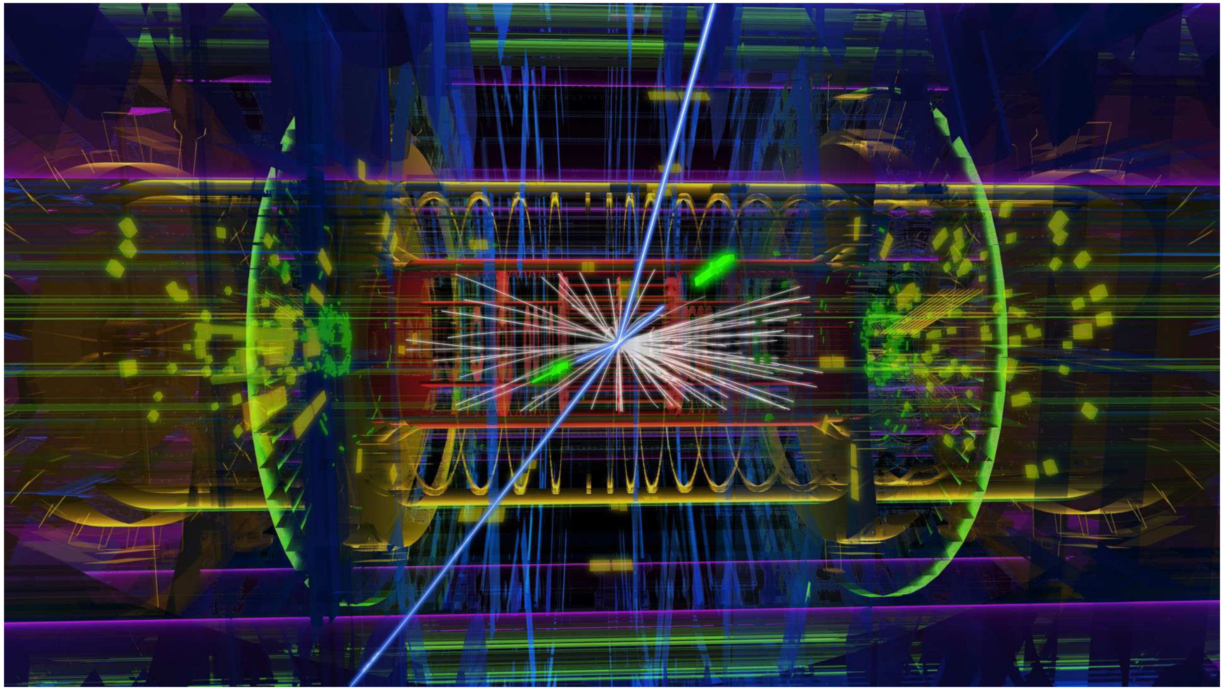
Electroweak Physics at the LHC

— Lecture 1 —

Electroweak Issues and Higher-Order Corrections



Stefan Dittmaier

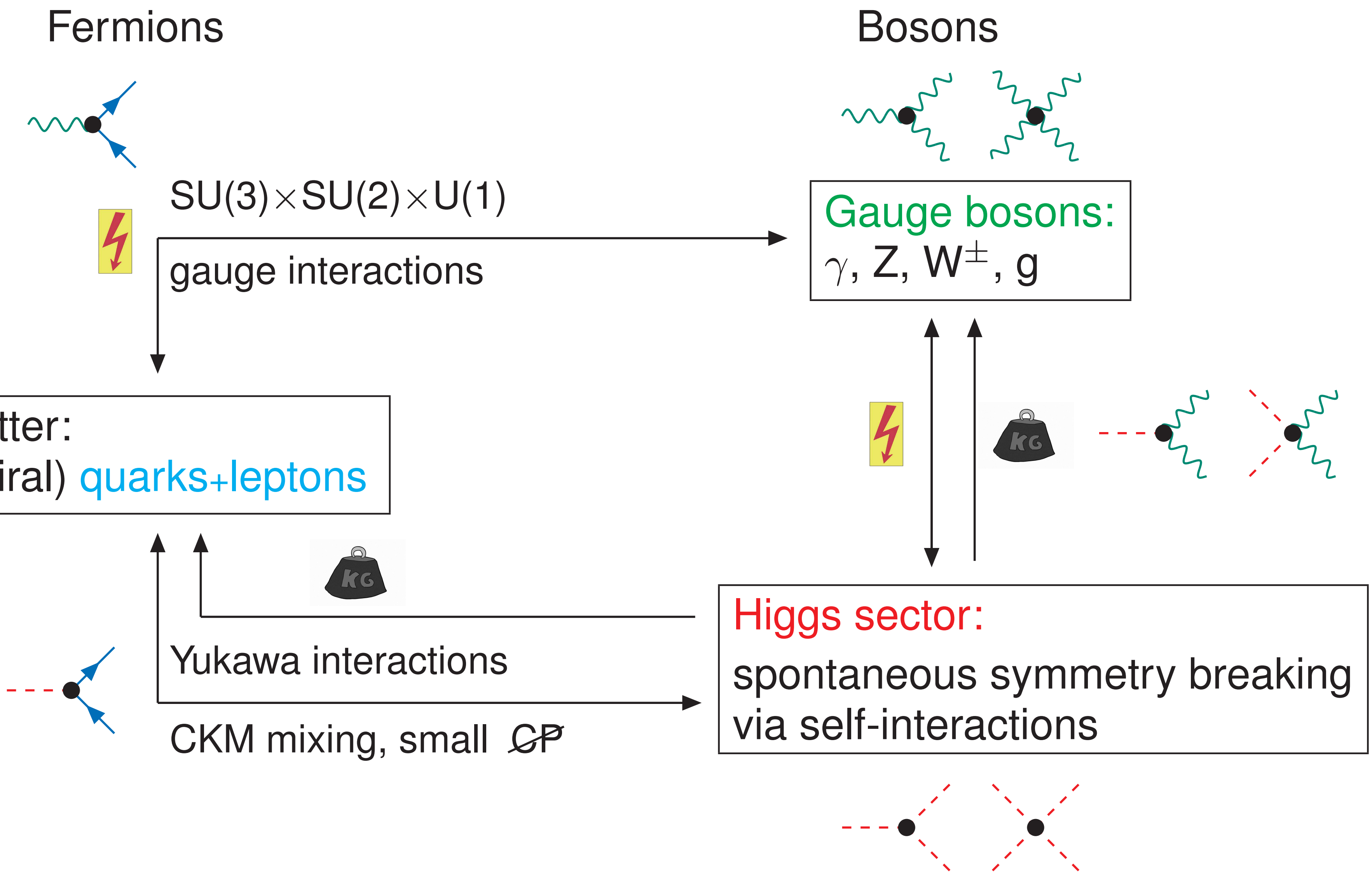
Albert-Ludwigs-Universität Freiburg

Masse und Symmetrien nach der
GRK 2044
Higgs-Boson-Entdeckung am LHC

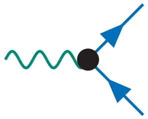
Contents

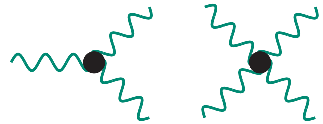
Recapitulation of the Standard Model

Structure and elementary interactions of the Standard Model



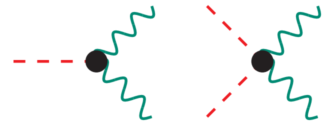
Structure and elementary interactions of the Standard Model

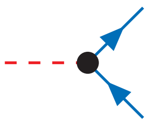


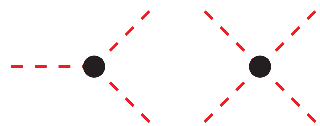


Test of the model

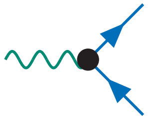
$\Leftrightarrow$ Exp. reconstruction of the $\underbrace{\text{elementary couplings}}_{\text{Feynman rules}}$

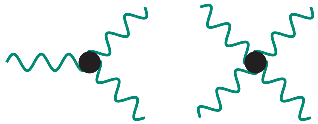






Structure and elementary interactions of the Standard Model

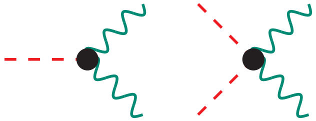


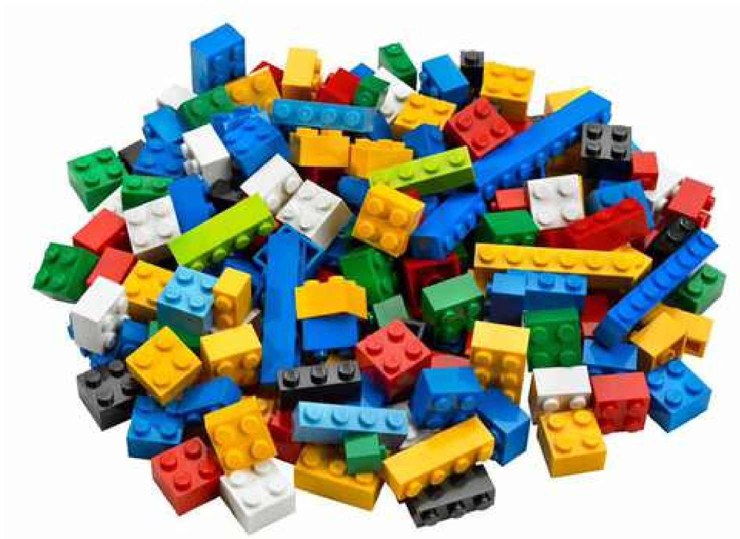


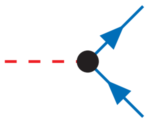
Test of the model

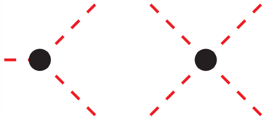
$\Leftrightarrow$ Exp. reconstruction of the $\underbrace{\text{elementary couplings}}_{\text{Feynman rules}}$

Building blocks for particle reactions

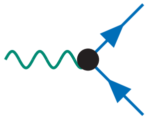


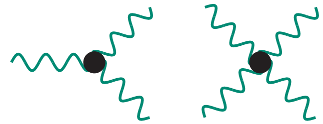






Structure and elementary interactions of the Standard Model

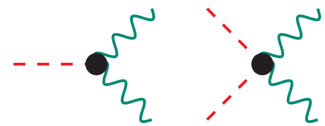


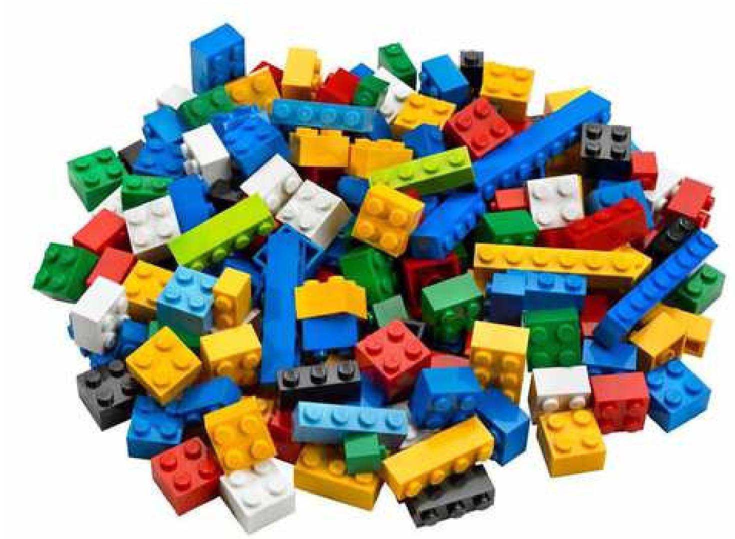


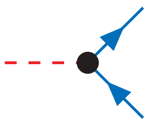
Test of the model

$\Leftrightarrow$ Exp. reconstruction of the $\underbrace{\text{elementary couplings}}_{\text{Feynman rules}}$

Building blocks for particle reactions







Standard Model extensions $\rightarrow$ more fields, more particles, more interactions, ...

Feynman rules derived from SM Lagrangian:

$\hookrightarrow$ Recapitulate EW gauge interactions !

Gauge-boson couplings to fermions

$\hookrightarrow$ induced by "minimal substitution" in free Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \sum_f \mathrm{i}\overline{\psi_f}\not{\partial}\psi_f$:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g_2 T_\mathrm{I}^a W_\mu^a + \mathrm{i}g_1 \frac{Y}{2} B_\mu$$

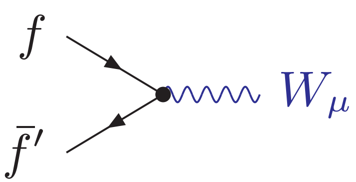
$$T_\mathrm{I}^a = \text{weak isopsin} = \begin{cases} \sigma^a/2 & \text{for left-handed } f \\ 0 & \text{for right-handed } f \end{cases}$$

Y = weak hypercharge, fixed by Gell-Mann–Nishijima relation $Q = T_\mathrm{I}^3 + Y/2$

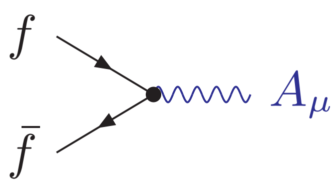
Identification of photon after "Weinberg rotation" about weak mixing angle θ_W:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\mathrm{W} & s_\mathrm{W} \\ -s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \quad \text{with } g_2 = \frac{e}{s_\mathrm{W}},\ g_1 = \frac{e}{c_\mathrm{W}}, \quad s_\mathrm{W} \equiv \sin\theta_\mathrm{W}$$

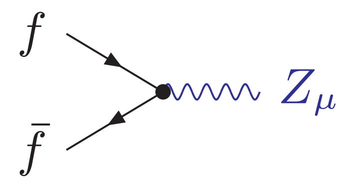
$\Rightarrow$ Interaction vertices:



$$\frac{\mathrm{i}e}{\sqrt{2}s_\mathrm{W}}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)$$

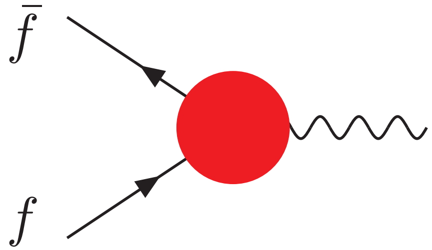


$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_{Vf} - g_{Af}\gamma_5), \qquad g_{Vf} = -\frac{s_\mathrm{W}}{c_\mathrm{W}}Q_f + \frac{T_{\mathrm{I},f}^3}{2c_\mathrm{W}s_\mathrm{W}}, \quad g_{Af} = \frac{T_{\mathrm{I},f}^3}{2c_\mathrm{W}s_\mathrm{W}}$$

Effective $\mathrm{Z}f\bar{f}$ couplings from $\mathrm{e^+e^-} \to \mathrm{Z}/\gamma^* \to f\bar{f}$ @ LEP1



$$\mathrm{Z}_\mu \quad = \quad \mathrm{i}e\gamma_\mu(\bar{g}_{Vf} - \bar{g}_{Af}\gamma_5)$$

Leptonic couplings from LEP1 asymmetry measurements, e.g.:

$$A_{\mathrm{FB}}^{0,f} = \frac{\sigma^0_{f,\mathrm{F}} - \sigma^0_{f,\mathrm{B}}}{\sigma^0_{f,\mathrm{F}} + \sigma^0_{f,\mathrm{B}}} = \frac{3}{4}\mathcal{A}_\mathrm{e}\mathcal{A}_f$$

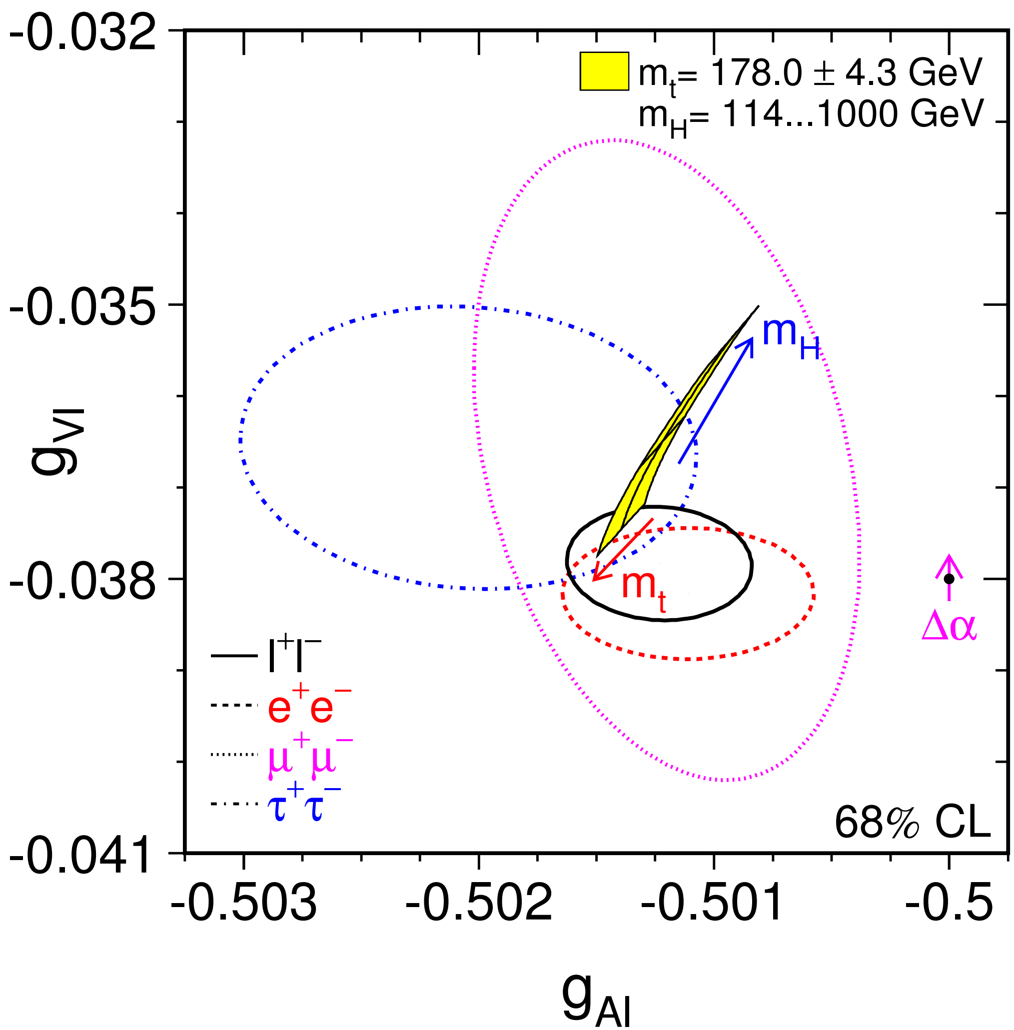
($\mathrm{F/B} =$ For/Backward hemisphere)

with $\mathcal{A}_f = \dfrac{2\bar{g}_{Vf}\bar{g}_{Af}}{\bar{g}^2_{Vf} + \bar{g}^2_{Af}}$

Good agreement with SM

- lepton universality confirmed
- constraints on m_t and M_H

LEPEWWG '05



Translation of effective couplings into effective weak mixing angle

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = \frac{1}{4}\left(1 - \mathrm{Re}\left\{\frac{g_{Vl}}{g_{Al}}\right\}\right)$$

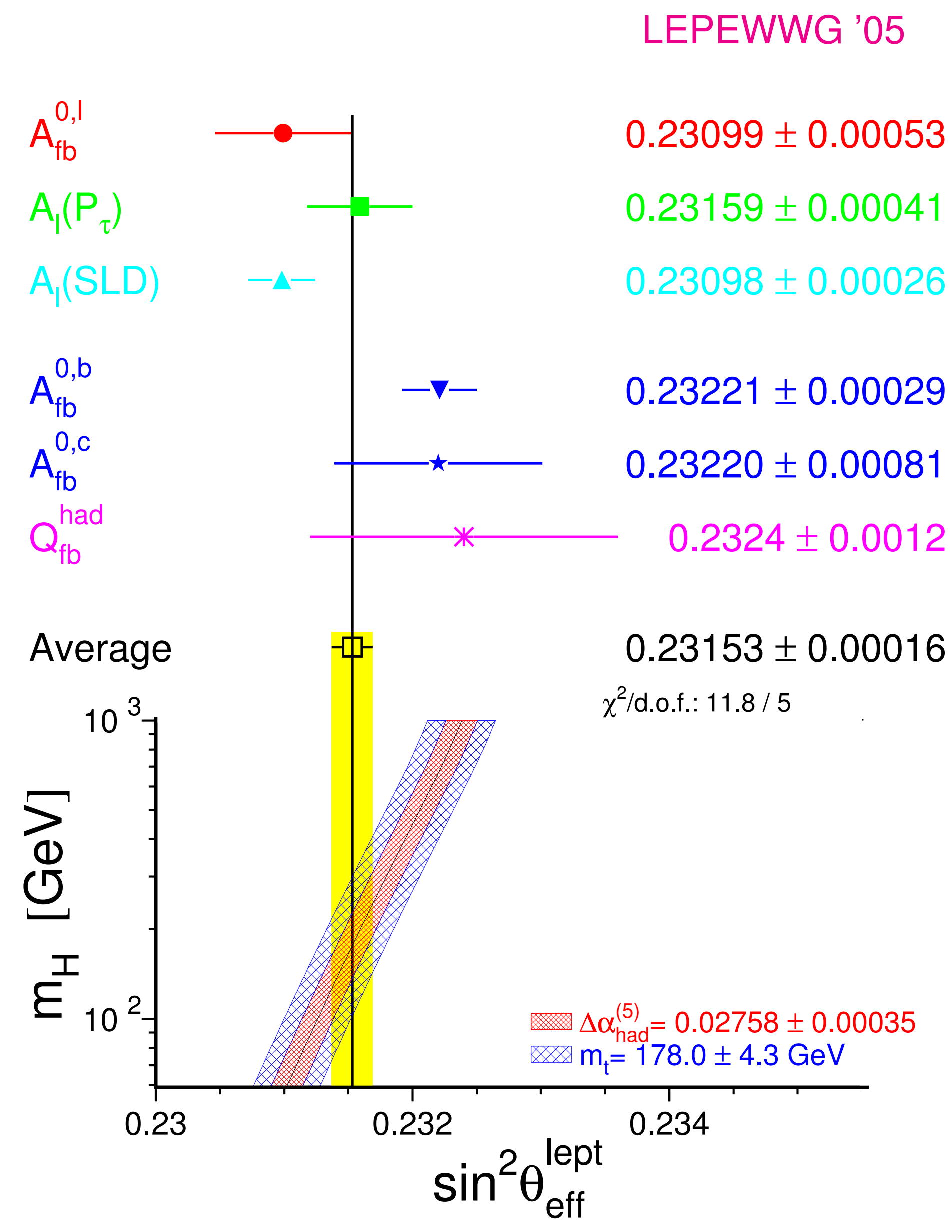
Important features:

- high sensitivity to M_{H}
- combination of very different observables
- $\sim 3\sigma$ difference between $A_{\mathrm{FB}}^{0,b}$(LEP) and $A_{\mathrm{LR}}^{0,l}$(SLD)

  with the initial-state pol. asymmetry

$$A_{\mathrm{LR}}^{0,l} = \frac{\sigma_{\mathrm{L}}^0 - \sigma_{\mathrm{R}}^0}{\sigma_{\mathrm{L}}^0 + \sigma_{\mathrm{R}}^0}\,\frac{1}{\langle|\mathcal{P}_{\mathrm{e}}|\rangle}$$

$\Rightarrow$ Precise LHC result on $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ highly desirable !



Gauge-boson self-interactions

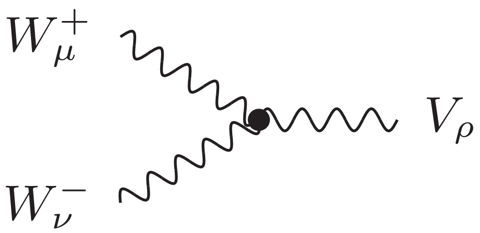
$\hookrightarrow$ induced by gauge-invariant Yang–Mills Lagrangian

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

with the field-strength tensors

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

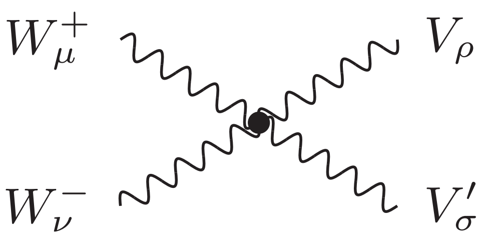
$\Rightarrow$ Feynman rules for gauge-boson self-interactions: (fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}$

$\rightarrow$ testable in di-boson production $\mathrm{ee/pp} \rightarrow VV$



$$\mathrm{i}e^2 C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

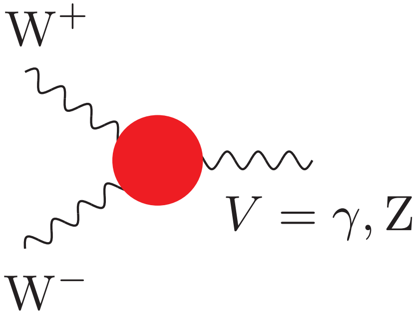
with $C_{W^2\gamma^2} = -1, C_{W^2\gamma Z} = \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}, C_{W^2Z^2} = -\frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}, C_{W^4} = \frac{1}{s_{\mathrm{W}}^2}$

$\rightarrow$ testable in tri-boson production $\mathrm{ee/pp} \rightarrow VVV$
and vector-boson scattering $\mathrm{pp}(VV \rightarrow VV) \rightarrow VV + 2\mathrm{jets}$

(Non-)standard TGCs

Gaemers, Gounaris ’79; Hagiwara, Hikasa, Peccei, Zeppenfeld ’87; Bilenky, Kneur, Renard, Schildknecht ’93; etc.

General parametrization (C- and P-conserving):



$$\mathcal{L}_{VWW} = -\mathrm{i}eg_{VWW}\Big\{ g_1^{\mathrm{V}}(W^+_{\mu\nu}W^{-,\mu}V^{\nu} - W^{-,\mu\nu}W^+_{\mu}V_{\nu}) + \kappa_{\mathrm{V}}W^+_{\mu}W^-_{\nu}V^{\mu\nu} + \frac{\lambda_{\mathrm{V}}}{M_{\mathrm{W}}^2}W^+_{\rho\mu}W^{-,\mu}_{\nu}V^{\nu\rho}\Big\}$$

Meaning for static W^+ bosons:

$$Q_{\mathrm{W}} = eg_1^{\gamma} = \text{electric charge} \quad (= e \text{ by charge conservation})$$

$$\mu_{\mathrm{W}} = \frac{e}{2M_{\mathrm{W}}}(g_1^{\gamma} + \kappa_{\gamma} + \lambda_{\gamma}) = \text{magnetic dipole moment}$$

$$q_{\mathrm{W}} = -\frac{e}{M_{\mathrm{W}}^2}(\kappa_{\gamma} - \lambda_{\gamma}) = \text{electric quadrupole moment}$$

Standard Model values:

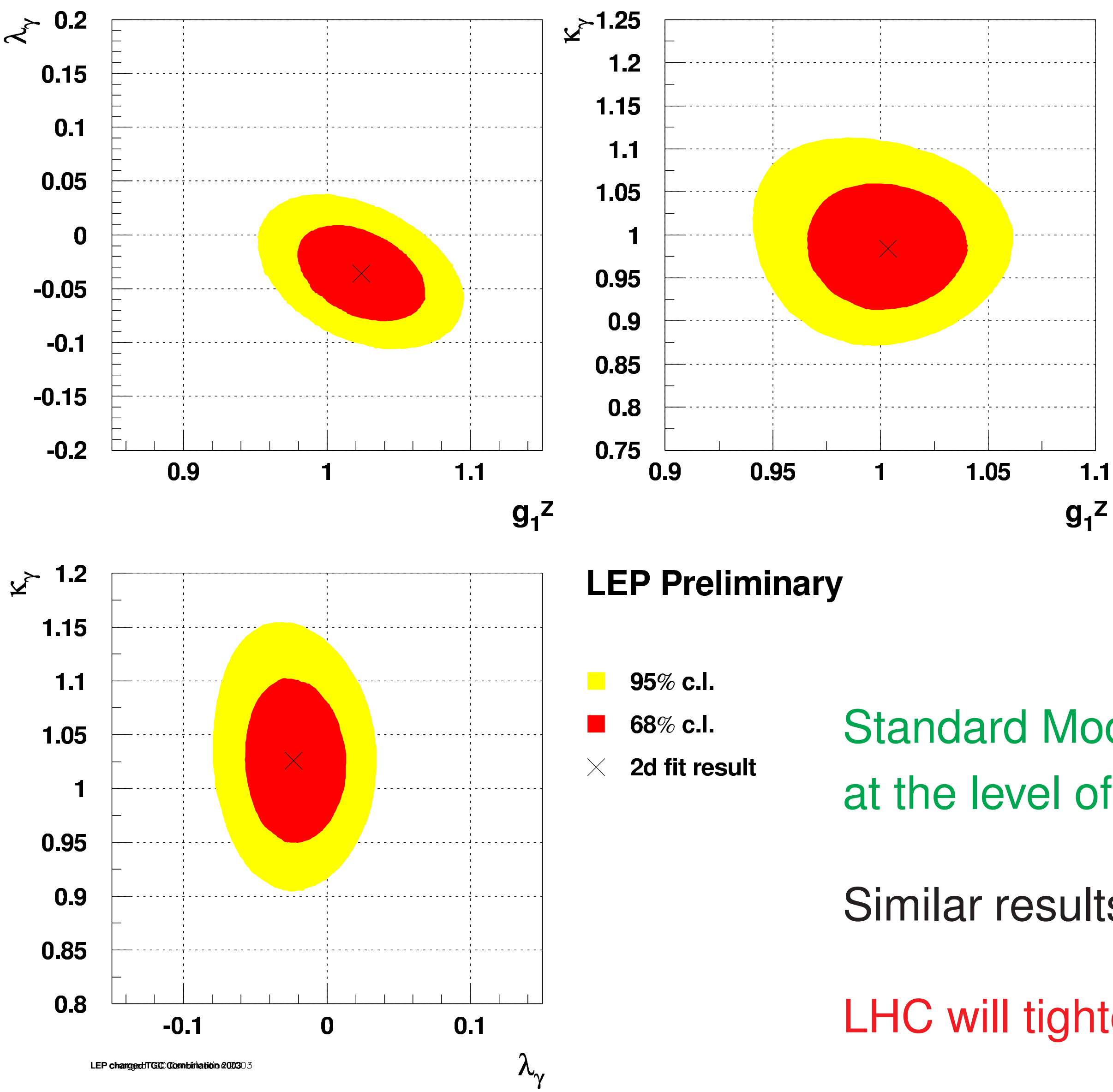
$$g_1^{\mathrm{V}} = \kappa_{\mathrm{V}} = 1, \qquad \lambda_{\mathrm{V}} = 0$$

Restriction to SU(2)×U(1)-symmetric dim-6 operators:

$$\kappa_{\mathrm{Z}} = g_1^{\mathrm{Z}} - (\kappa_{\gamma} - 1)\tan^2\theta_{\mathrm{W}}, \qquad \lambda_{\mathrm{Z}} = \lambda_{\gamma}$$

LEP2 constraints on charged TGCs from $e^+e^- \to WW \to 4f$

LEPEWWG '04



$$\Delta g_1^Z = -0.009^{+0.022}_{-0.021}$$

$$\Delta \kappa_\gamma = -0.016^{+0.042}_{-0.047}$$

$$\lambda_\gamma = -0.016^{+0.021}_{-0.023}$$

Standard Model values verified at the level of 2–4%

Similar results from Tevatron and LHC Run 1

LHC will tighten limits further !

Generic features of electroweak corrections

Relevance of EW corrections @ LHC

- 2015: LHC restarts @ $13-14\,\mathrm{TeV}$

  $\hookrightarrow$ energy reach extends deeper into TeV range

  $\hookrightarrow$ $\delta_{\mathrm{EW}} \sim$ some 10%

- integrated LHC luminosity will reach some $100\,\mathrm{fb}^{-1}$

  $\hookrightarrow$ many measurements at several-% level

  $\hookrightarrow$ typical size of δ_{EW}

- planned high-precision measurements: XS ratios, M_{W}, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$

  $\hookrightarrow$ δ_{EW} is crucial ingredient

Spirit of this lecture

- describe salient features of EW corrections, in particular enhancement effects
- prepare the ground for the discussion of W/Z production processes coming in the follow-up lectures
- give some recommendations from a theorist's point of view

Features of and issues in EW precision calculations

Relevance and size of EW corrections

generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$ suggests NLO EW $\sim$ NNLO QCD

but systematic enhancements possible, e.g.

- by photon emission
  $\hookrightarrow$ kinematical effects, mass-singular log's $\propto \alpha \ln(m_\mu/Q)$ for bare muons, etc.
- at high energies
  $\hookrightarrow$ EW Sudakov log's $\propto (\alpha/s_W^2)\ln^2(M_W/Q)$ and subleading log's

EW corrections to PDFs at hadron colliders

induced by factorization of collinear initial-state singularities, new: photon PDF

Instability of W and Z bosons

- realistic observables have to be defined via decay products (leptons, γ's, jets)
- off-shell effects $\sim \mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$ are part of the NLO EW corrections

Combining QCD and EW corrections in predictions

- how to merge results from different calculations
- reweighting procedures in MC's

Input parameter schemes

SM input parameters: (natural choice)

α_s, α, M_W, M_Z, M_H, m_f, V_{CKM}

Issues:

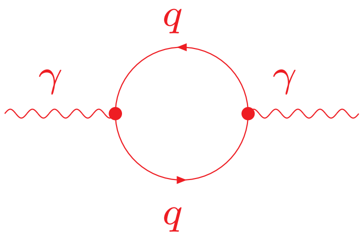
- Setting of α: process-specific choice to
  - ◇ avoid sensitivity to non-preturbative light-quark masses
  - ◇ minimize universal EW corrections

  Schemes: fix M_W, M_Z and α
  - ◇ $\alpha(0)$-scheme: relevant for external photon
  - ◇ $\alpha(M_Z)$-scheme: relevant for internal photons at high energies (γ^*)
  - ◇ G_μ-scheme: $\alpha_{G_\mu} = \sqrt{2} G_\mu M_W^2 (1 - M_W^2/M_Z^2)/\pi$, relevant for W, Z

- Warnings / pitfalls:
  - ◇ α must not be set diagram by diagram, but global factors like $\alpha(0)^m \alpha_{G_\mu}^n$ in gauge-invariant contributions mandatory !
  - ◇ weak mixing angle: $s_W \neq$ free parameter if M_W and M_Z are fixed !
  - ◇ Yukawa couplings are uniquely fixed by fermion masses !

The universal radiative corrections $\Delta\alpha$ and $\Delta\rho$

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q
for $|s|$ in GeV range and below (non-perturbative regime)

$\hookrightarrow$ charge-renormalization constant δZ_e sensitive to m_q

Solution:

fit hadronic part of $\Delta\alpha(s) = -\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,ren}}(s)/s\}$ and thus of δZ_e

via dispersion relations to $R(s) = \dfrac{\sigma(\mathrm{e^+e^-} \to \text{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)}$

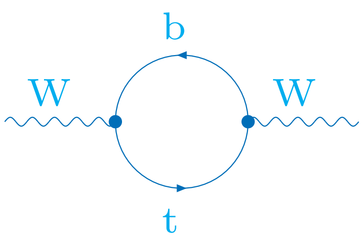
Jegerlehner et al.

$\Rightarrow$ Running elmg. coupling: $\quad \alpha(s) = \dfrac{\alpha(0)}{1-\Delta\alpha_{\mathrm{ferm}\neq\mathrm{top}}(s)}$

Leading correction to the ρ-parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

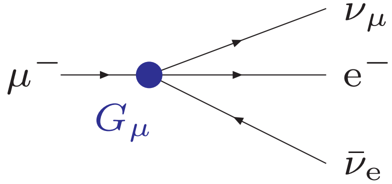
$\hookrightarrow$ large effects from bottom–top loops in W self-energy Veltman '77



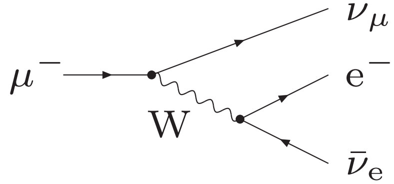
$$\Delta\rho_{\mathrm{top}} \sim \frac{\Sigma^{ZZ}_{\mathrm{T}}(0)}{M_{\mathrm{Z}}^2} - \frac{\Sigma^{WW}_{\mathrm{T}}(0)}{M_{\mathrm{W}}^2} \sim \frac{3G_\mu m_{\mathrm{t}}^2}{8\sqrt{2}\pi^2}$$

Fermi constant G_μ as input parameter – the quantity Δr

μ decay including higher-order corrections



+ QED corrections $\Longleftrightarrow$



+ EW corrections

$\hookrightarrow$ Relation between G_μ, $\alpha(0)$, M_W, and M_Z including corrections:

$$\alpha_{G_\mu} \equiv \frac{\sqrt{2}}{\pi} G_\mu M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \alpha(0)(1 + \Delta r)$$

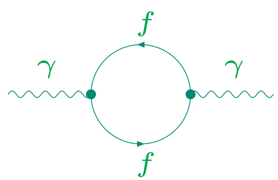
Δr comprises quantum corrections to μ decay

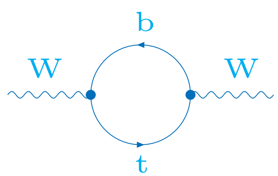
(beyond electromagnetic corrections in Fermi model) Sirlin '80, Marciano, Sirlin '80

$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha(M_\mathrm{Z}^2) \quad - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top} \quad + \quad \Delta r_\mathrm{rem}(M_\mathrm{H})$$

$$\sim 6\% \qquad \sim 3\% \qquad \sim 1\%$$

$$\alpha \ln(m_f/M_\mathrm{Z}) \qquad G_\mu m_\mathrm{t}^2 \qquad \alpha \ln(M_\mathrm{H}/M_\mathrm{Z})$$



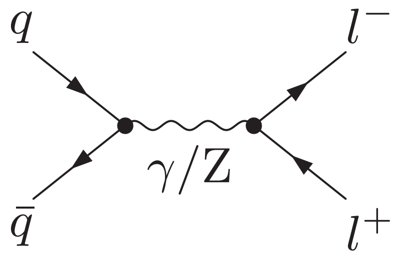


Input-parameter schemes including electroweak NLO corrections

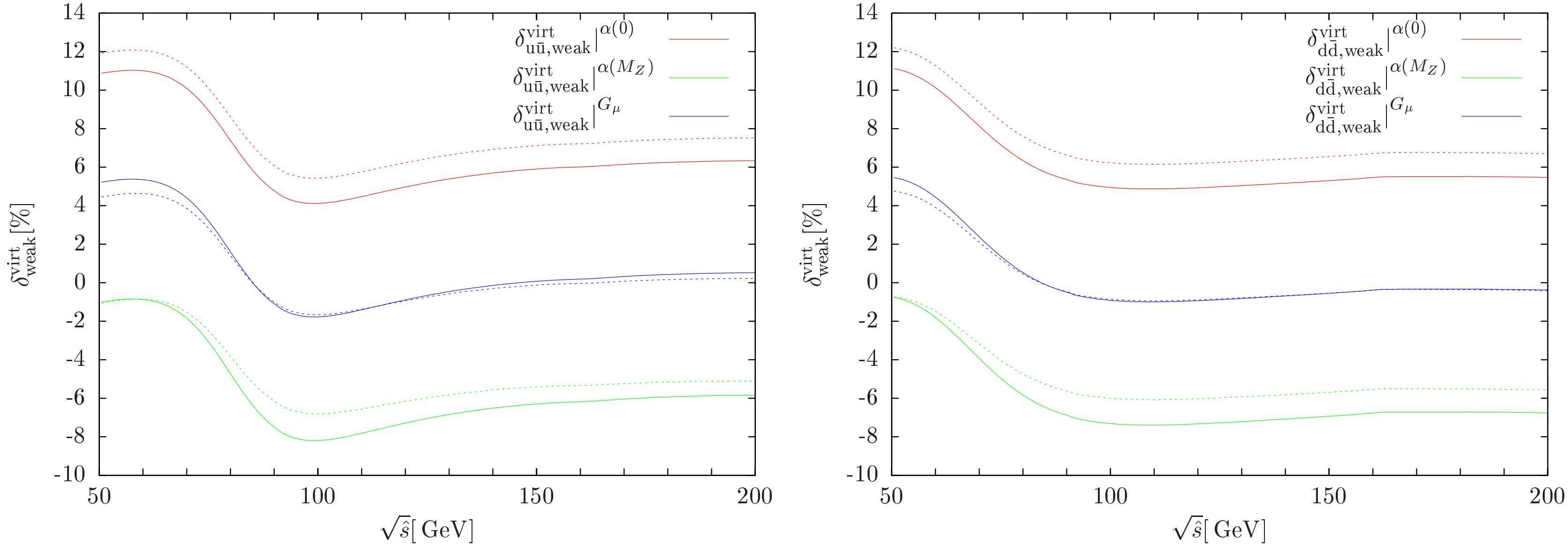
Cross section: $\sigma_{\mathrm{NLO}} = \alpha^N A_{\mathrm{LO}}\,(1+\delta_{\mathrm{EW}}), \qquad \delta_{\mathrm{EW}} = \mathcal{O}(\alpha)$

- $\alpha(0)$-scheme: $\sigma_{\mathrm{LO}} = \alpha(0)^N A_{\mathrm{LO}}$
- $\alpha(M_{\mathrm{Z}})$-scheme: $\sigma_{\mathrm{LO}} = \alpha(M_{\mathrm{Z}})^N A_{\mathrm{LO}}, \quad \delta_{\mathrm{EW}}^{\alpha(M_{\mathrm{Z}})} = \delta_{\mathrm{EW}}^{\alpha(0)} + N\Delta\alpha(M_{\mathrm{Z}}) + \dots$
- G_μ-scheme: $\sigma_{\mathrm{LO}} = \alpha(G_\mu)^N A_{\mathrm{LO}}, \quad \delta_{\mathrm{EW}}^{G_\mu} = \delta_{\mathrm{EW}}^{\alpha(0)} + N\Delta r + \dots$
- Mixed scheme: $N = n + n_\gamma, \quad n_\gamma =$ # external photons
  $\sigma_{\mathrm{LO}} = \alpha(G_\mu)^n \alpha(0)^{n_\gamma} A_{\mathrm{LO}}, \quad \delta_{\mathrm{EW}}^{\mathrm{mix}} = \delta_{\mathrm{EW}}^{\alpha(0)} + n\Delta r + \dots$
  - ◇ absorbs all $\Delta\alpha$ terms in LO to all orders
  - ◇ absorbs $\Delta\rho$ terms in LO (all for Ws up to 2 loops, parts for Zs)
  - ◇ factor α in δ_{EW} can still be adjusted appropriately
    (e.g. $\alpha\to\alpha(0)$ if γ radiation dominates, $\alpha\to\alpha_{G_\mu}$ if weak corrections dominate)
  - ◇ example: $q\bar{q}' \to \mathrm{W}\gamma, \quad n = n_\gamma = 1$

Example: weak corrections to Z production



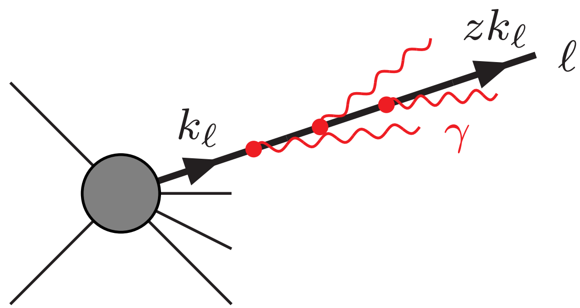
S.D., Huber ’09



- off-sets between NLO EW corrections in different schemes
- dashed lines include leading 2-loop effects from $\Delta\alpha$ and $\Delta\rho$
  $\hookrightarrow$ highest stability against h.o. corrections in G_μ scheme here

Photon radiation off leptons

Collinear final-state radiation (FSR) off leptons



Leading logarithmic effect is universal:

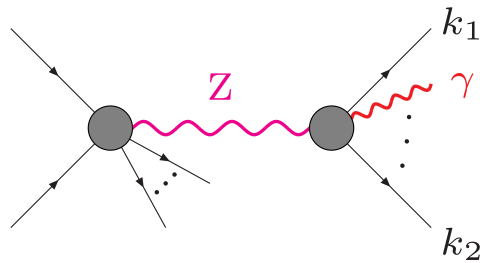
$$\sigma_{\mathrm{LL,FSR}} = \int \underbrace{\mathrm{d}\sigma^{\mathrm{LO}}(k_l)}_{\text{hard scattering}} \int_0^1 \mathrm{d}z \;\; \underbrace{\Gamma_{\ell\ell}^{\mathrm{LL}}(z,Q^2)}_{\substack{\text{leading-log structure}\\ \text{function, } Q \text{ = typ. scale}}} \;\; \Theta_{\mathrm{cut}}(zk_l)$$

- $\Gamma_{\ell\ell}^{\mathrm{LL}}(z,Q^2)$ known to $\mathcal{O}(\alpha^5)$ + soft exponentiation, equivalent description by QED parton showers
- $\mathcal{O}(\alpha)$ approximation: $\Gamma_{\ell\ell}^{\mathrm{LL},1}(z,Q^2) = \frac{\alpha(0)}{2\pi}\left[\ln\left(\frac{Q^2}{m_\ell^2}\right) - 1\right]\left(\frac{1+z^2}{1-z}\right)_+$
- Alternative approach: QED parton shower
  $\hookrightarrow$ advantage: photons described with finite p_{T} and definite multiplicity

Impact on predictions:

- log-enhanced corrections for “bare” leptons (muons) $\rightarrow$ large radiative tails
- KLN theorem: mass-singular FSR effects cancel if $(\ell\gamma)$ system is inclusive
  (full integration over z)
- full FSR not universal, in general not even separable from other EW corrections

Radiative tail from final-state radiation



results if resonances reconstructed from decay products

Typical situations: $\quad \mathrm{e}^+\mathrm{e}^- \to \mathrm{WW/ZZ} \to 4f,$

$\qquad\qquad\qquad\qquad \mathrm{pp} \to \mathrm{Z} \to f\bar{f} + X$

Final-state radiation:

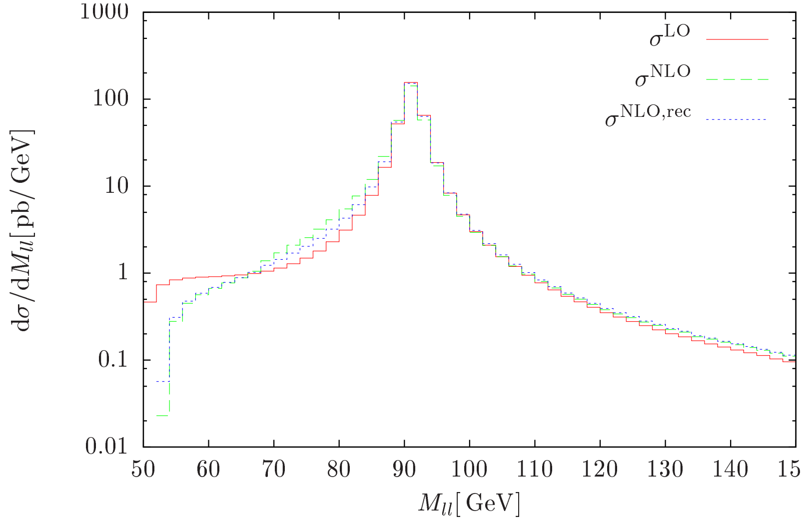
resonance for

$M^2 = (k_1+k_2)^2 < (k_1+k_2+k_\gamma)^2 \sim M_\mathrm{Z}^2$

$\hookrightarrow$ radiative tail in distribution $\frac{\mathrm{d}\sigma}{\mathrm{d}M}$

of reconstructed invariant mass M

for $M < M_\mathrm{Z}$

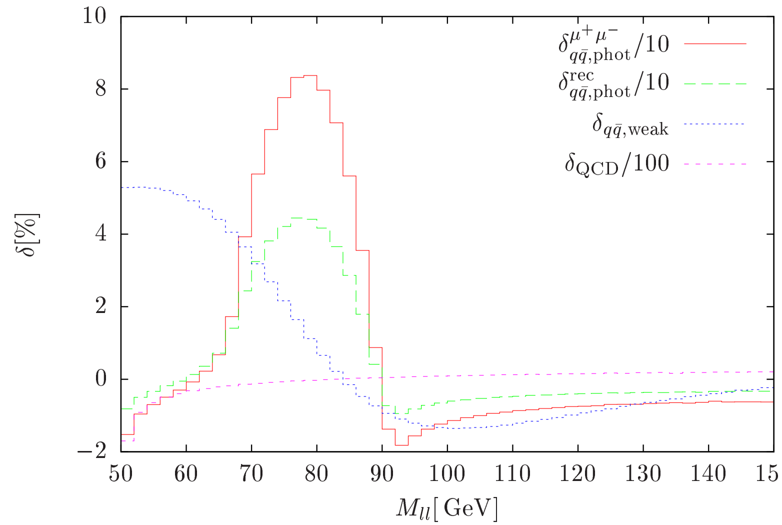


S.D., Huber '09

Example: Single-Z production

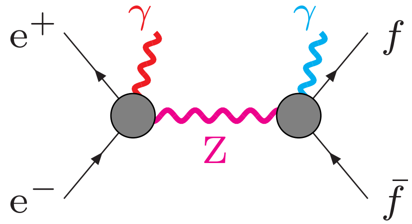
- radiative tail with corrections up to $\sim 80\%$
- FSR effect drastically reduced
  by photon recombination ("rec"):

  If $R_{l\gamma} < 0.1$ then $(l\gamma) \to \tilde{l}$ with $p_{\tilde{l}} = p_l + p_\gamma$.



Comparison with radiative tail from initial-state radiation

appears if initial state is fixed



Typical situations: $e^+e^- \to Z \to f\bar{f}$,
$\mu^+\mu^- \to Z, H, ? \to f\bar{f}$

$\hookrightarrow$ scan over s-channel resonance in $\sigma_{\rm tot}(s)$ by changing CM energy $\sqrt{s}$

Initial-state radiation:

Z can become resonant for $s = (p_+ + p_-)^2 \;>\; (p_+ + p_- - k_\gamma)^2 \sim M_Z^2$

$\hookrightarrow$ radiative tail for $s > M_Z^2$ due to "radiative return"

Final-state radiation:

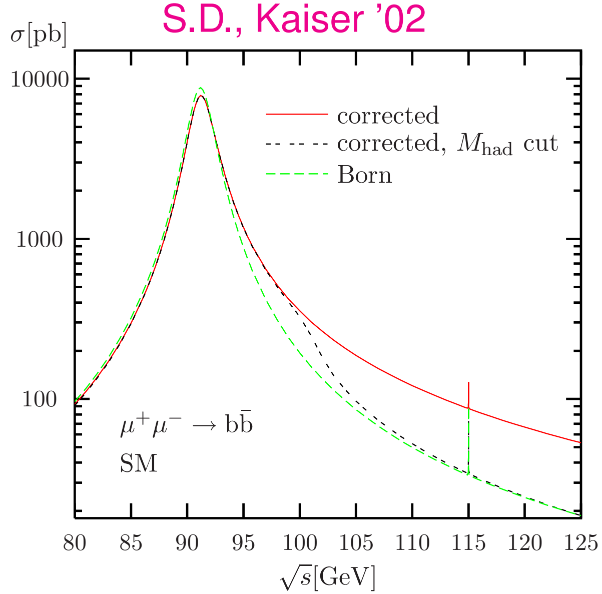
$s = k_Z^2 \sim M_Z^2$ for FSR

$\hookrightarrow$ only rescaling of resonance

Example:

cross section for $\mu^-\mu^+ \to b\bar{b}$ in lowest order and including photonic and QCD corrections, with and without invariant-mass cut $\sqrt{s} - M(b\bar{b}) < 10\,\mathrm{GeV}$

S.D., Kaiser '02



Recommendations to experimentalists:

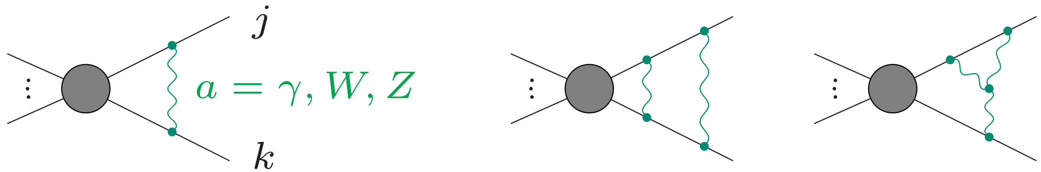
- no unfolding or subtraction of FSR effects !
  $\hookrightarrow$ would introduce untransparent conventions for non-universal EW corrections

- use concept of “dressed leptons” if reduction of large FSR effects is desirable
  (recombination of collinear $\ell\gamma$ configurations, analogous to QCD jet algorithms)

Electroweak corrections at high energies

Electroweak corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



etc.

+ sub-leading logarithms from collinear singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta_{\mathrm{LL}}^{1-\mathrm{loop}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta_{\mathrm{NLL}}^{1-\mathrm{loop}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta_{\mathrm{LL}}^{2-\mathrm{loop}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta_{\mathrm{NLL}}^{2-\mathrm{loop}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still relevant at 2-loop level

Note: differences to QED / QCD where Sudakov log's cancel

- massive gauge bosons W, Z can be reconstructed
  $\hookrightarrow$ no need to add "real W, Z radiation"
- non-Abelian charges of W, Z are "open" $\to$ Bloch–Nordsieck theorem not applicable

Extensive theoretical studies at fixed perturbative (1-/2-loop) order and suggested resummations via evolution equations

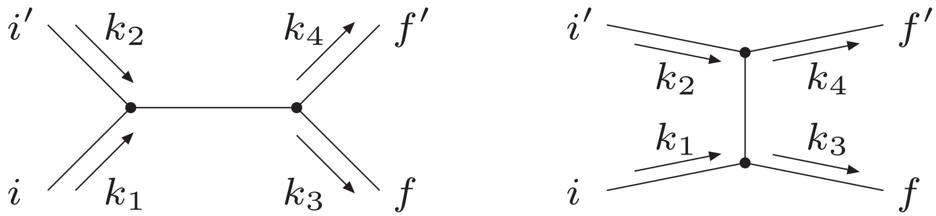
Beccaria et al.; Beenakker, Werthenbach; Ciafaloni, Comelli; Denner, Pozzorini; Fadin et al.; Hori et al.; Melles; Kühn et al., Denner et al. '00–'08

High-energy limit – Sudakow versus Regge regime

Sudakov regime: all invariants $k_i \cdot k_j \gg M_W^2$!

Example:

$2 \to 2$ particle process



Kinematic variables in centre-of-mass frame in high-energy limit ($k_j^2 \to 0$):

$$s = (k_1 + k_2)^2 \sim 4E^2, \qquad E = \text{beam energy},$$

$$t = (k_1 - k_3)^2 \sim -4E^2 \sin^2(\theta/2), \qquad \theta = \text{scattering angle},$$

$$M_{34} = \sqrt{s} \sim 2E,$$

$$k_\mathrm{T} = k_{3,\mathrm{T}} \sim E \sin\theta$$

High-energy limits in distributions:

- $\dfrac{\mathrm{d}\sigma}{\mathrm{d}k_\mathrm{T}}$: $k_\mathrm{T} \gg M_\mathrm{W} \quad \Rightarrow \quad s, |t| \gg M_\mathrm{W}^2 \quad \Rightarrow$ Sudakov domination
- $\dfrac{\mathrm{d}\sigma}{\mathrm{d}M_{34}}$: $M_{34} \gg M_\mathrm{W} \quad \Rightarrow$ small $|t|$ possible $\quad \Rightarrow$ in general no Sudakov domination (i.e. typically smaller corrections)

Example: Drell–Yan production

Neutral current: $\mathrm{pp} \to \ell^+\ell^-$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on S.D./Huber arXiv:0911.2329)

$M_{\ell\ell}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	$738.733(6)$	$32.7236(3)$	$1.48479(1)$	$0.0809420(6)$	$0.00679953(3)$	$0.000303744(1)$
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q},phot}}/\%$	-1.81	-4.71	-2.92	-3.36	-4.24	-5.66
$\delta_{\mathrm{q\bar{q},weak}}/\%$	-0.71	-1.02	-0.14	-2.38	-5.87	-11.12
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.27	0.54	-1.43	-7.93	-15.52	-25.50
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.00046	-0.0067	-0.035	0.23	1.14	3.38

no Sudakov domination!

Charged current: $\mathrm{pp} \to \ell^+\nu_\ell$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on Brensing et al. arXiv:0710.3309)

$M_{\mathrm{T},\nu_\ell\ell}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	$4495.7(2)$	$27.589(2)$	$1.7906(1)$	$0.084697(4)$	$0.0065222(4)$	$0.00027322(1)$
$\delta^{\mu^+\nu_\mu}_{\mathrm{q\bar{q}}}/\%$	$-2.9(1)$	$-5.2(1)$	$-8.1(1)$	$-14.8(1)$	$-22.6(1)$	$-33.2(1)$
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q}}}/\%$	$-1.8(1)$	$-3.5(1)$	$-6.5(1)$	$-12.7(1)$	$-20.0(1)$	$-29.6(1)$
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.0005	0.5	-1.9	-9.5	-18.5	-29.7
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.0002	-0.023	-0.082	0.21	1.3	3.8

Sudakov domination!

Electroweak corrections at high energies (continued)

- NLO EW high-energy logs – an approximation for full NLO EW ?
  - – miss finite contributions of $\mathcal{O}(\alpha)$ and photonic radiation effects
  - + simple approximation in Sudakov regime:

    s and $|t|$ large for $2 \to 2$ $\quad\Rightarrow$ large p_{T} or M_{T} !
  - – fail in non-Sudakov regime:

    e.g. s large, but $|t|$ NOT large for $2 \to 2$ $\quad\Rightarrow$ e.g. large M_{ll} in Drell–Yan !
  - + generically included in ALPGEN Chiesa, Montagna, Piccinini et al. '13
- Real W and Z emission processes
  - ◇ not fully separable from underlying process

    (e.g. hadronically decaying W/Z's in jet environment)
  - ◇ partially compensate negative virtual EW corrections

    $\hookrightarrow$ strongly dependent on W/Z reconstruction / separation

Recommendations:

- full NLO EW corrections whenever possible
- careful validations of logarithmic approximations against full results
- real W/Z emission: full ME calculations via multipurpose LO MC's

Electroweak corrections in PDFs

Electroweak effects in PDFs

Analogy to QCD-improved parton model:

Collinear splittings $q \to q\gamma$, $\gamma \to q\bar{q}$ lead to quark mass singularities

- absorption of $\alpha \ln m_q$ singularities via factorization into redefined PDFs
- $\mathcal{O}(\alpha)$ corrections to all PDFs & new photon PDF

2004: MRST2004QED = first PDF set with $\mathcal{O}(\alpha)$ corrections

Martin, Roberts, Stirling, Thorne '04

- typical impact on PDFs: $\Delta(\mathrm{PDF}) \lesssim 0.3\%\ (1\%)$ for $x \lesssim 0.1\ (0.4)$, $\mu_{\mathrm{fact}} \sim M_\mathrm{W}$
- photon PDF from analytical ansatz; uncertainty $\sim \mathcal{O}(20\%)$ or more
- additional real corrections from photons in initial state
  $\hookrightarrow$ typically $\mathcal{O}(1\%)$, but with large uncertainties
- included QED corrections are not full NLO EW
  (missing corrections in PDF fit, EW evolution in LO)
  $\hookrightarrow$ small uncertainties of $\mathcal{O}(\alpha)$
  (DIS fact. scheme for QED corrections recommended,
  but scheme choice is part of intrinsic uncertainty) Diener, S.D., Hollik '05

Electroweak effects in PDFs

Analogy to QCD-improved parton model:

Collinear splittings $q \to q\gamma$, $\gamma \to q\bar{q}$ lead to quark mass singularities

- absorption of $\alpha \ln m_q$ singularities via factorization into redefined PDFs
- $\mathcal{O}(\alpha)$ corrections to all PDFs & new photon PDF

2004: MRST2004QED = first PDF set with $\mathcal{O}(\alpha)$ corrections

Martin, Roberts, Stirling, Thorne '04

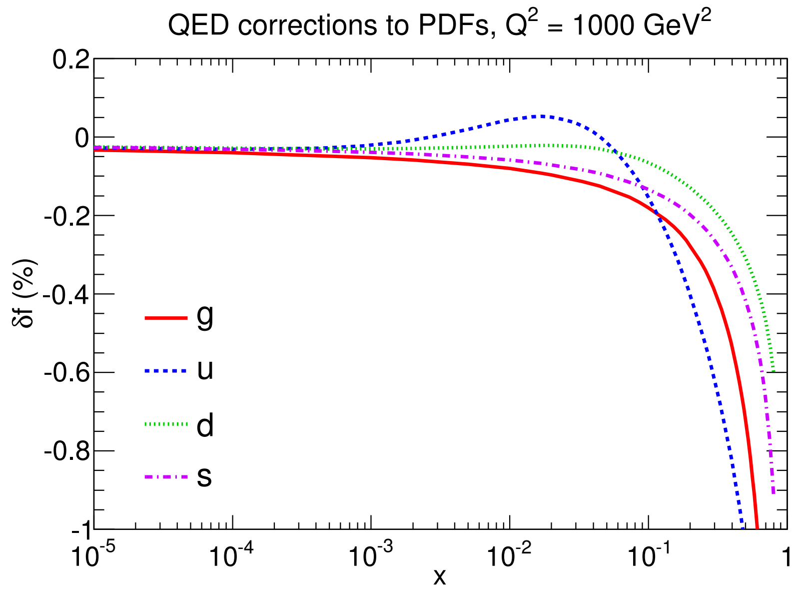
2013: NNPDF2.3QED = NNPDF set with $\mathcal{O}(\alpha)$ corrections

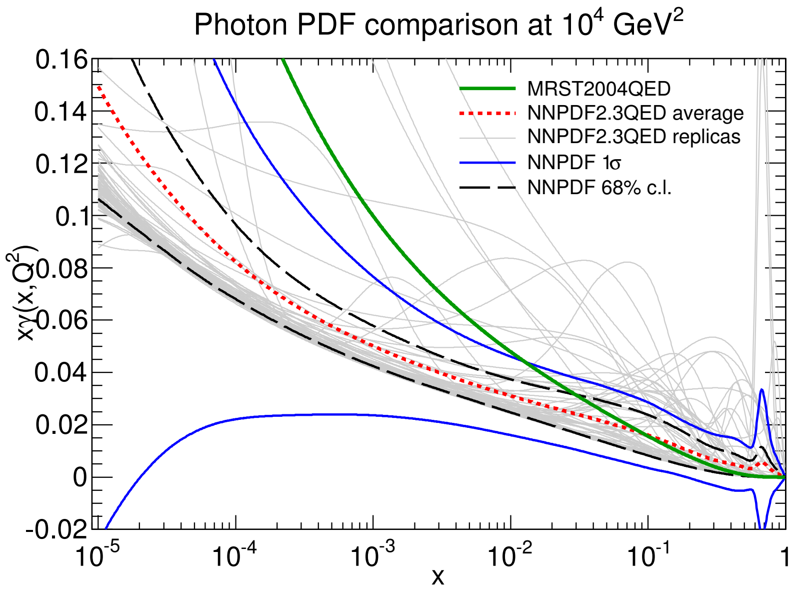
Ball et al. [NNPDF collaboration] '13

- currently best PDF prediction at (N)NLO QCD + NLO EW
- PDF samples for error estimate provided
- photon PDF fitted to DIS and Drell–Yan data $(10^{-5} \lesssim x \lesssim 10^{-1})$
- small $\mathcal{O}(\alpha)$ ambiguity still remains

Electroweak effects in PDFs (continued)

NNPDF2.3QED PDF set





Photon PDF:

- agreement with old $\gamma_{\mathrm{MRST}}(x)$ for $x \gtrsim 0.03$, but $\gamma_{\mathrm{NNPDF}}(x) < \gamma_{\mathrm{MRST}}(x)$ for smaller x
- lack of experimental information for $x \gtrsim 0.1$
  $\hookrightarrow$ constrained via $\gamma\gamma \to \mu^+\mu^-, \mathrm{W^+W^-}$ for larger x in the future ?

Literature

For more details see “Dictionary for electroweak corrections” in

J. Butterworth, *et al.*, “Les Houches 2013: Physics at TeV Colliders: Standard Model Working Group Report,” arXiv:1405.1067 [hep-ph], page 11,

and original references therein.