The ultimate uncertainty of the top quark pole mass

Freiburg, 25 January 2017

M. Beneke (TU München)

(1) Introduction & motivation

(2) Pole mass to all orders

(3) Background on asymptotic behaviour

(4) Beyond the ultimate accuracy

TECHNISCHE UNIVERSITÄT MÜNCHEN

MB, P. Marquard, P. Nason, M. Steinhauser [arXiv:1605.03609]

Introduction & motivation

Top mass from mass reconstruction and kinematics in hadron collisions.

ATLAS	$m_t = 172.84 \pm 0.34 \pm 0.61\,\mathrm{GeV}$
CMS	$m_t = 172.44 \pm 0.13 \pm 0.47\,\mathrm{GeV}$
	(b-JES, colour-rec, UE, scale-dep)
CDF	$m_t = 172.85 \pm 0.52 \pm 0.99\,\mathrm{GeV}$
D0	$m_t = 174.98 \pm 0.41 \pm 0.63\,\mathrm{GeV}$
	Tevatron com. $174.30 \pm 0.37 \pm 0.54\,\mathrm{GeV}$

Combination dominated by lepton+jet channel.

Few permille accuracy.
Best-known quark mass!

Table 1: List of systematic uncertainties for the muon+jets and electron+jets final states, and for the combined fit to the entire data set

	μ+jets		e+jets		ℓ+jets	
	$\delta^{\mu}_{m_t}$ (GeV)	$\delta^{\mu}_{\text{JES}}$	$\delta^{\text{e}}_{m_t}$ (GeV)	$\delta^{\text{e}}_{\text{JES}}$	$\delta^{\ell}_{m_t}$ (GeV)	$\delta^{\ell}_{\text{JES}}$
Fit calibration	0.08	0.001	0.09	0.001	0.06	0.001
b-JES	0.60	0.000	0.62	0.000	0.61	0.000
p_{T}- and η-dependent JES	0.30	0.001	0.28	0.001	0.28	0.001
Lepton energy scale	0.03	0.000	0.04	0.000	0.02	0.000
Missing transverse momentum	0.05	0.000	0.07	0.000	0.06	0.000
Jet energy resolution	0.22	0.004	0.24	0.004	0.23	0.004
b tagging	0.11	0.001	0.15	0.001	0.12	0.001
Pileup	0.07	0.002	0.08	0.001	0.07	0.001
Non-$\text{t}\bar{\text{t}}$ background	0.10	0.001	0.16	0.000	0.13	0.001
Parton distribution functions	0.07	0.001	0.07	0.001	0.07	0.001
Renormalization and factorization scales	0.23	0.004	0.41	0.005	0.24	0.004
ME-PS matching threshold	0.17	0.000	0.15	0.001	0.18	0.001
Underlying event	0.26	0.002	0.24	0.001	0.15	0.002
Color reconnection effects	0.66	0.004	0.39	0.003	0.54	0.004
Total	1.06	0.008	1.00	0.007	0.98	0.008

[CMS, 1209.2319 – 2011 data]

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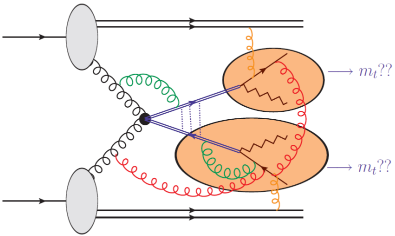
Which top mass is measured here?

- pole mass?
- Monte Carlo parameter mass
- any mass (since LO)

Which mass?



[from A. Signer, Top Quark Physics]

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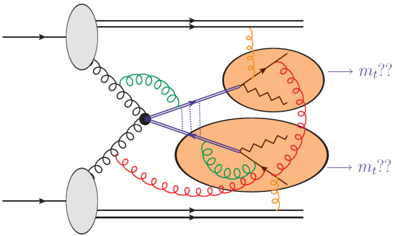

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- Pole mass appears natural ...
- ... if top self-energy diagrams are set to zero.
- Intrinsic *non-perturbative* $\mathcal{O}(\Lambda_{\rm QCD})$ uncertainty remains.

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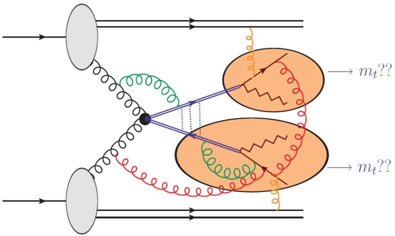

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Two theoretical issues

- Error in the extracted mass due to inaccuracies in the theoretical description of the process, perturbative or non-perturbative.

  Some understanding for boosted top in e^+e^- annihilation [Fleming et al., 0711.2079; Butenschön et al., 1608.01318]

  Possibly $\mathcal{O}(1\,\text{GeV})$.

- Principal limitation of accuracy of the mass definition itself (due to infrared sensitivity & renormalon divergence of the series) of $\mathcal{O}(\Lambda_{\rm QCD})$ [MB, Braun, 1994; Bigi et al., 1994]

  $\mathcal{O}(1\,\text{GeV})$ or $\mathcal{O}(100\,\text{MeV})$?

In a nut-shell ...

$$\underbrace{m_P}_{\text{pole mass}} = \underbrace{m(\mu_m)}_{\overline{\text{MS}}\text{ mass}} \left[1 + \sum_{n=1}^{\infty} c_n(\mu, \mu_m, m(\mu_m))\, \alpha_s^n(\mu) \right]$$

Know

- Four-loop conversion c_4 exactly [Marquard, Smirnov, Smirnov, Steinhauser, 1502.01030]
- All orders at order n with accuracy $\mathcal{O}(1/n^3, 1/2^n)$ [MB, 1994]

$$c_{n+1} = N \frac{\mu}{m(\mu_m)} (2b_0)^n \, n! \, n^b \left(1 + \frac{\tilde{s}_1}{n+b} + \frac{\tilde{s}_2}{(n+b)(n+b-1)} + \mathcal{O}(1/n^3, 1/2^n) \right)$$

GIVEN N ($b_0, b, \tilde{s}_1, \tilde{s}_2$ known).

Determine N from $n = 4$ with few percent accuracy by matching both.

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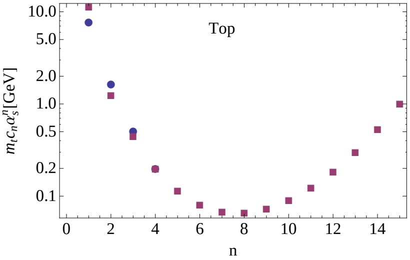


On the four-loop computation [Marquard, Smirnov, Smirnov, Steinhauser, 1502.01030, 1606.06754]

Pole mass m_P is defined as the zero of the inverse propagator $S^{-1}(p) = \not{p} - m_0 - \Sigma(p, m_0)$ at $\not{p} = m_P$, i.e.

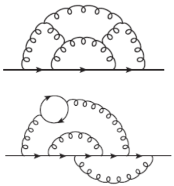
$$m_P - m_0 = \Sigma_{|\not{p}=m_P}$$

- Perturbative solution exists to any order
- Infrared finite (Note: $\Sigma_{|\not{p}=m_0}$ is NOT finite)

$$m_0 = Z_P\, m_P = Z_{\overline{\rm MS}}\, m_{\overline{\rm MS}}$$

$Z_{\overline{\rm MS}}$ needs pole parts only, known to four-loop [Chetyrkin 1997, Larin et al. 1997] and even five-loop [Baikov et al. 2014]. Z_P involves finite part of 4-loop on-shell integrals [Marquard et al., 1502.01030]

- Generate diagrams with QGRAF, translate to FORM code with Q2E/EXP – approx 100 four-loop topologies
- Reduction to 386 master integrals with FIRE5 and Crusher (IBP)
- Calculate masters with FIESTA + Mellin-Barnes. Partly numerical.



$$m_P = 163.643 + 7.557 + 1.617 + 0.501 + 0.195\,\text{GeV}$$

for $\overline{\text{MS}}$ top mass $m(m) = 163.643$ GeV and $\mu_m = \mu = m(m)$.

- Four-loop term 200 MeV
- For bottom 4-loop $\approx$ 3-loop $\approx$ 150 MeV
- For charm 4-loop $\approx$ 0.5 GeV and smallest terms at two-loop $\approx$ 200 MeV

Asymptotic behaviour

$$c_n(\mu, \mu_m, m(\mu_m)) \underset{n\to\infty}{\longrightarrow} N\, c_n^{(\mathrm{as})}(\mu, m(\mu_m)) \equiv N \frac{\mu}{m(\mu_m)}\, \tilde{c}_n^{(\mathrm{as})}$$

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- Dependence on μ and μ_m known and simple.
- Except for unknown normalization N, everything determined from the QCD beta-function

$$\beta_0 = \frac{1}{4\pi}\,(11 - \frac{2}{3} n_l) \qquad b = \frac{\beta_1}{2\beta_0^2} \qquad s_1 = \frac{1}{2\beta_0}\left(\frac{\beta_1^2}{2\beta_0^3} - \frac{\beta_2}{2\beta_0^2}\right)$$

s_2 (s_3) needs four-loop (five-loop) beta-function (known), and so on.

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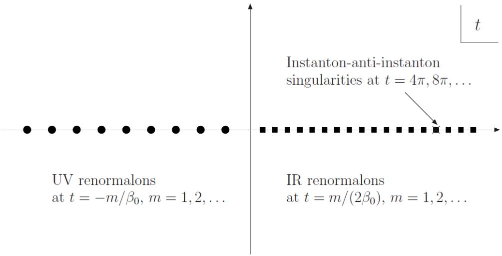
Series diverges.
How do we know this?
What does it mean?

Asymptotic behaviour (II)

Borel transform and integral

$$B(t) = \sum_{n=0}^{\infty} \frac{c_{n+1}}{n!}\, t^n \qquad \Longleftrightarrow \qquad m_P = m(\mu_m)\left(1 + \int_0^\infty dt\, e^{-t/\alpha_s(\mu)}\, B(t)\right)$$



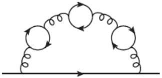
- If the Borel integral exists, it may reproduce the true result, and the series is "Borel-summable". (Series may still be asymptotic, if not summable.)
- Usually, the integral does not exist. The above *leading* asymptotic behaviour corresponds to the cut starting at $t = 1/(2\beta_0)$ *closest to the origin* in the Borel plane.
- By deforming the contour, one can define a Borel sum and an "ambiguity".

Asymptotic behaviour (III)

$$\tilde{c}_{n+1}^{(\mathrm{as})} = (2\beta_0)^n \, \frac{\Gamma(n+1+b)}{\Gamma(1+b)} \qquad \Longleftrightarrow \qquad B(t) = \frac{1}{(1-2\beta_0 t)^{1+b}}$$

$$\Longleftrightarrow \qquad \delta \int_0^\infty dt\, e^{-t/\alpha_s} \, \frac{1}{(1-2\beta_0 t)^{1+b}} \propto \alpha_s^{-b} \, e^{-1/(2\beta_0\alpha_s)} \sim \Lambda_{\mathrm{QCD}}/\mu$$

- Divergence and ambiguity is a consequence of *power corrections* from long distances.
- For the pole mass loop integrals are finite, but $\int_0^{\Lambda_{\mathrm{QCD}}} dk \Longrightarrow \mathcal{O}(\Lambda_{\mathrm{QCD}})$ from infrared/strong coupling region.
- This causes an IR renormalon pole/cut at $t = 1/(2\beta_0)$. Quite generally: pole/cut at $a/(2\beta_0)$ implies power correction $(\Lambda_{\mathrm{QCD}}/\mu)^a$.



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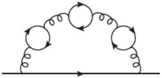

- The following is equivalent: knowledge of the ambiguity, knowledge of the singularity nature of the Borel transform, knowledge of the asymptotic behaviour of the perturbation expansion.

IR renormalons and factorization

$$\text{Obs} = C_{\text{LP}} \times ME_{\text{LP}} + \underbrace{C_{\text{NLP}} \times ME_{\text{NLP}}}_{(\Lambda_{\text{QCD}}/\mu)^a \text{ suppressed}} + \ldots$$

- C_{LP} has IR ambiguity of $\mathcal{O}(\text{NLP})$.
- ME_{NLP} has non-summable UV renormalon related to power-like UV divergemce
- The two cancel exactly to render Obs unambiguous. Similar to cancellation of *logarithmic* renormalization scale dependence within a given power term.

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- Obs = meson mass = pole mass + $\mathcal{O}(\Lambda_{\rm QCD})$
- Obs = off-shell quark self-energy expanded around $p^2 \equiv (mv+k)^2 = m_P^2$.

$$P_v S(p)^{-1} P_v = m_P - m(\mu_m) + [v \cdot k - \Sigma_{\rm HQET}] + \ldots$$

Relates pole mass IR renormalon to linear UV divergence of the static self energy.
Corresponding operator $\bar{h}_v h_v$ *has no anomalous dimension*.
Ambiguity is *exactly* $\Lambda_{\rm QCD}$ with no logs of m.

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right) [\text{ambiguity}] = 0$$

$$\Longrightarrow \qquad [\text{ambiguity}] = \text{const} \times \mu \times \exp\left(\int_{\alpha_s} d\alpha' \frac{1}{2\beta(\alpha')}\right)$$

Hence obtain the complete nature of the cut in terms of the beta function up to the constant $\equiv N$.

This works uniquely for the pole mass, because

- Leading renormalon is determined by a single operator and single unknown constant.
- Next renormalons at $\pm 1/\beta_0$ are $1/2^n$ suppressed, and normalization is probably smaller (bubble graphs)

Note: relate m_P and $m(\mu_m)$. Which one has the ambiguity? $\overline{\text{MS}}$ parameters are IR renormalon-free, because they are related to bare parameters by pure poles.

How ambiguous is the top pole mass really?

- Physics $\rightarrow \mathcal{O}(\Lambda_{\rm QCD})$ — No free quark, not a physical quantity, meson mass differs by $\mathcal{O}(\Lambda_{\rm QCD})$

  What is $\mathcal{O}(\Lambda_{\rm QCD})$? 1 GeV, 100 MeV? — Depends on value of N.

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- Simple idea:

$$N = \frac{c_4^{\rm exact}(\mu,\mu_m,m(\mu_m))}{\frac{\mu}{m(\mu_m)}\,\tilde{c}_4^{(\rm as)}}$$

  up to corrections $1/2^4, 1/4^3$ — few percent.
  Should be independent of $\mu,\mu_m,m(\mu_m)$.

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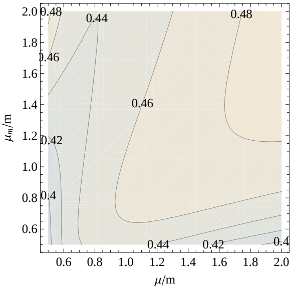
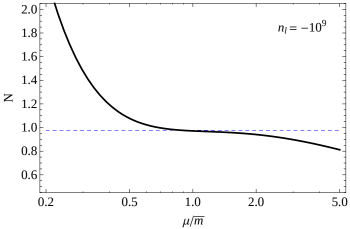
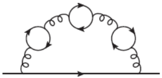
$$N = 0.4616^{+0.027}_{-0.070}\,(\mu \text{ and } \mu_m)$$

Checks



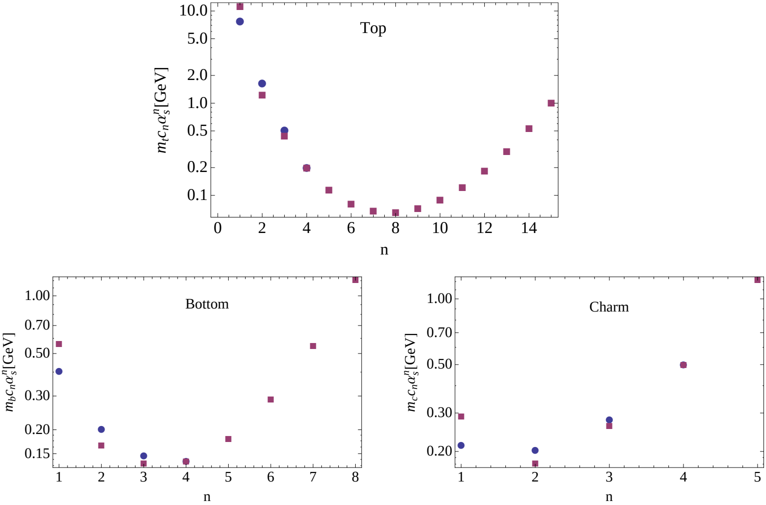


$$\lim_{|n_l|\to\infty} N = \frac{C_F}{\pi} \times e^{\frac{5}{6}} = 0.97656\ldots$$



- Scale (in)dependence
- Leaving out s_1 ($\to$ 0.4573) and s_1, s_2 ($\to$ 0.4584)
- Procedure works for different number of colours and flavours N_c, n_l.
- Works even for $n = 3$.
- Apply to large-n_l limit where N is known.

$$m_P\,[\text{GeV}] = 163.643 + 7.557 + 1.617 + 0.501 + 0.195 + \underbrace{\text{remainder}}_{5-\text{loop and beyond}} \pm \underbrace{\text{ambiguity}}_{\text{minimal term}}$$



Final result [MB, Marquard, Nason, Steinhauser, 1605.03609]

$$\delta^{(5+)} m_P = \underbrace{0.250^{+0.015}_{-0.038}\,(N)}_{\text{5 loops and beyond}} \pm 0.010\,(\alpha_s) \underbrace{\pm\, 0.071\ \text{(ambiguity)}}_{\text{intrinsic uncertainty}}\ \text{GeV}$$

$$m_P^{\text{c}}/\overline{m} = 1.06164^{+0.00009}_{-0.00023}\,(N) \pm 0.00086\,(\alpha_s) \pm 0.00043\ \text{(ambiguity)}$$

- Ultimate intrinsic uncertainty of the top pole mass is only about 70 MeV.

  [Up to 110 MeV when internal bottom and charm mass effects are accounted for.]

- Given the $\overline{\text{MS}}$ mass, the top quark pole mass is determined with an accuracy of 0.92 per mil.

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DISCLAIMER: This is the ultimate theoretical limitation, if no experimental uncertainties and best possible computations. It does NOT mean that the top mass quoted by ATLAS/CMS/CDF/D0 equals the pole mass within their stated errors of approx. 600 MeV!

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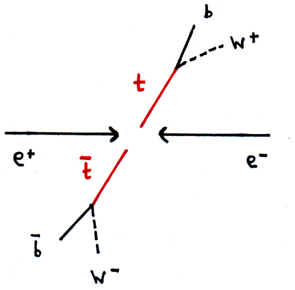
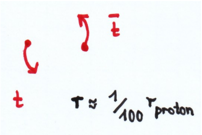




Tune $E_{\text{beam}} \approx m_t$, such that the top velocity is small, $v/c \approx 1/10$

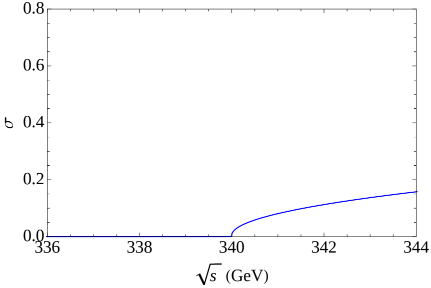
- Chromostatic force forces $t\bar{t}$ into a bound orbit
- Size $r \approx 10^{-17}$ m

  Smallest non-elementary structure in the Standard Model of particle physics.
- Revolution time
  $t \approx 2\pi r/v \approx 10^{-24}$ s $\approx 2 \times \tau_{\text{top}}$

  Very short-lived due to top decay.

Pair production threshold – Coulomb force and Weak Decay

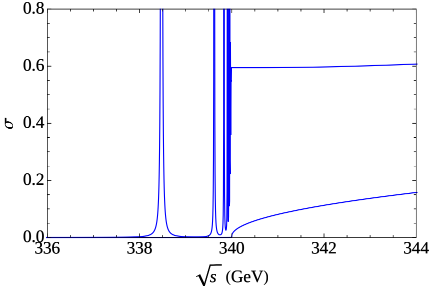
Stable top quark, no strong interaction



Pair production threshold – Coulomb force and Weak Decay

Unique QCD dyanmics

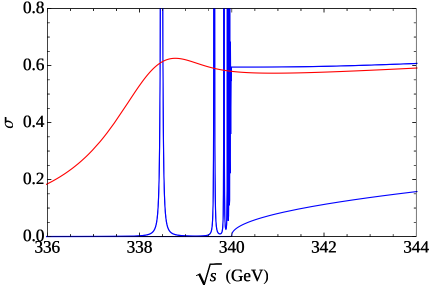
Stable top quark, with strong Coulomb force



Pair production threshold – Coulomb force and Weak Decay

Unique QCD dyanmics

Unstable top quark, with strong Coulomb force



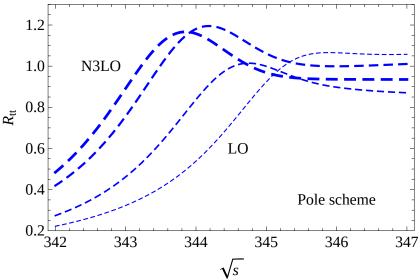
Ultra-precise mass measurement.
Direct "spectroscopic" width measurement.
Match $[\delta m_t]_{\rm exp} \approx 30\,\mathrm{MeV}$???

Which mass?

$$\sigma_{t\bar{t}} = \sigma_{t\bar{t}}(m_P, \alpha_s)$$

- Pole mass leads to large shifts in the peak position of the $t\bar{t}$ cross section
- Theoretical uncertainty several hundred MeV at third order, limited by ultimate uncertainty.
- Problem arises from the long-range part of the strong Coulomb force [MB, 1998]

$$\int_{|\vec{q}\,|<\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\, \tilde{V}_{\text{Coulomb}}(\vec{q}\,)$$



Not the pole mass.

- Solution ("Kill two birds with one stone"): intermediate mass definition $m_{\rm PS}$, which can be related precisely to the $\overline{\rm MS}$ mass ($\rightarrow$ top Yukawa coupling) AND avoids large, spurious corrections to the cross section.

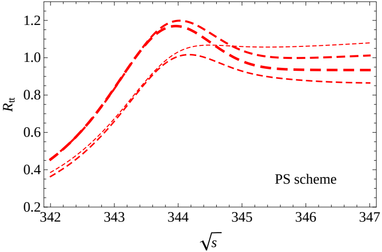
$$\sigma_{t\bar{t}} = \sigma_{t\bar{t}}(m_{\rm PS}, \alpha_s)$$

- Potential-subtracted mass [MB, 1998]

$$m_{\rm PS}(\mu_f) \equiv m_{\rm pole} + \frac{1}{2}\int_{|\vec{q}\,|<\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\,\tilde{V}_{\rm Coulomb}(\vec{q}\,)$$

$$m_{\rm PS}(\mu_f) - \overline{m}(\overline{m}) = \underbrace{[m_{\rm PS}(\mu_f) - m_{\rm pole}]}_{\text{known to } \mathcal{O}(\mu_f\alpha_s^4)\ \text{[hep-ph/0501289]}} + \underbrace{[m_{\rm pole} - \overline{m}(\overline{m})]}_{\text{known to } \mathcal{O}(m_t\alpha_s^4)\ \text{[1502.01030]}}$$

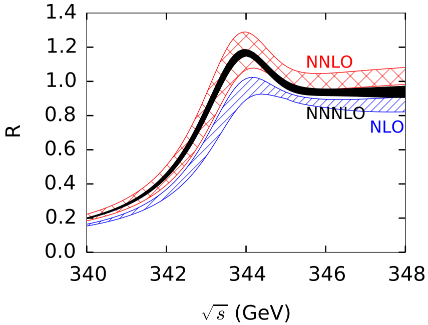
Conversion precision $\approx 20\,$MeV.
Cancellation of large perturbative contributions from the IR.



$t\bar{t}$ threshold in e^+e^- at NNNLO

NNNLO + summation in (PNR)QCD [MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]

Photon exchange and Z-vector coupling only.
$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
$\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.2229$,
$\mu = (50\ldots80\ldots350)\,\mathrm{GeV}$, $\mu_w = 350\,\mathrm{GeV}$.



Complete NNNLO QCD combines many third-order pieces [MB, Kiyo, Schuller, hep-ph/0501289; MB, Kiyo, 0804.4004, 1312.4791; MB, Piclum, Rauh, 1312.4792; Marquard, Piclum, Seidel, Steinhauser, 0904.0920, 1401.3004]

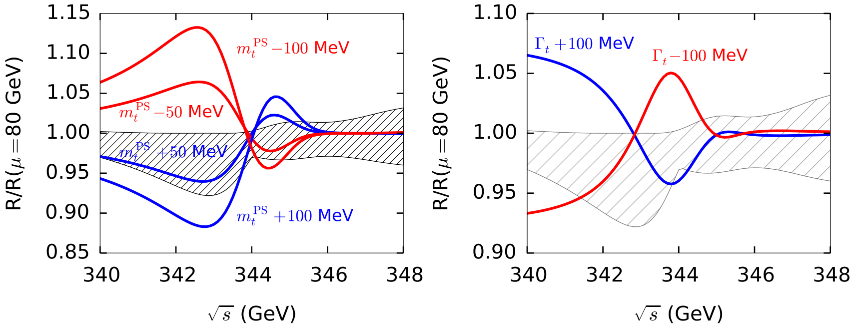
Including Higgs, QED and non-resonant process $W^+W^-b\bar{b}$ ("single-top") [MB, Jantzen, Ruiz-Femenia, 1004.2188, MB, Maier, Piclum, Rauh, 1506.06865]
Electroweak corrections and code release [MB, Kiyo Maier, Piclum, 1605.03010]

Position shift: 310 MeV (LO to NLO) 150 MeV (to NNLO) 64 MeV (to NNNLO)
Improvement of factor 3 in uncertainty in peak height.

Sensitivity to m_t vs. theoretical uncertainty

NNNLO $$\frac{\delta\sigma}{\sigma} = \pm\,(2\ldots 3.5)\%$$

Shaded band — Relative scale uncertainty, superimposed variation with shifted top mass input normalized to reference.



[MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]

Summary

I The top pole mass is a unique case where multi-loop results can be matched to well-understood and simple asymptotic behaviour to obtain a perturbative all-order result.

II Ultimate uncertainty is about

$$100\,\text{MeV}$$

not 1 GeV. Irrelevant for hadron collider physics.

III This does NOT mean that the top pole mass is known to 600 MeV (present experimental error). The limiting factor is the precise perturbative computation including renormalization of the process from which the mass is extracted.

IV In e^+e^- collisions a well-defined top mass (eventually $\overline{\text{MS}}$) can be determined from the threshold with controlled errors of less than 50 MeV.