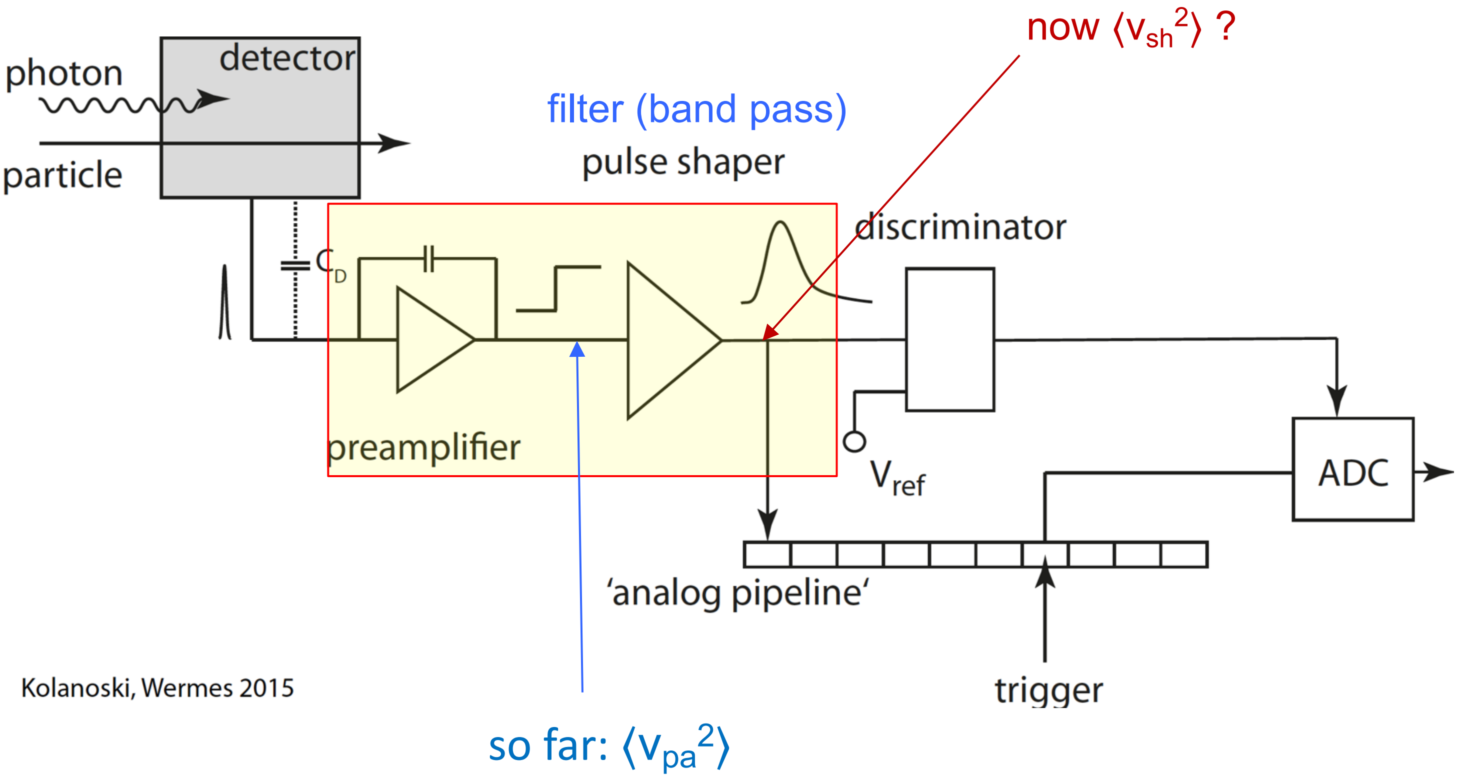
$$\frac{d\left\langle v_{\rm pa}^2\right\rangle}{d\omega} = \frac{eI_0}{\pi\omega^2 C_f^2}$$

$$2eI_0 \rightarrow \frac{4kT}{R_f}$$

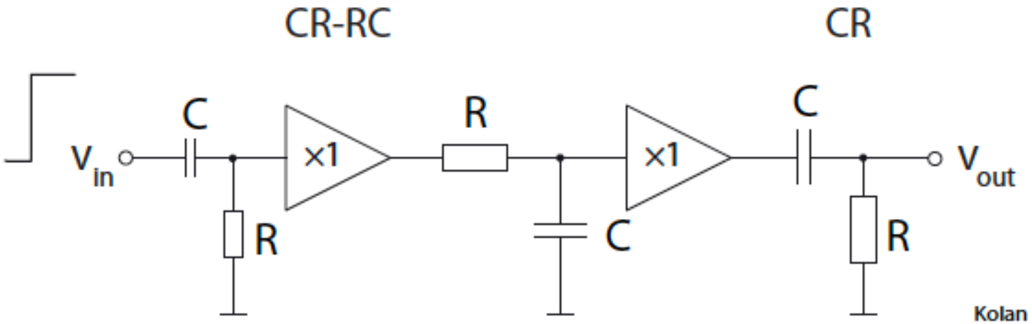
$$\frac{d\left\langle v_{\rm pa}^2\right\rangle_{R_f}}{d\omega} = \frac{2kT}{R_f}\frac{1}{\pi\omega^2 C_f^2}$$

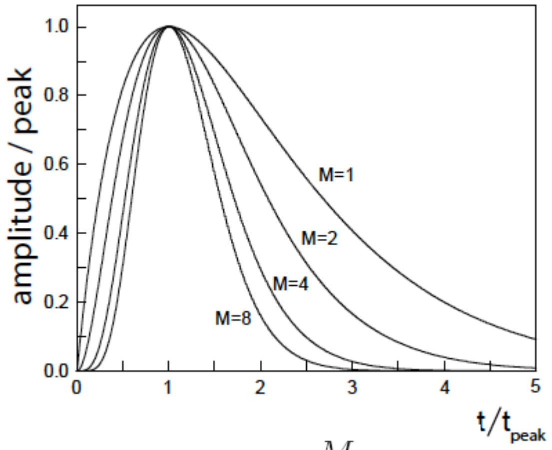
Its magnitude is usually small in comparison to the other contributions, in particular to the leakage-current-induced shot noise.

Noise in a preamp + shaper system

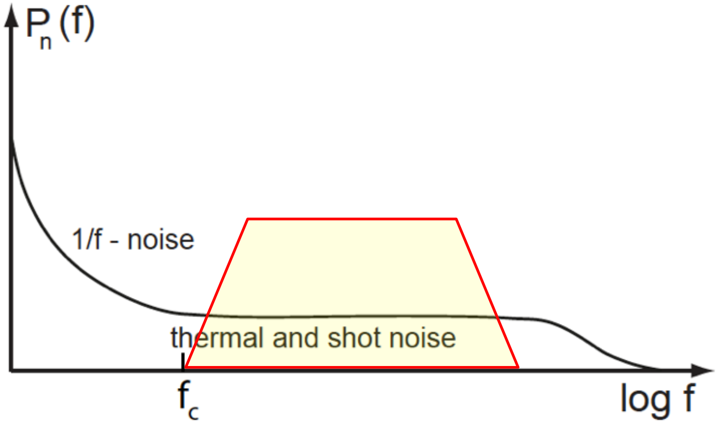


Shaper function





$$f^{(1,M)}(t) = \frac{1}{M!}\left(\frac{t}{\tau}\right)^M e^{-t/\tau}$$

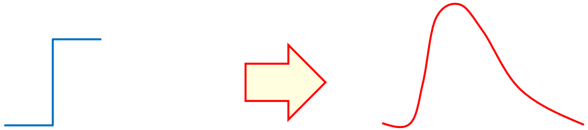


consequences for noise

BW limitation => lower noise
on the expense of speed

Shaper

in the time domain



$$v_{sh}(t) = A\frac{t}{\tau}\,\mathrm{e}^{-\frac{t}{\tau}}$$

peaks at τ with height A/2.71

in the frequency domain $\quad 1/s$

$$|H(\omega)|^2 = A^2\left(\frac{\omega\tau}{1+\omega^2\tau^2}\right)^2 \quad (\text{with } s \to i\omega)$$

for noise
need square

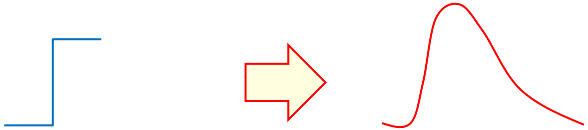
$$\langle v_{\rm sh}^2\rangle = \int_0^\infty \frac{d\langle v_{\rm pa}^2\rangle}{d\omega}\,|H(\omega)|^2\,d\omega$$

$$\langle v_{\rm sh}^2\rangle = \sum_{k=-2}^{0}\int_0^\infty c_k\omega^k\,|H(\omega)|^2\,d\omega$$

$$= A^2\frac{1}{2}\sum_{k=-2}^{0} c_k\tau^{-k-1}\,\Gamma\left(1+\frac{k+1}{2}\right)\Gamma\left(1-\frac{k+1}{2}\right)$$

$$\Gamma(x+1) = x\Gamma(x), \qquad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}, \qquad \Gamma(1) = 1$$

in the time domain



$$v_{sh}(t) = A\frac{t}{\tau}\,\mathrm{e}^{-\frac{t}{\tau}}$$

peaks at τ with height A/2.71

in the frequency domain $\quad 1/s \quad |H(\omega)|^2 = A^2\left(\frac{\omega\tau}{1+\omega^2\tau^2}\right)^2 \quad (\text{with } s \rightarrow i\omega)$

$$\boxed{\langle v_{\mathrm{sh}}^2\rangle = \frac{\pi}{4}A^2\left(c_{-2}\,\tau + \frac{2}{\pi}c_{-1} + c_0\frac{1}{\tau}\right)}$$

with

$$c_{-2} = \frac{e}{\pi}I_0\frac{1}{C_f^2}\,, \qquad c_{-1} = K_f\frac{1}{C'_{ox}WL}\frac{C_D^2}{C_f^2}\,, \qquad c_0 = \frac{4}{3\pi}kT\frac{1}{g_m}\frac{C_D^2}{C_f^2}$$

Equivalent Noise Charge (ENC)

$$\mathrm{ENC} = \frac{\text{noise output voltage (V)}}{\text{output voltage of a signal of } 1\,\mathrm{e}^- \ (\mathrm{V}/\mathrm{e}^-)}$$

$$\mathrm{ENC}^2 = \frac{\langle v_{\mathrm{sh}}^2 \rangle}{v_{\mathrm{sig}}^2}$$

Equivalent noise charge

for 1e at the input we get

$$v_{\mathrm{sig}} = \frac{A}{2.71}\,\frac{e}{C_f}$$

peak of shaper pulse

$$\Rightarrow \quad \mathrm{ENC}^2 = \frac{\langle v_{\mathrm{sh}}^2 \rangle}{v_{\mathrm{sig}}^2}$$

$$\boxed{\mathrm{ENC}^2(e^{-2}) = \frac{(2.71)^2}{4e^2}\left(eI_0\tau + 2C_D^2\,K_f\,\frac{1}{C'_{ox}WL} + \frac{4}{3}\frac{kT}{g_m}\frac{C_D^2}{\tau}\right)}$$

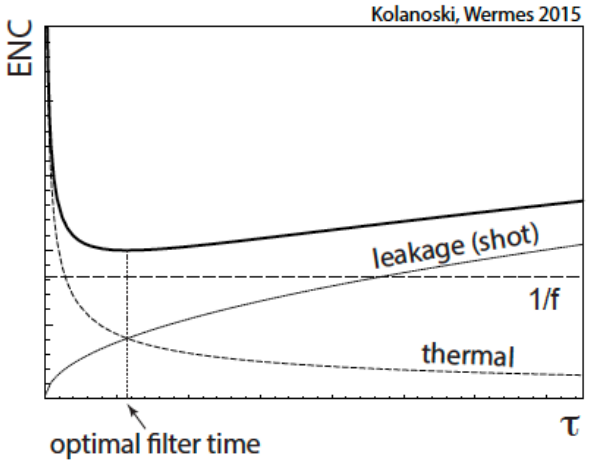
$$\mathrm{ENC}^2 = a_{\mathrm{shot}}\,\tau + a_{1/\mathrm{f}}\,C_D^2 + a_{\mathrm{therm}}\,\frac{C_D^2}{\tau}$$

$$\boxed{\frac{\mathrm{ENC}^2}{e^2} = 11\,\frac{I_0}{\mathrm{nA}}\,\frac{\tau}{\mathrm{ns}} + 740\,\frac{1}{WL/(\mu\mathrm{m}^2)}\,\frac{C_D^2}{(100\,\mathrm{fF})^2} + 4000\,\frac{1}{g_m/\mathrm{mS}}\,\frac{C_D^2/(100\,\mathrm{fF})^2}{\tau/\mathrm{ns}}}$$

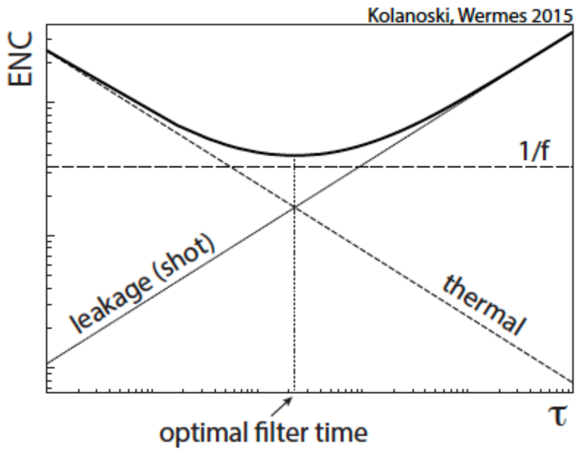
Optimal filter time

$$\mathrm{ENC}^2 = a_{\mathrm{shot}}\,\tau + a_{1/\mathrm{f}}\,C_D^2 + a_{\mathrm{therm}}\,\frac{C_D^2}{\tau}$$

$$\tau_{\mathrm{opt}} = \left(\frac{a_{\mathrm{therm}}}{a_{\mathrm{shot}}}\,C_D^2\right)^{1/2} = \left(\frac{4kT}{3\,eI_0 g_m}\,C_D^2\right)^{1/2}$$



(a) Linear representation.



(b) Double logarithmic representation.

Examples

Pixel detector. As an example featuring small electrodes and correspondingly small input capacitances we choose a silicon pixel detector (section 8.7) with parameters $C_D = 200\,\text{fF}$, $I_0 = 1\,\text{nA}$, $\tau = 50\,\text{ns}$, $W = 20\,\mu\text{m}$, $L = 0.5\,\mu\text{m}$, $g_m = 0.5\,\text{mS}$, where we assumed a typical leakage current before the detector received substantial radiation damage. With (17.110) an equivalent noise charge of

$$\text{ENC}^2 \approx (24\,e^-)^2(\text{shot}) + (17\,e^-)^2(1/\text{f}) + (25\,e^-)^2(\text{therm}) \approx \boxed{(40\,e^-)^2}$$

Strip detector. For a typical silicon microstrip detector (see section 8.6.2) after radiation damage one obtains with $C_D = 20\,\text{pF}$, $I_0 = 1\,\mu\text{A}$, $\tau = 50\,\text{ns}$, $W = 2000\,\mu\text{m}$, $L = 0.4\,\mu\text{m}$, $g_m = 5\,\text{mS}$:

$$\text{ENC}^2 \approx (750\,e^-)^2(\text{shot}) + (200\,e^-)^2(1/\text{f}) + (800\,e^-)^2(\text{therm}) = \boxed{(1100\,e^-)^2}.$$

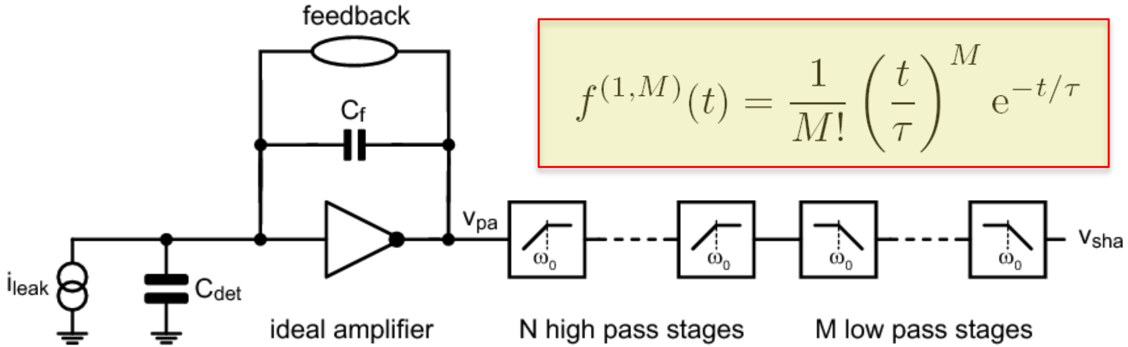
Liquid argon calorimeter. As an example of a detector with a large electrode capacitance we take a liquid argon calorimeter cell with typical values as given by the ATLAS electromagnetic calorimeter (see section 15.5.3.2 on page 597) in the central region. With the parameters $C_D = 1.5\,\text{nF}$, $I_0 = <2\,\mu\text{A}$, $\tau = 50\,\text{ns}$, $W = 3000\,\mu\text{m}$, $L = 0.25\,\mu\text{m}$, $g_m = 100\,\text{mS}$, i.e. assuming only a small (negligible) parallel shot noise (leakage current), one obtains:

$$\text{ENC}^2 \approx (1000\,e^-)^2(\text{shot}) + (15000\,e^-)^2(1/\text{f}) + (13500\,e^-)^2(\text{therm}) \approx \boxed{(20200\,e^-)^2}.$$

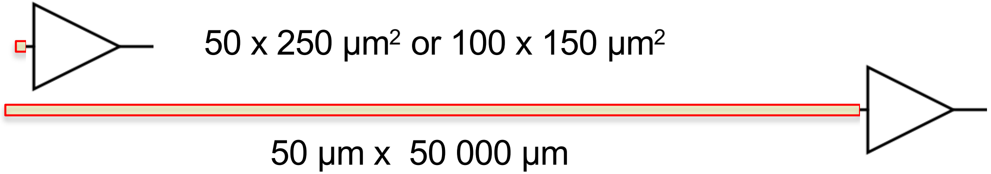
Noise in a pixel/strip/liq.Ar detector (ionisation detector)

… with CSA preamplifier & shaper

$$f^{(1,M)}(t) = \frac{1}{M!}\left(\frac{t}{\tau}\right)^{M} e^{-t/\tau}$$



comparing pixels
and strips



	C_D	I_0	τ	W	L	g_m	ENC therm	ENC 1/f	ENC shot	ENC tot
pixel	200 fF	1 nA	50 ns	20 μm	0.5 μm	0.5 mS	25 e-	17 e-	24 e-	40 e-
strip	20 pF	1 μA	50 ns	2000 μm	0.4 μm	5 mS	800 e-	200 e-	750 e-	1100 e-
liq. Ar	1.5 nF	2 μA	50 ns	3000 μm	0.25 μm	100 mS	1000 e-	15 000 e-	13 500 e-	20200 e-

Some selected literature … Detectors and/or Readout Techniques

UNIVERSITÄT BONN

- Rossi, Fischer, Rohe, Wermes, Pixel Detectors: from Fundamentals to Applications, Springer 2006

- Spieler, Semiconductor Detector Systems, Oxford Univ. Press. 2005
- Blum, Riegler, Rolandi, Particle Detection with Wire Chambers, 2010

- Leroy and Rancoita, Radiation Interaction in Matter and Detection, World Scientific, 4^{th} ed, 2016
- Leroy and Rancoita, Silicon Solid State Devices and Radiation Detection, 2012

- Grupen and Shwartz, Particle Detectors, Cambridge Univ. Press, 2008

- Kolanoski, Wermes, Teilchendetektoren … Grundlagen und Anwendungen (2016) in German
- Kolanoski, Wermes, Particle Detectors ... Fundamentals and Applications (2nd Edition in English, available in 2019)

- M. Garcia-Sciveres, N. Wermes, Review of advances in pixel detectors for experiments with high rate and radiation, Reports on Progress in Physics 81 (2018) no.6, 066101