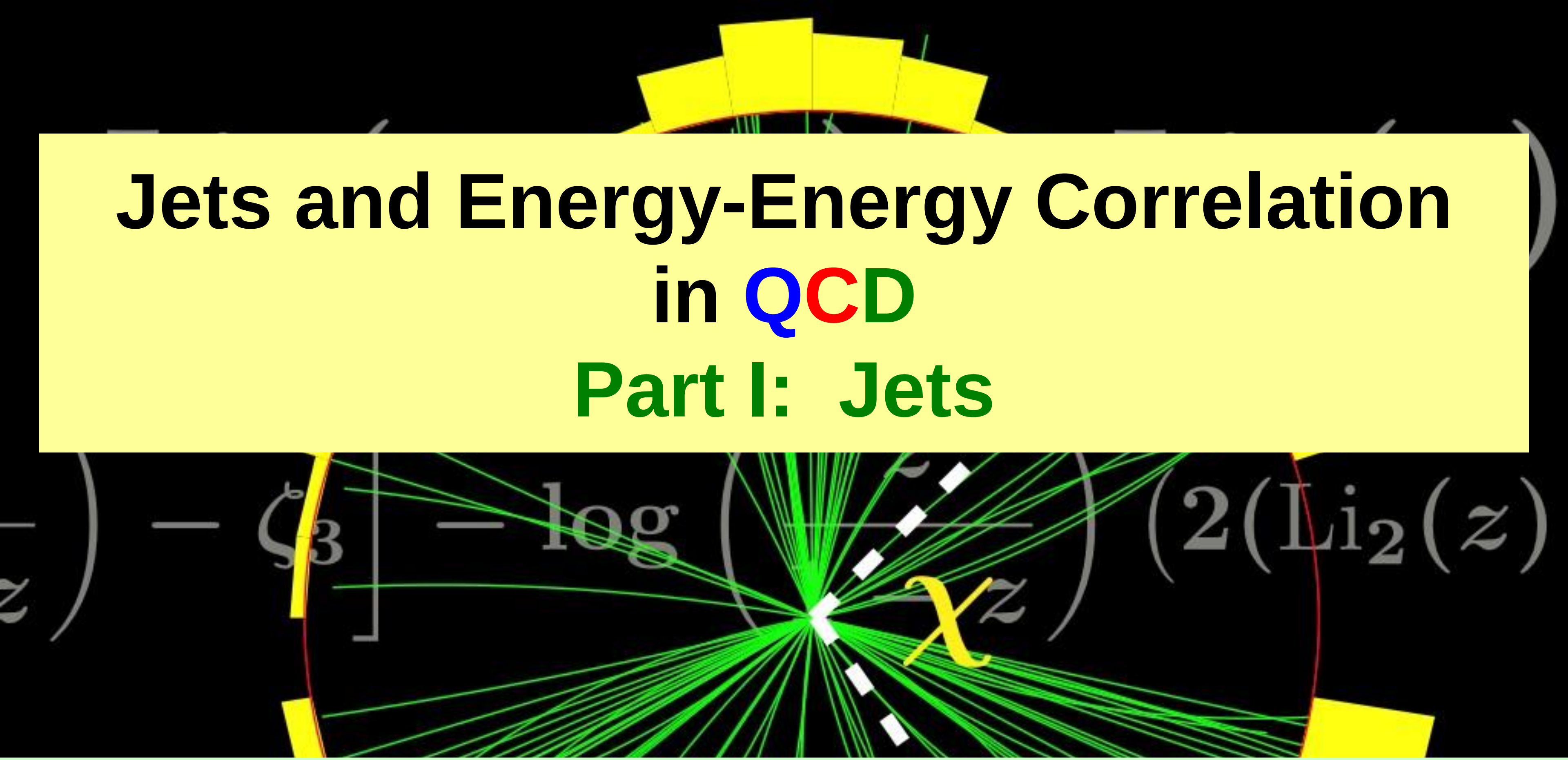


Jets and Energy-Energy Correlation in QCD

Part I: Jets

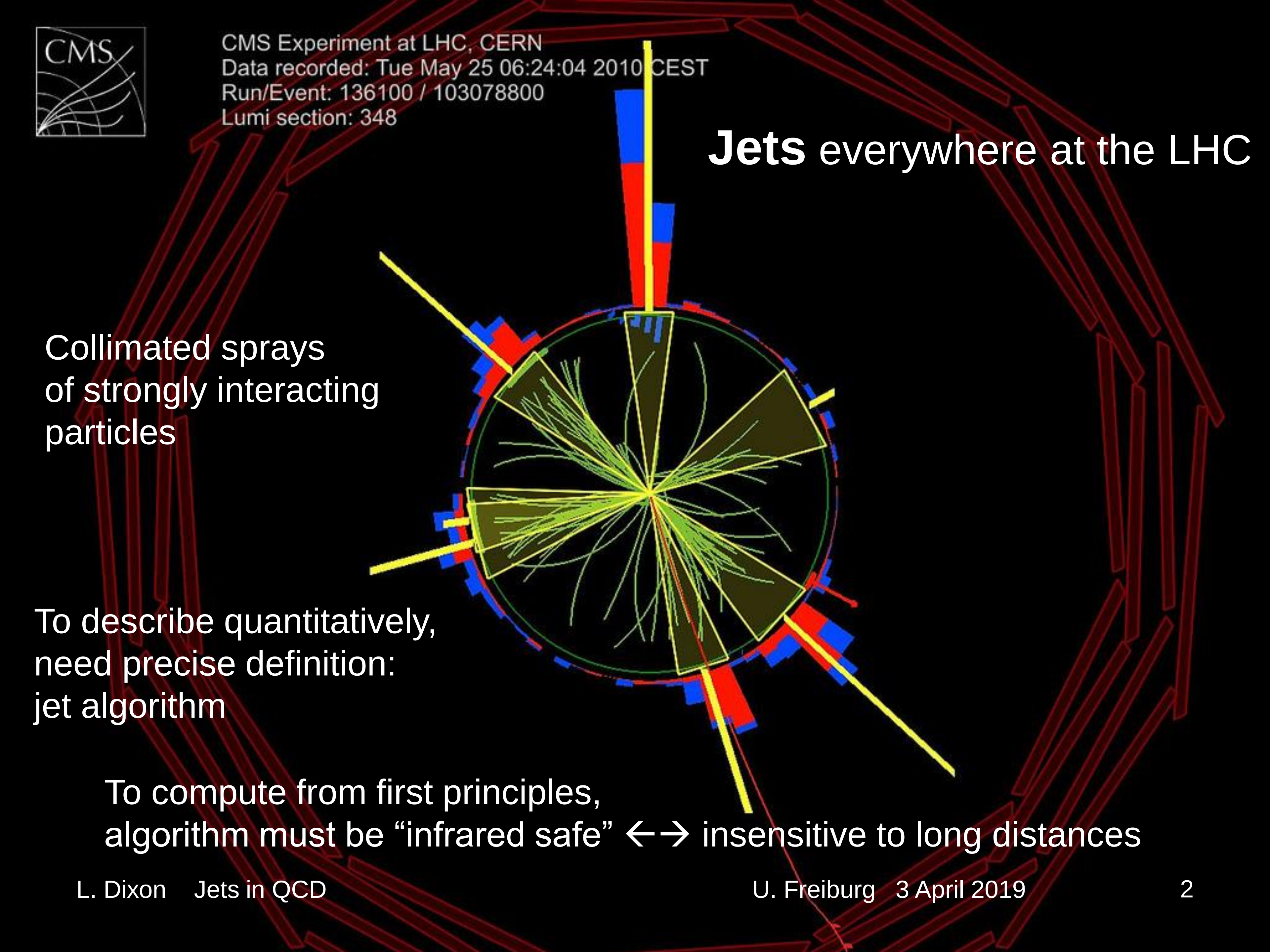
Lance Dixon (SLAC)

Graduate College

University of Freiburg

3 April, 2019

SLAC NATIONAL ACCELERATOR LABORATORY



Jets everywhere at the LHC

Collimated sprays
of strongly interacting
particles

To describe quantitatively,
need precise definition:
jet algorithm

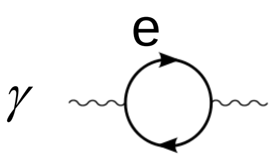
To compute from first principles,
algorithm must be “infrared safe” ←→ insensitive to long distances

The Key of Asymptotic Freedom

Gross, Wilczek, Politzer (1973)

Quantum fluctuations of massless virtual particles polarize vacuum

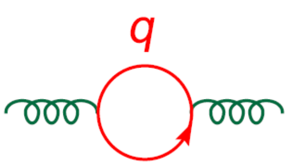
QED: electrons screen charge (e larger at short distances)



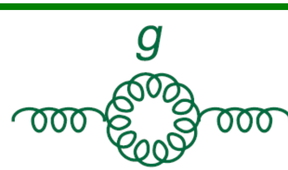
$> 0 \;\rightarrow\; e^2(r) = \dfrac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} ln \frac{r}{r_0}}$

$$\beta_0 \approx -r \frac{d}{dr}\left(\frac{\pi}{e^2}\right) = -\frac{2}{3}$$

QCD: gluons anti-screen charge (g_s smaller at short distances)



> 0



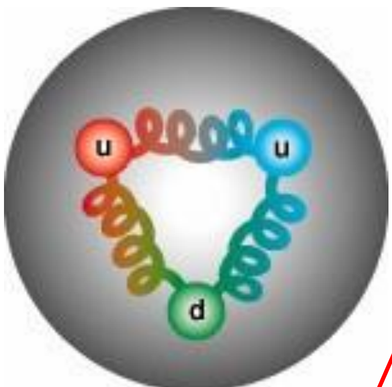
< 0

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f$$

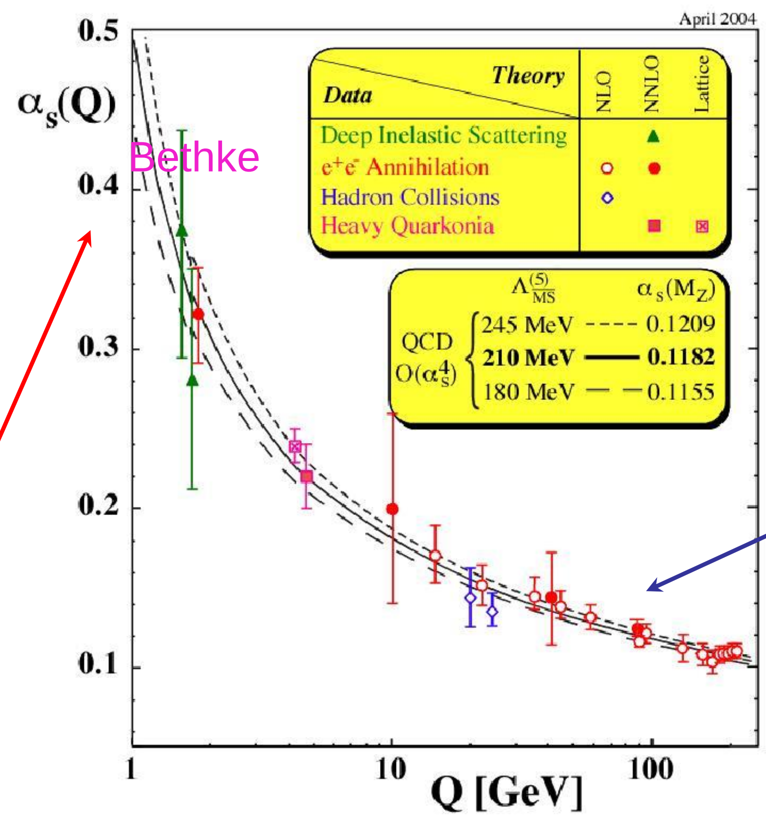
Gluon self-interactions make quarks almost free, and make QCD calculable at short distances (high energies): $\alpha_s \rightarrow$ **0** asymptotically

Short-distance calculability

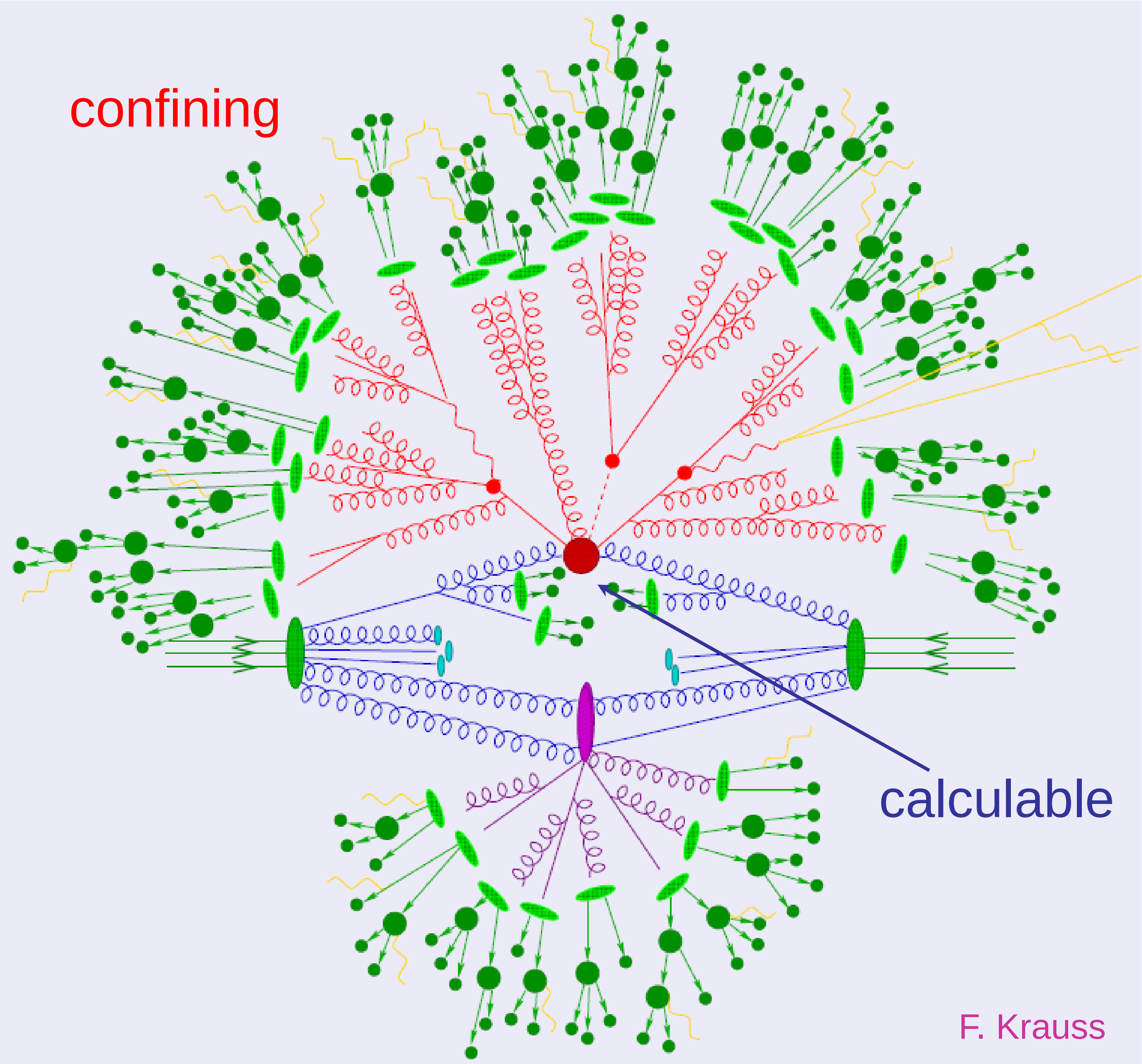
Running of α_s is *logarithmic*, *slow* at short distances (large Q)



confining

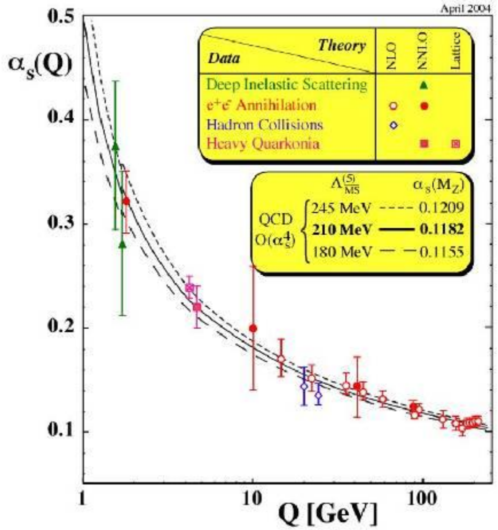


calculable


confining
calculable
F. Krauss

QCD factorization & parton model

- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- They are sampled "one at a time" in hard collisions.
- Leads to QCD-improved parton model:



"suitable" final state

Parton distribution function: prob. of finding parton a in proton 1, carrying fraction x_1 of its momentum

factorization scale ("arbitrary")

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Partonic cross section, computable in perturbative QCD

partonic CM energy^2

renormalization scale ("arbitrary")

Parton evolution

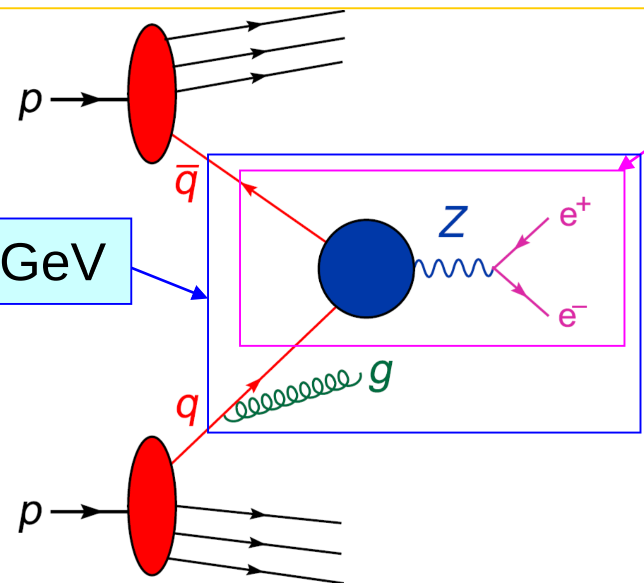
- partons in the proton are not quite free
- distributions $f_a(x,\mu_F)$ evolve as scale μ_F at which they are resolved varies

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) \;=\; \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 \; f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Example:
Z production at LHC

large μ_F, ~ m_Z

small μ_F, ~ few GeV

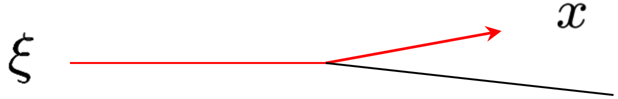


$p_T(g)$ ~ 10 GeV, say

Parton evolution (cont.)

- parton distributions are nonperturbative
- must be measured experimentally
- experimental data at much lower μ_F^2 than scale of interest at LHC (100-1000 GeV)^2
- fortunately, evolution at μ_F > 1-2 GeV is perturbative
- DGLAP equation:

$$\mu^2 \frac{\partial}{\partial \mu^2} f_a(x,\mu) = \frac{\alpha_s(\mu)}{2\pi} \sum_b \int_x^1 \frac{d\xi}{\xi} P_{ab}(x/\xi, \alpha_s(\mu)) \, f_b(\xi,\mu)$$



$$x = \frac{x}{\xi} \times \xi$$

$$P_{ab}(x,\alpha_s) \;=\; P_{ab}^{(0)}(x) + \frac{\alpha_s}{2\pi} P_{ab}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(2)}(x) + \cdots$$

LO (1974) NLO (1980) NNLO (2004)

Short-Distance Cross Section in Perturbation Theory

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha} \left[\underbrace{\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R)}_{\text{NLO}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R)}_{\text{NNLO}} + \cdots \right]$$

- **Problem:** Leading-order (LO) predictions only qualitative due to **poor convergence** of expansion in $\alpha_s(\mu) \sim 0.1$
- Can easily get ~ 50-100% uncertainties in LO predictions

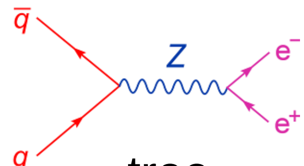
- Uncertainties brought under much better control with NLO corrections: ~ 50-100% → ~ 15-20%

- NNLO becoming increasingly available
→ ~ 3-8% uncertainties

Overall structure of higher-order QCD corrections

Example of *Z* production at hadron colliders

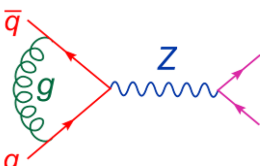
LO $\hat{\sigma}^{(0)}$



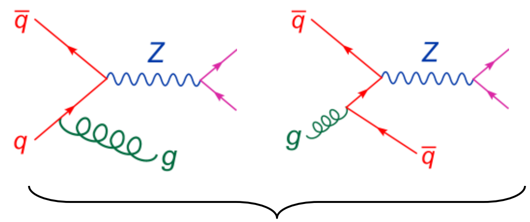
tree

convolute with pdfs
apply cuts

NLO $\hat{\sigma}^{(1)}$



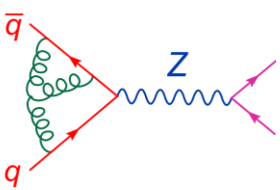
1 loop



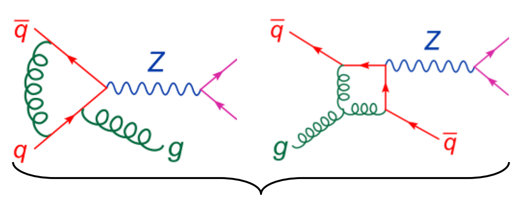
tree + 1 parton

dim. reg. $D = 4 - 2\epsilon$
first, cancel infrared divergences ($1/\epsilon^2$)
between virtual & real

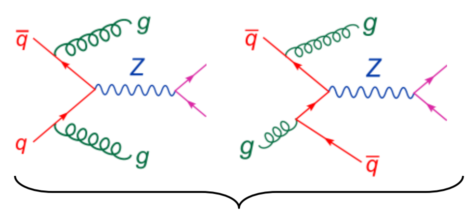
NNLO $\hat{\sigma}^{(2)}$



2 loops



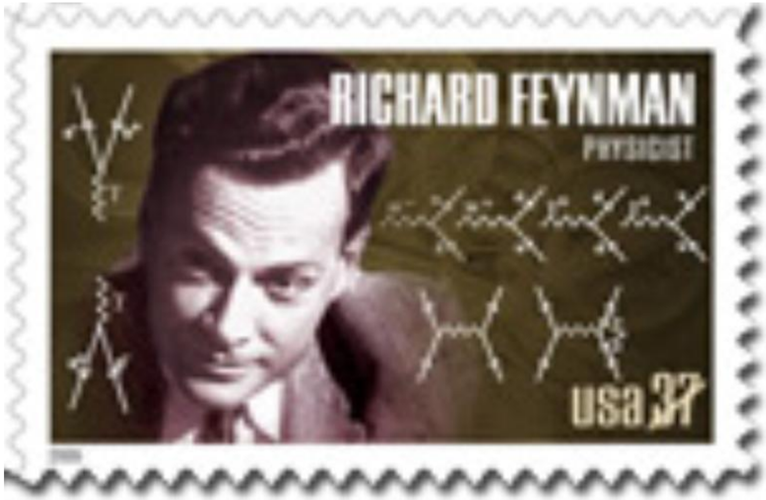
1 loop + 1 parton

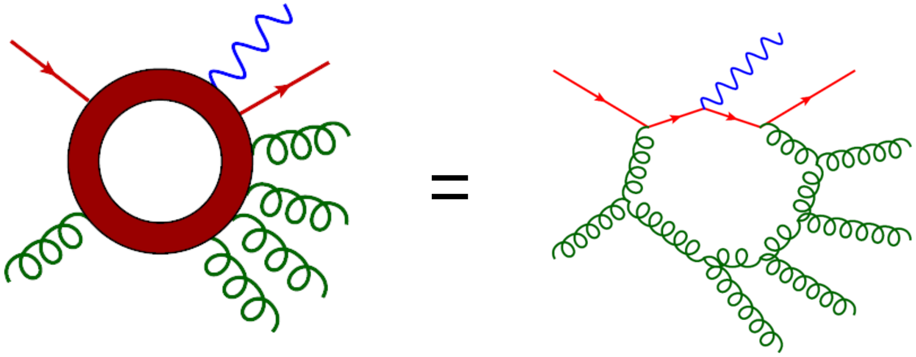


tree + 2 partons

intricate ($1/\epsilon^4$) IR cancellations

Short-distance cross section perturbative → expansion in Feynman diagrams



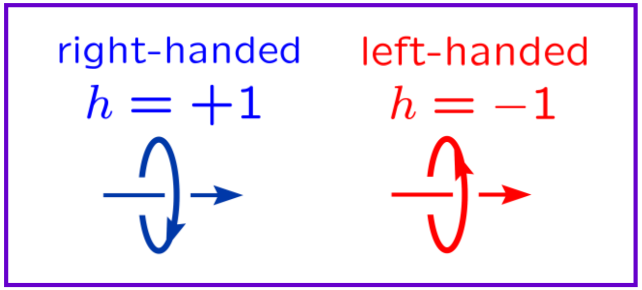


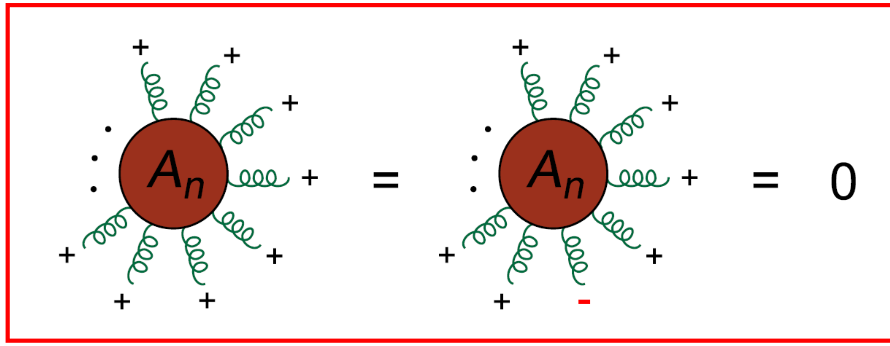
+ 256,264 more

- Feynman diagrams, while very general and **powerful**, are not optimized for such processes
- **Much more efficient methods based on helicity formalism, generalized unitarity,…**

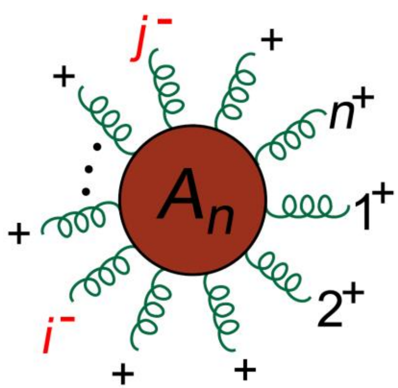
Helicity Formalism Exposes Tree-Level Simplicity in QCD

Many tree-level helicity amplitudes either vanish or are very short





Very opaque from Feynman diagrams



$$= \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n\, 1\rangle}$$

Parke-Taylor formula (1986)

Spinor helicity formalism

Scattering amplitudes for massless plane waves of definite momentum:
Lorentz 4-vectors k_i^μ $\quad$ $k_i^2=0$

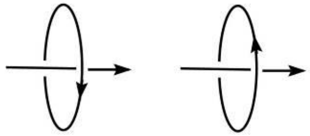
Natural to use Lorentz-invariant products (invariant masses): $s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2$

But for elementary particles with **spin** (*e.g.* all observed ones!) **there is a better way:**

Take "square root" of 4-vectors k_i^μ (spin 1)
use Dirac (Weyl) spinors $u_\alpha(k_i)$ (spin ½)

right-handed: $(\lambda_i)_\alpha = u_+(k_i)$ $\quad$ left-handed: $(\tilde{\lambda}_i)_{\dot{\alpha}} = u_-(k_i)$

q,g,γ, all have 2 helicity states, $h = \pm$



Spinor products

Instead of Lorentz products: $s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2$

Use spinor products: $\bar{u}_-(k_i)u_+(k_j) = \varepsilon^{\alpha\beta}(\lambda_i)_\alpha(\lambda_j)_\beta = \langle i\, j\rangle$

$$\varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\bar{u}_+(k_i)u_-(k_j) = \varepsilon^{\dot\alpha\dot\beta}(\tilde\lambda_i)_{\dot\alpha}(\tilde\lambda_j)_{\dot\beta} = [i\, j]$$

Identity $k_i^\mu(\sigma_\mu)_{\alpha\dot\alpha} = (\not{k}_i)_{\alpha\dot\alpha} = u_+(k_i)\bar{u}_+(k_i) = (\lambda_i)_\alpha(\tilde\lambda_i)_{\dot\alpha}$

$\Rightarrow$ These are **complex square roots** of Lorentz products:

$$\langle i\, j\rangle[j\, i] = \frac{1}{2}\mathrm{Tr}\left[\not{k}_i\ \not{k}_j\right] = 2k_i \cdot k_j = s_{ij}$$

$$\boxed{\langle i\, j\rangle = \sqrt{s_{ij}}\, e^{i\phi_{ij}} \qquad [j\, i] = \sqrt{s_{ij}}\, e^{-i\phi_{ij}}}$$

Most iconic Feynman diagram turns 70

Phys. Rev. 76, 769 (1949)

772 R. P. FEYNMAN

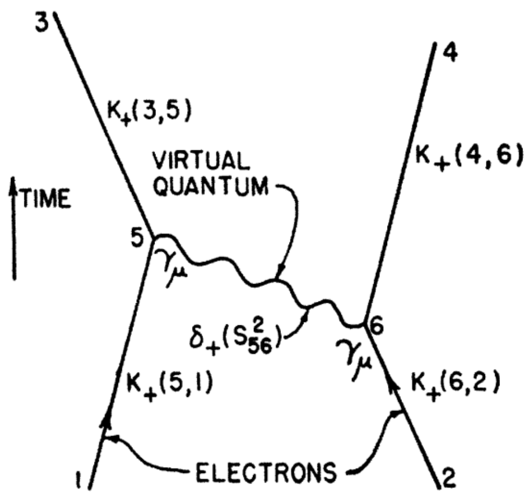


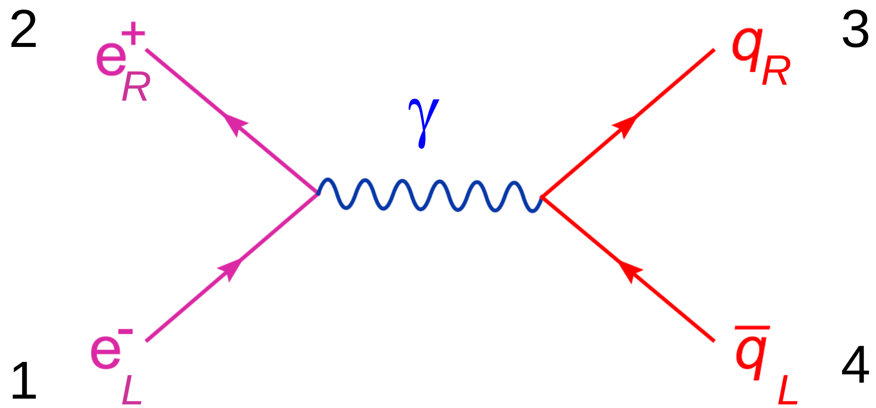
FIG. 1. The fundamental interaction Eq. (4). Exchange of one quantum between two electrons.

electron-electron scattering in QED

Can repurpose to describe the most important processes in the Standard Model

Carved in stone in Tuva, Central Asia, next to Mongolia
[courtesy Glen Cowan, Ralph Leighton]

That diagram as a helicity amplitude



add helicity information, numeric labels

$$\mathcal{A}_4 = 2ie^2 Q_e Q_q \delta_{i_3}^{\bar{\imath}_4} A_4$$

$$A_4 = \frac{1}{2\,s_{12}} \overline{v_-}(k_2)\gamma^\mu u_-(k_1)\ \overline{u_+}(k_3)\gamma_\mu v_+(k_4)$$

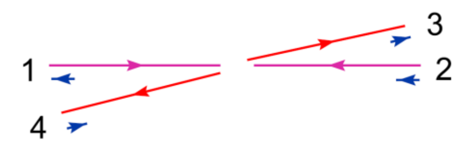
$$= \frac{1}{2\,s_{12}} (\sigma^\mu)_{\alpha\dot{\alpha}}(\lambda_2)^\alpha(\tilde{\lambda}_1)^{\dot{\alpha}}\ (\sigma_\mu)^{\dot{\beta}\beta}(\tilde{\lambda}_3)_{\dot{\beta}}(\lambda_4)_\beta$$

$$= \frac{1}{s_{12}} (\lambda_2)^\alpha(\tilde{\lambda}_1)^{\dot{\alpha}}(\lambda_4)_\alpha(\tilde{\lambda}_3)_{\dot{\alpha}}$$

Fierz identity

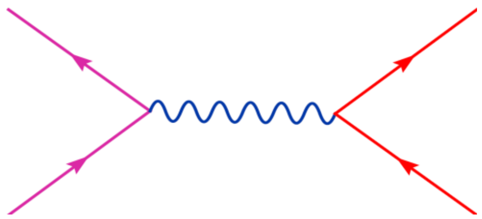
$$(\sigma^\mu)_{\alpha\dot{\alpha}}(\sigma_\mu)^{\dot{\beta}\beta} = 2\ \delta_\alpha^\beta\,\delta_{\dot{\alpha}}^{\dot{\beta}}$$

$$\boxed{A_4 = \frac{\langle 2\,4\rangle\,[1\,3]}{s_{12}}} = e^{i\phi}\frac{s_{13}}{s_{12}} = \frac{-e^{i\phi}}{2}(1-\cos\theta)$$



helicity suppressed as 1 || 3 or 2 || 4

Sometimes useful to rewrite answer



Crossing symmetry more manifest
if we switch to all-outgoing helicity labels
(flip signs of incoming helicities)

$$A_4 = \frac{\langle 2\,4\rangle\,[1\,3]}{s_{12}}$$

$$= \frac{\langle 2\,4\rangle\,[1\,3]}{\langle 1\,2\rangle[2\,1]}\,\frac{\langle 1\,3\rangle}{\langle 1\,3\rangle}$$

$$= -\frac{\langle 2\,4\rangle\,[2\,4]\langle 2\,4\rangle}{\langle 1\,2\rangle[2\,4]\langle 4\,3\rangle}$$

$$\boxed{A_4 = \frac{\langle 2\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}}$$

"holomorphic"

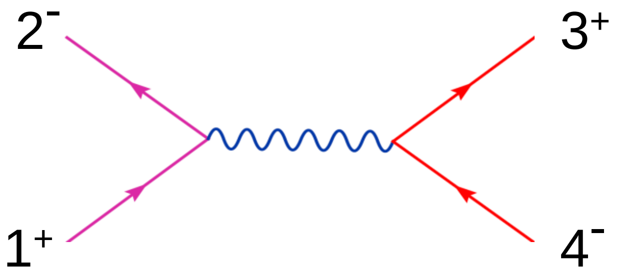
or $$\boxed{A_4 = \frac{[1\,3]^2}{[1\,2][3\,4]}}$$

"antiholomorphic"

useful identities

$$\langle i\,j\rangle = -\langle j\,i\rangle$$
$$[i\,j] = -[j\,i]$$
$$\langle i\,i\rangle = [i\,i] = 0$$
$$\langle i\,j\rangle\,[j\,i] = s_{ij}$$
$$\sum_{j=1}^{4} \langle i\,j\rangle\,[j\,k] = 0$$
$$s_{12} = s_{34}$$
$$s_{13} = s_{24}$$

Symmetries for all other helicity config's

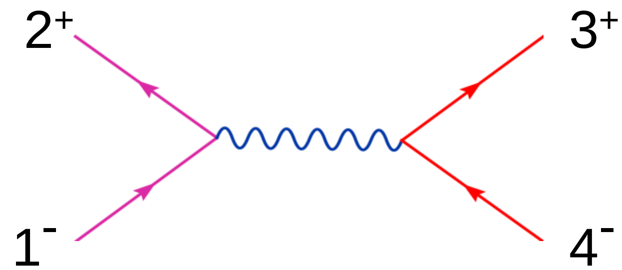


$$A_4 = \frac{\langle 2\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}$$

C

$1 \leftrightarrow 2$

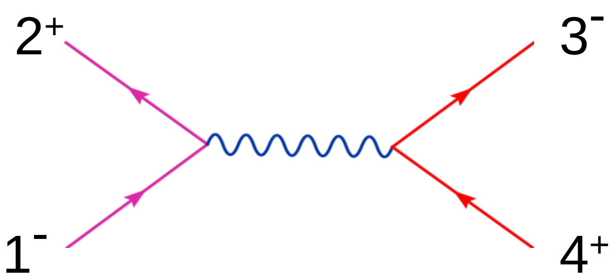
$\Rightarrow$



$$A_4 = -\frac{\langle 1\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}$$

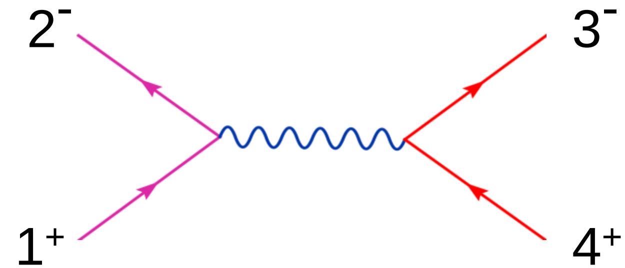
P $\Downarrow$ $\langle\,\rangle \leftrightarrow [\,]$

$\Downarrow$



$$A_4 = \frac{[2\,4]^2}{[1\,2][3\,4]}$$

$\Rightarrow$



$$A_4 = -\frac{[1\,4]^2}{[1\,2][3\,4]}$$

Unpolarized, helicity-summed cross sections

(the norm in QCD)

$$\frac{d\sigma(e^+e^- \to q\bar{q})}{d\cos\theta} \propto \sum_{\text{hel.}} |A_4|^2 = 2\left\{\left|\frac{\langle 2\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}\right|^2 + \left|\frac{\langle 1\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}\right|^2\right\}$$

$$= 2\,\frac{s_{24}^2 + s_{14}^2}{s_{12}^2}$$

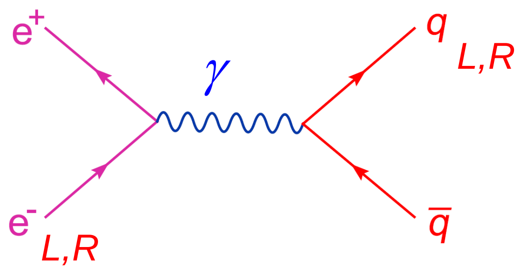
$$= \frac{1}{2}[(1-\cos\theta)^2 + (1+\cos\theta)^2]$$

$$= 1 + \cos^2\theta$$

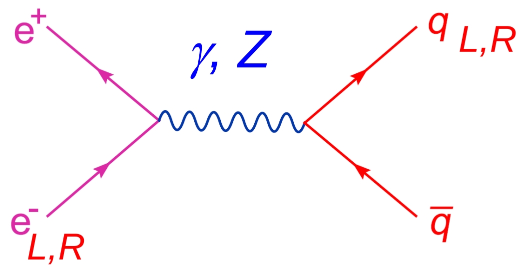
$$[2e^2 Q_e Q_q]^2 N_c$$

Reweight helicity amplitudes → electroweak/QCD processes

For example, *Z* exchange



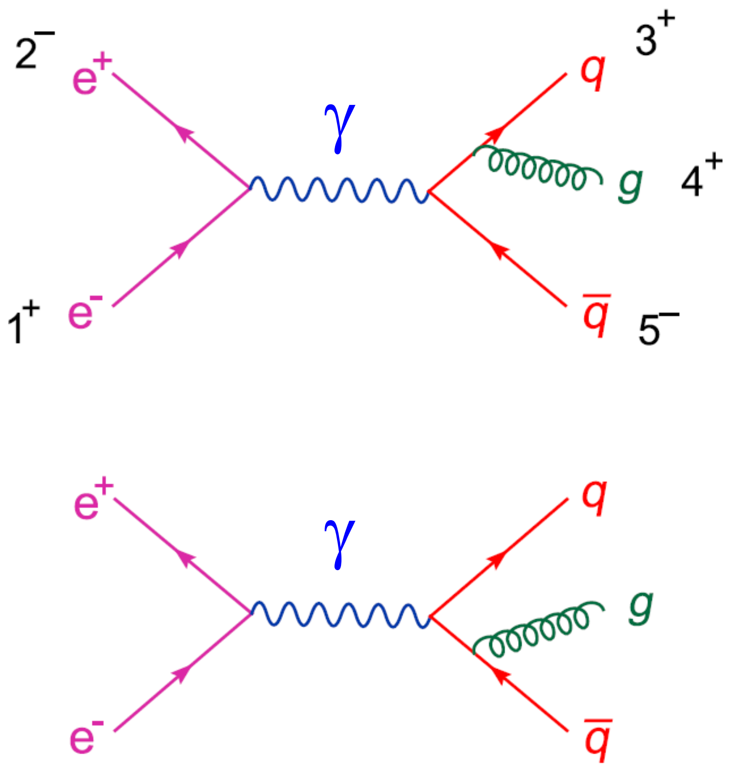
$\Rightarrow$



$$Q_e Q_q \quad \Rightarrow \quad Q_e Q_q + \frac{v^e_{L,R} v^q_{L,R} \; s}{s - M_Z^2 + i\Gamma_Z M_Z}$$

$$v_L^f = \frac{2 I_3^f - 2 Q_f \sin^2\theta_W}{\sin 2\theta_W} \qquad v_R^f = -\frac{2 Q_f \sin^2\theta_W}{\sin 2\theta_W}$$

QCD real radiative corrections to this process



$$\mathcal{A}_5 = 2ie^2 g Q_e Q_q (T^{a_4})_{i_3}^{\bar{\imath}_5} A_5$$

$$A_5 = \frac{\langle 2\,5\rangle}{s_{12}} \frac{\langle 1^+|(\not{k}_3 + \not{k}_4)\not{\varepsilon}_4^+|3^-\rangle}{\sqrt{2}\, s_{34}} + \frac{[1\,3]}{s_{12}} \frac{\langle 2^-|(\not{k}_4 + \not{k}_5)\not{\varepsilon}_4^+|5^+\rangle}{\sqrt{2}\, s_{45}}$$

Helicity formalism for massless vectors

Berends, Kleiss, De Causmaecker, Gastmans, Wu (1981); De Causmaecker, Gastmans, Troost, Wu (1982); Xu, Zhang, Chang (1984); Kleiss, Stirling (1985); Gunion, Kunszt (1985)

$$\boxed{\begin{aligned}(\varepsilon_i^+)_\mu &= \varepsilon_\mu^+(k_i, q) = \frac{\langle i^+|\gamma_\mu|q^+\rangle}{\sqrt{2}\,\langle i\, q\rangle} \\ (\not\varepsilon_i^+)_{\alpha\dot\alpha} &= \not\varepsilon_{\alpha\dot\alpha}^+(k_i, q) = \frac{\sqrt{2}\,\tilde\lambda_i^{\dot\alpha}\lambda_q^\alpha}{\langle i\, q\rangle}\end{aligned}}$$

reference vector q^μ is null, $q^2 = 0$

$\not q|q^\pm\rangle = 0$

obeys $\varepsilon_i^+ \cdot k_i = 0$ (required transversality)

$\varepsilon_i^+ \cdot q = 0$ (bonus)

under azimuthal rotation about k_i axis, helicity +1/2 $\tilde\lambda_i^{\dot\alpha} \to e^{i\phi/2}\tilde\lambda_i^{\dot\alpha}$

helicity -1/2 $\lambda_i^\alpha \to e^{-i\phi/2}\lambda_i^\alpha$

so $\not\varepsilon_i^+ \propto \dfrac{\tilde\lambda_i^{\dot\alpha}}{\lambda_i^\alpha} \to e^{i\phi}\, \not\varepsilon_i^+$ as required for helicity +1

$e^+e^- \to qg\bar{q}$ (cont.)

$$A_5 = \frac{\langle 2\,5\rangle}{s_{12}} \frac{\langle 1^+|(\not k_3 + \not k_4)\not\varepsilon_4^+|3^-\rangle}{\sqrt{2}\, s_{34}} + \frac{[1\,3]}{s_{12}} \frac{\langle 2^-|(\not k_4 + \not k_5)\not\varepsilon_4^+|5^+\rangle}{\sqrt{2}\, s_{45}}$$

$$= \frac{\langle 2\,5\rangle}{s_{12}} \frac{\langle 1^+|(\not k_3 + \not k_4)|q^+\rangle [4\,3]}{s_{34}\langle 4\,5\rangle} + \frac{[1\,3]}{s_{12}} \frac{\langle 2^-|(\not k_4 + \not k_5)|4^-\rangle \langle q\,5\rangle}{s_{45}\langle 4\,5\rangle}$$

Choose $q = k_5$ to remove 2nd graph

$$= \frac{\langle 2\,5\rangle}{s_{12}} \frac{\langle 1^+|(\not k_3 + \not k_4)|5^+\rangle [4\,3]}{s_{34}\langle 4\,5\rangle}$$

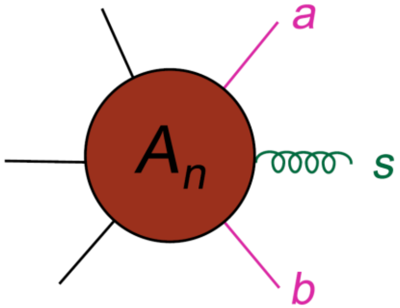
$$= -\frac{\langle 2\,5\rangle [1\,2] \langle 2\,5\rangle [4\,3]}{\langle 1\,2\rangle [2\,1] \langle 3\,4\rangle [4\,3] \langle 4\,5\rangle}$$

$$\boxed{A_5 = \frac{\langle 2\,5\rangle^2}{\langle 1\,2\rangle \langle 3\,4\rangle \langle 4\,5\rangle}}$$

Properties of $\mathcal{A}_5(e^+e^- \to q g \bar{q})$

1. Soft gluon behavior $\quad k_4 \to 0$

$$A_5 = \frac{\langle 2\,5\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle\langle 4\,5\rangle} = \frac{\langle 3\,5\rangle}{\langle 3\,4\rangle\langle 4\,5\rangle} \times \frac{\langle 2\,5\rangle^2}{\langle 1\,2\rangle\langle 3\,5\rangle}$$
$$\to \mathcal{S}(3,4^+,5) \times A_4(1^+,2^-,3^+,5^-)$$



Universal "eikonal" factors
for emission of soft gluon *s*
between two hard partons *a* and *b*

$$\mathcal{S}(a,s^+,b) = \frac{\langle a\,b\rangle}{\langle a\,s\rangle\langle s\,b\rangle}$$
$$\mathcal{S}(a,s^-,b) = -\frac{[a\,b]}{[a\,s][s\,b]}$$

Soft emission is from the classical chromoelectric current:
independent of parton type (*q* *vs.* *g*) and helicity
– only depends on momenta of *a*,*b*, and color charge

Properties of $\mathcal{A}_5(e^+e^- \to q g \bar{q})$ (cont.)

2. Collinear behavior

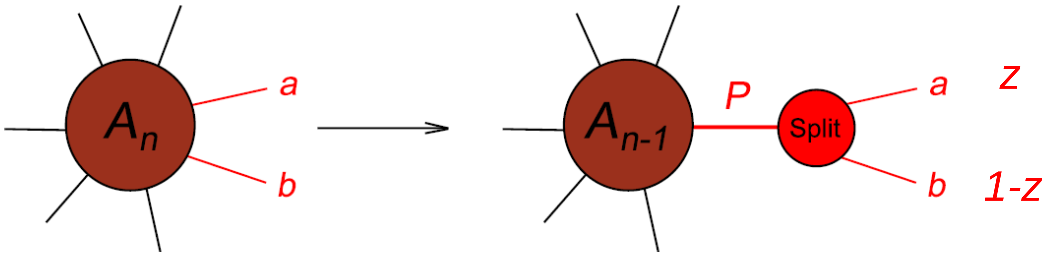
$k_3 \,||\, k_4$: $\quad k_3 = z\, k_P, \quad k_4 = (1-z)\, k_P$

$k_P \equiv k_3 + k_4, \quad k_P^2 \to 0$

$\lambda_3 \approx \sqrt{z}\lambda_P, \quad \lambda_4 \approx \sqrt{1-z}\lambda_P,$ etc.

$$A_5 = \frac{\langle 2\,5\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle\langle 4\,5\rangle} \approx \frac{1}{\sqrt{1-z}\,\langle 3\,4\rangle} \times \frac{\langle 2\,5\rangle^2}{\langle 1\,2\rangle\langle P\,5\rangle}$$

$$\to \text{Split}_-(3_q^+, 4_g^+) \times A_4(1^+, 2^-, P^+, 5^-)$$



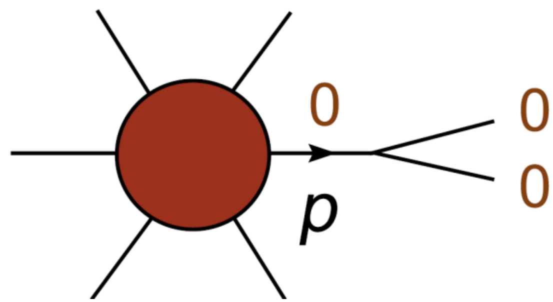
Time-like kinematics (fragmentation).
Space-like (parton evolution)
related by crossing

Universal collinear factors, or splitting amplitudes
$\text{Split}_{-h_P}(a^{h_a}, b^{h_b})$ depend on parton type and helicity h

Spinor Magic

Spinor products precisely capture
square-root + phase behavior in **collinear limit**.
Excellent variables for **helicity amplitudes**

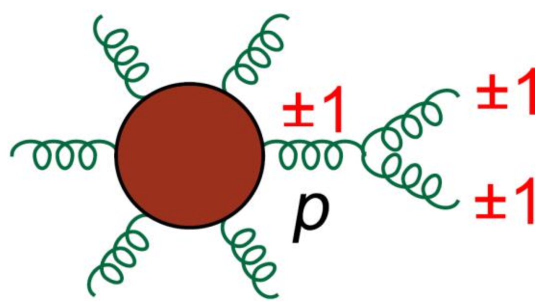
scalars



$$\sim \frac{1}{p^2} \sim \frac{1}{s_{ij}}$$

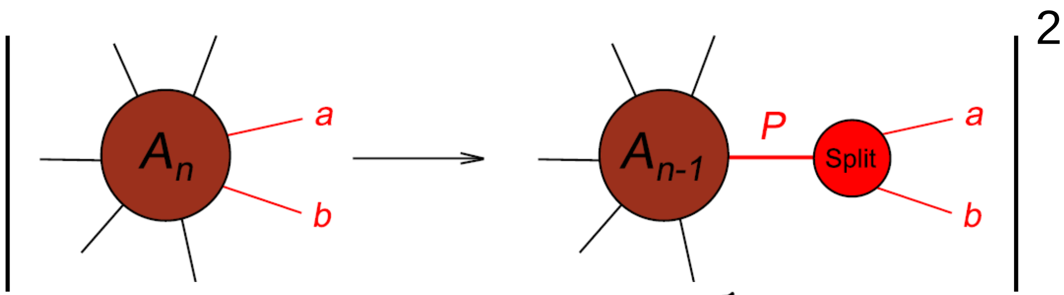
gauge theory

angular momentum mismatch



$$\sim \frac{e^{\pm i\phi}}{\sqrt{p^2}} \sim \frac{1}{\langle i\, j\rangle} \text{ or } \frac{1}{[i\, j]}$$

From splitting amplitudes to probabilities



$$d\sigma_n \sim d\sigma_{n-1} \times \frac{1}{s_{ab}} \times P(z)$$

$$P(z) \propto \sum_{h_P, h_a, h_b} \left| \text{Split}_{-h_P}(a^{h_a}, b^{h_b}) \right|^2 s_{ab}$$

$q \to qg$:

$$P_{qq}(z) \propto C_F \left\{ \left| \frac{1}{\sqrt{1-z}} \right|^2 + \left| \frac{z}{\sqrt{1-z}} \right|^2 \right\} \qquad C_F = \frac{N_c^2-1}{2N_c}$$

$$= C_F \frac{1+z^2}{1-z} \qquad z < 1$$

Note soft-gluon singularity as $z_g = 1 - z \to 0$

Similarly for gluons

Exercise: Extract from Parke-Taylor formula

$g \to gg$:

$$P_{gg}(z) \propto C_A \left\{ \left| \frac{1}{\sqrt{z(1-z)}} \right|^2 + \left| \frac{z^2}{\sqrt{z(1-z)}} \right|^2 + \left| \frac{(1-z)^2}{\sqrt{z(1-z)}} \right|^2 \right\}$$

$$= C_A \frac{1 + z^4 + (1-z)^4}{z(1-z)} \qquad C_A = N_c$$

$$= 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right] \qquad z < 1$$

Again a soft-gluon singularity. Gluon number not conserved.
But momentum is. Momentum conservation mixes $g \to gg$ with

$g \to q\bar{q}$:

$$\boxed{P_{qg}(z) = T_R \left[z^2 + (1-z)^2 \right]} \qquad T_R = \frac{1}{2}$$

Exercise: deduce, up to color factors, by taking
$e^+ \,||\, e^-$ in $\mathcal{A}_5(e^+ e^- \to q g \bar{q})$

Gluon splitting (cont.)

$g \to gg$:

Applying momentum conservation,

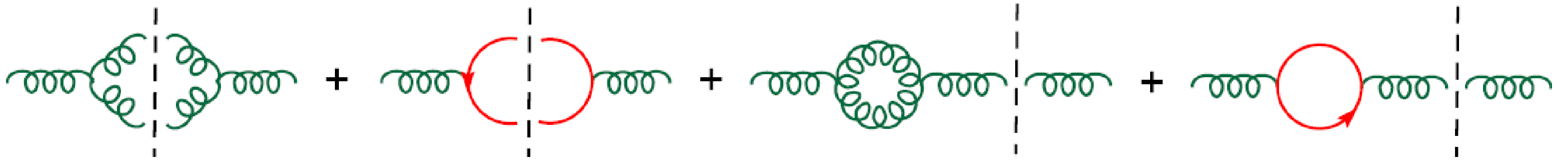
$$\int_0^1 dz\, z\, \left[P_{gg}(z) + 2n_f P_{qg}(z)\right] \;=\; 0$$

gives

$$P_{gg}(z) \;=\; 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] \;+\; b_0\,\delta(1-z)$$

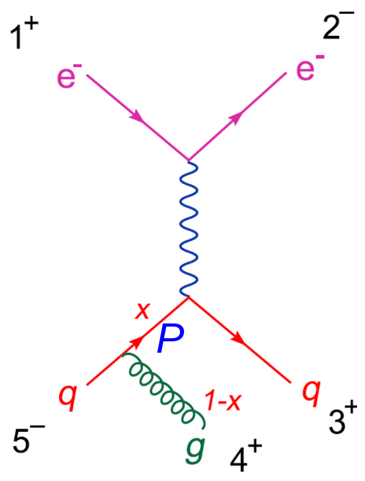
$$b_0 \;=\; \frac{11C_A - 4n_f T_R}{6}$$

Amusing that first β-function coefficient enters,
since no loops were done, except implicitly via unitarity:



Space-like splitting

- The case relevant for parton evolution
- Related by crossing to time-like case
- Have to watch out for flux factor, however



$$q \to qg: \quad k_P = x\, k_5, \quad k_4 = (1-x)\, k_5$$

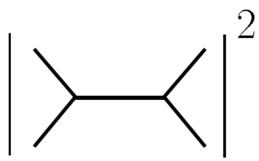
$$A_5 = \frac{\langle 2\,5\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle\langle 4\,5\rangle} \approx \frac{\frac{1}{x}}{\sqrt{\frac{1-x}{x}}\,\langle 4\,5\rangle} \times \frac{\langle 2\,P\rangle^2}{\langle 1\,2\rangle\langle 3\,P\rangle}$$

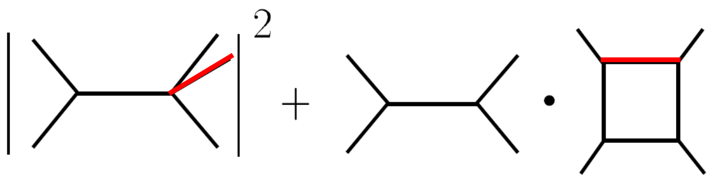
$$= \frac{1}{\sqrt{x}} \frac{1}{\sqrt{1-x}\,\langle 4\,5\rangle} \times \frac{\langle 2\,P\rangle^2}{\langle 1\,2\rangle\langle 3\,P\rangle}$$

absorb into flux factor: $d\sigma_5 \propto \frac{1}{s_{15}}$ $\quad d\sigma_4 \propto \frac{1}{s_{1P}} = \frac{1}{x\, s_{15}}$

When dust settles, get exactly the same splitting kernels (at LO)

Infrared divergences in QCD

LO 

NLO 

real virtual

Usually regulate dimensionally,
$D = 4 - 2\epsilon$

$d^4k \;\to\; d^{4-2\epsilon}k$
in all phase-space and loop integrals

soft singularities: $k_s \to 0$ $\qquad \sigma^{\rm real} \sim \int \frac{dk_s^2}{k_s^{2(1+\epsilon)}} \sigma^{\rm LO}(k_s = 0)$

collinear singularities: $k_{ab}^2 \to 0 \quad (k_a \,||\, k_b)$ $\qquad \sim \int \frac{dk_{ab}^2}{k_{ab}^{2(1+\epsilon)}} \sigma^{\rm LO}(k_P)$

virtual soft/collinear singularities: $\qquad \sigma^{\rm virt} \sim \Big[-\frac{1}{\epsilon^2} \sum_i C_i - \frac{1}{\epsilon} \sum_{i,j} D_{ij} \ln\Big(\frac{\mu^2}{-s_{ij}}\Big) \Big] \sigma^{\rm LO}$

• Virtual corrections cancel real singularities, but only for quantities insensitive to soft/collinear radiation → **infrared-safe observables** ***O***

Infrared safety

Infrared-safe observables *O*:

- Behave smoothly in soft limit as any parton momentum → 0
- Behave smoothly in collinear limit as any pair of partons → parallel (||)

$$O_n(\ldots,k_s,\ldots) \;\to\; O_{n-1}(\ldots,\cancel{k_s},\ldots) \qquad k_s \to 0$$

$$O_n(\ldots,k_a,k_b,\ldots) \;\to\; O_{n-1}(\ldots,k_P,\ldots) \qquad k_a \,||\, k_b$$

- Cannot predict perturbatively any infrared-unsafe quantity, such as:
  - the number of partons (hadrons) in an event
  - observables requiring no radiation in some region (rapidity gaps or overly strong isolation cuts)
  - p_T(*W, Z* or Higgs) precisely at $p_T = 0$

Infrared safety (cont.)

Examples of IR safe quantities at LHC:

- most kinematic distributions of "electroweak" objects, *W, Z, Higgs* (photons tricky because of collinear issues)
- jets, defined by cluster or (suitable) cone algorithm

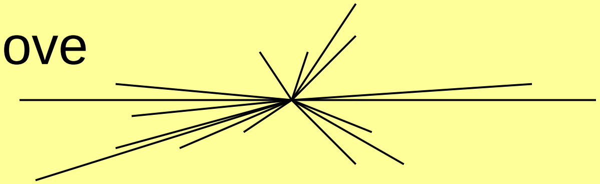
jet cluster algorithm n = -2, 0, 2 ←→ **anti-k_T , CA, k_T**

- Construct list of objects, starting with particles i, plus "the beam" b
- Define "distance" between objects, vanishing in soft/collinear limits:

$$d_{ij} = \min\{k_T^{(i)}, k_T^{(j)}\}^n \, [(\eta^{(i)} - \eta^{(j)})^2 + (\phi^{(i)} - \phi^{(j)})^2]/R^2 \qquad d_{ib} = [k_T^{(i)}]^n$$

- If a d_{ij} is smallest, cluster together i and j.

If a d_{ib} is smallest, declare i to be a jet and remove it from the list of particles

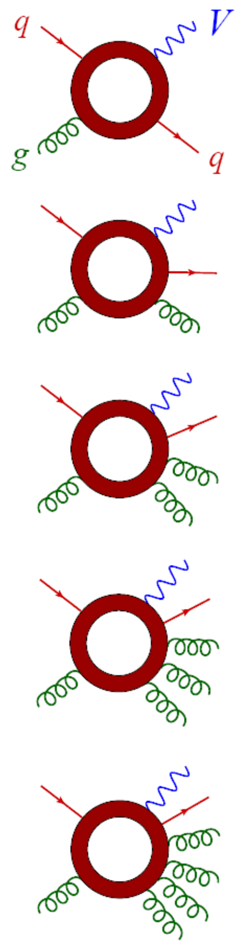


- Repeat until all objects are jets

Incredibly versatile, powerful concept – but hard to compute analytically beyond LO

Loop difficulty increases rapidly with number of jets

***pp* → *W* + *n* jets**   (just cases with most gluons)



# of jets	# 1-loop Feynman diagrams
1	11
2	110
3	1,253
4	16,648
5	256,265

- Total number of diagrams not the biggest problem.

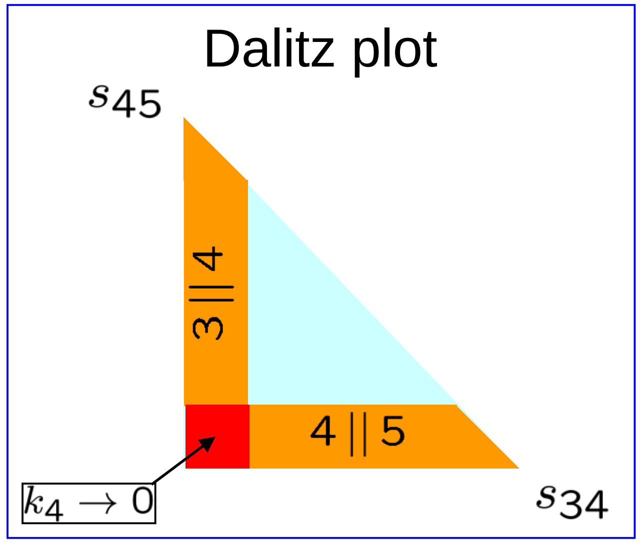
- Some of them have very complicated tensor structures and can develop numerical instabilities in various kinematic regions.

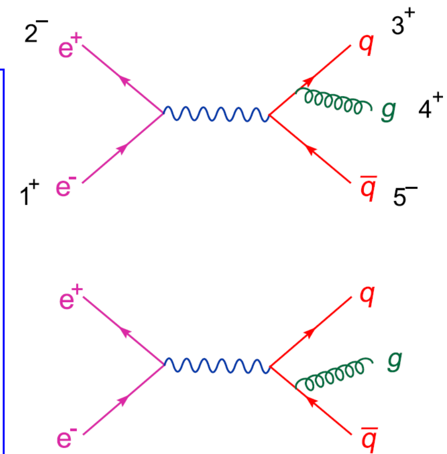
Real radiation prototype: infrared cancellations in e^+e^-

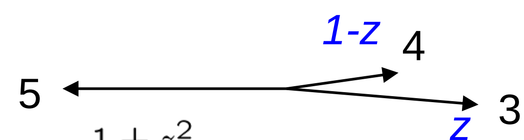
$$s_{34} + s_{45} + s_{35} \;=\; s_{12}$$

Or can use as variables:

- 1 invariant mass, say s_{34}
- CM polar angle for “decay” of (3,4) pair, $\cos\theta$, equivalent to z







$$\sigma_{\rm real}^{\rm tot} = \mu^{2\epsilon}\int d^{4-2\epsilon}{\rm LIPS}_{3,4,5}\,|\mathcal{A}_5|^2 \;\propto\; \left(\frac{\mu^2}{s_{12}}\right)^{\epsilon}|\mathcal{A}_4|^2\frac{\alpha_s}{2\pi}\int_0\frac{ds_{34}}{s_{34}^{1+\epsilon}}\int_0^1 dz(1-z)^{-\epsilon}C_F\frac{1+z^2}{1-z}$$

$$= |\mathcal{A}_4|^2\frac{\alpha_s}{2\pi}C_F\left(\frac{\mu^2}{s_{12}}\right)^{\epsilon}\left(-\frac{1}{\epsilon}\right)\int_0^1 dz\left[\frac{2}{(1-z)^{1+\epsilon}}-1-z\right]$$

$$= |\mathcal{A}_4|^2\frac{\alpha_s}{2\pi}C_F\left(\frac{\mu^2}{s_{12}}\right)^{\epsilon}\left(-\frac{1}{\epsilon}\right)\left[-\frac{2}{\epsilon}-\frac{3}{2}\right]$$

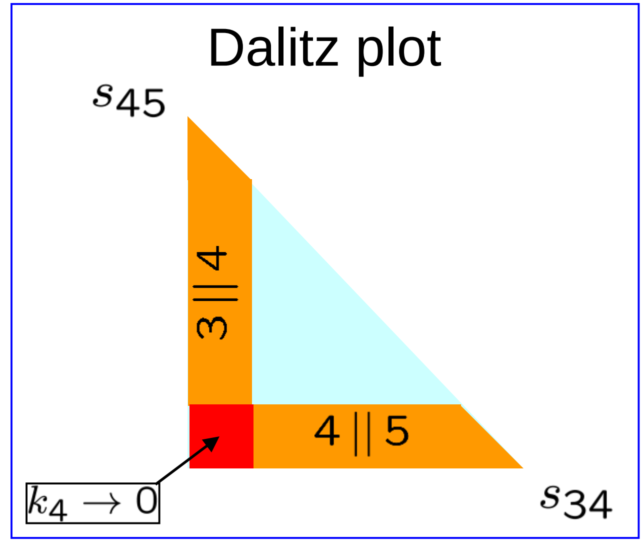
$$= |\mathcal{A}_4|^2\frac{\alpha_s}{\pi}C_F\left(\frac{\mu^2}{s_{12}}\right)^{\epsilon}\left[\frac{1}{\epsilon^2}+2\frac{3}{4\epsilon}+\text{finite}\right]$$

after doubling $z < 1$ term to account for 4 || 5 region

Real radiation in general case

Cannot perform the phase-space integral analytically in D=4-2ε, especially not for generic experimental cuts

Also can't do it numerically, because of $1/\varepsilon^2$ poles



2 solutions:

1. Slice out singular regions of phase-space, with (thin) width s_{min} Perform integral there approximately. Rest of integral done numerically. Check cancellation of s_{min} dependence.
2. Subtract a function that mimics the soft/collinear behavior of the radiative cross section, and which you can integrate (analytically). Integral of the difference can be done numerically.

Dipole formalism

Catani, Seymour, hep-ph/9602277, hep-ph/9605323

Popular (stable) version of the subtraction method

Build dipole subtraction function $D_{ij,k}$ for each pair of partons i,j that can get singular, and for each "spectator" parton k

$$\mathcal{D}_{ij,k}(p_1,...,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} \cdot {}_m\!< 1,..,\widetilde{ij},..,\widetilde{k},..,m+1| \frac{\boldsymbol{T}_k\cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2} \boldsymbol{V}_{ij,k} |1,..,\widetilde{ij},..,\widetilde{k},..,m+1>_m$$

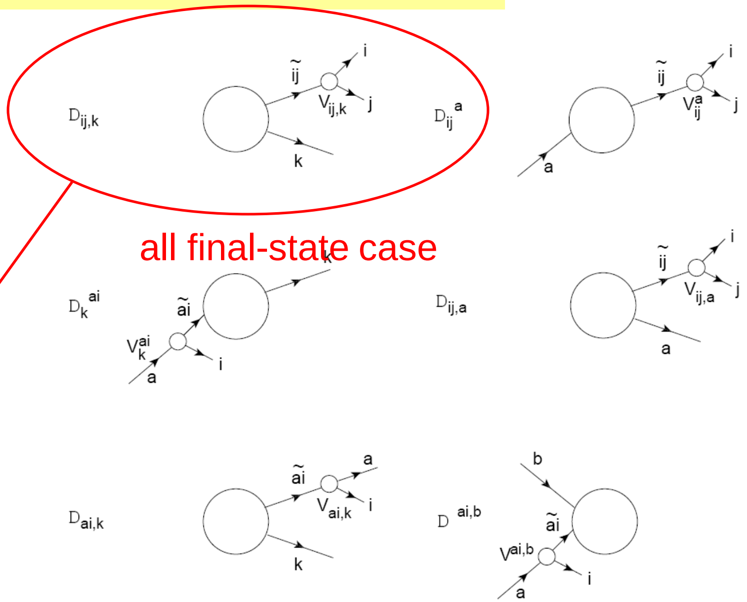


Figure 3: Effective diagrams for the different dipole formulae introduced in Sect. 5. The blobs denote the m-parton matrix element. Incoming and outgoing lines respectively stand for initial-state and final-state partons.

The $D_{ij,k}$ multiply the LO cross section, at a boosted phase-space point:

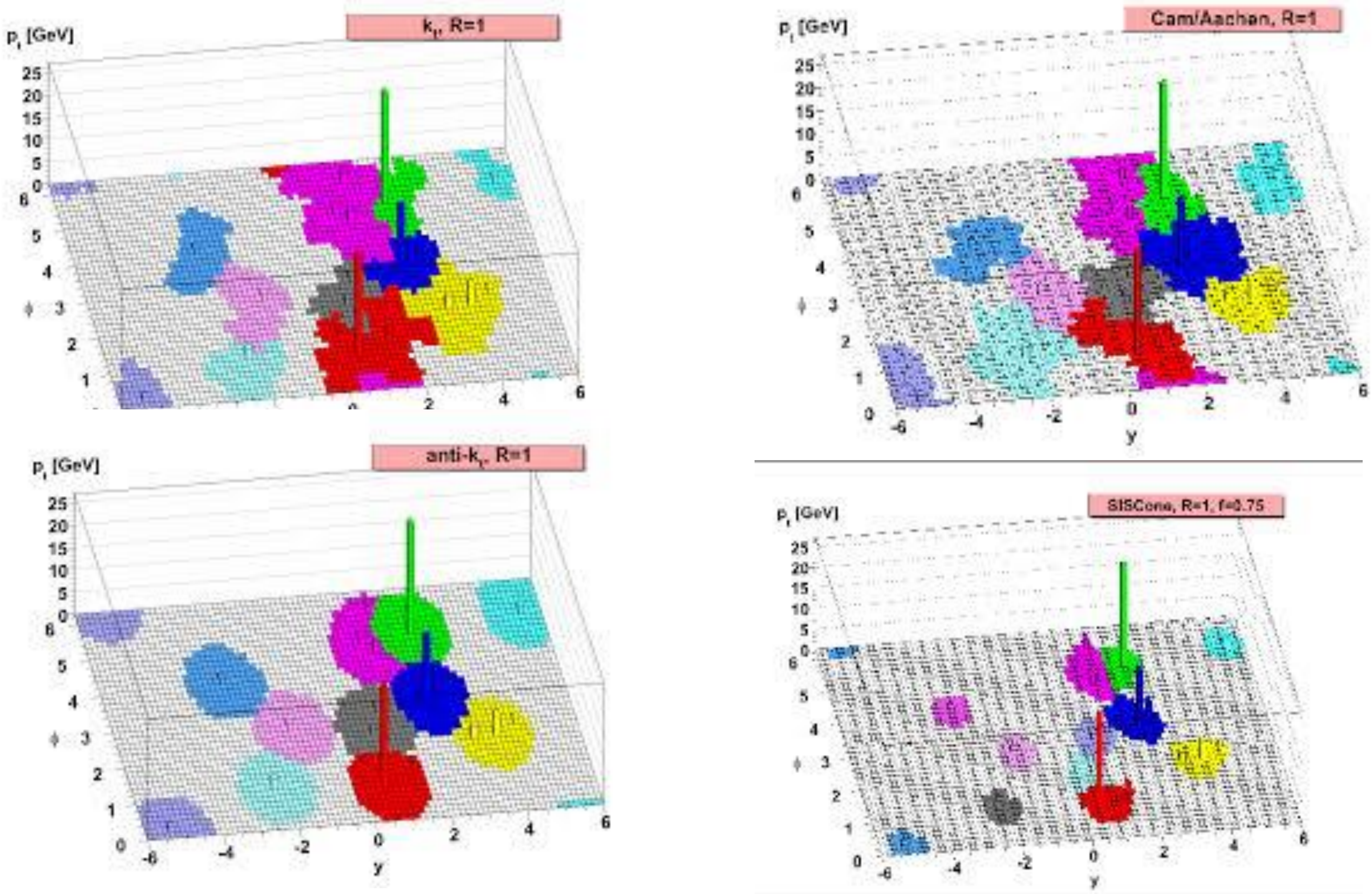
$$\widetilde{p}_k^{\mu} = \frac{1}{1-y_{ij,k}}\, p_k^{\mu} \quad , \quad \widetilde{p}_{ij}^{\mu} = p_i^{\mu} + p_j^{\mu} - \frac{y_{ij,k}}{1-y_{ij,k}}\, p_k^{\mu}$$

All dipole integrals can be done analytically

Hundreds of dipoles for NLO pp → 3 jets

Jet Catchment Areas

Cacciari, Salam, Soyez, 0802.1188

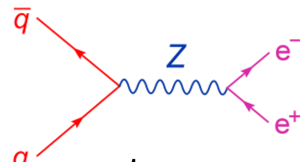


Overall structure of higher-order QCD corrections

Example of *Z* production at hadron colliders

convolute with pdfs
apply cuts

LO $\hat{\sigma}^{(0)}$

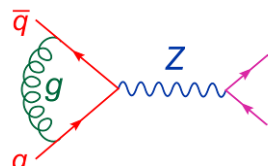


tree

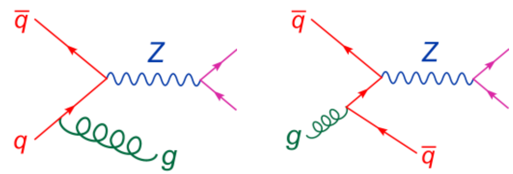
dim. reg. $D = 4 - 2\epsilon$

first, cancel infrared divergences ($1/\epsilon^2$)
between virtual & real

NLO $\hat{\sigma}^{(1)}$

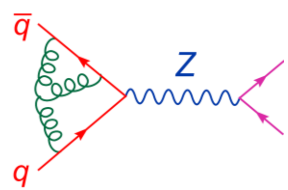


1 loop

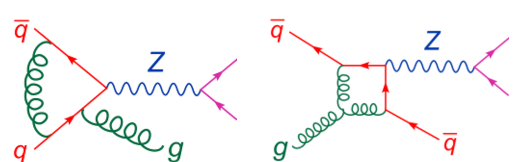


tree + 1 parton

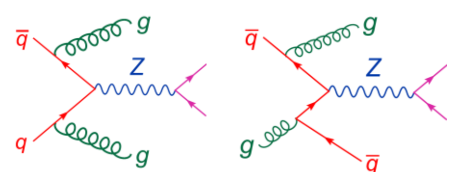
NNLO $\hat{\sigma}^{(2)}$



2 loops



1 loop + 1 parton

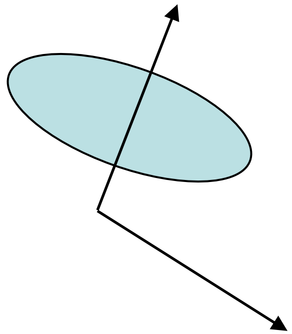


tree + 2 partons

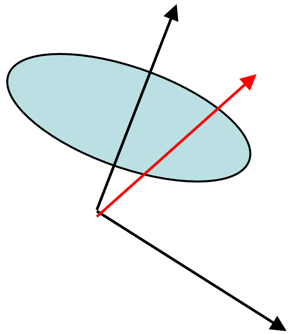
intricate ($1/\epsilon^4$) IR cancellations

Fixed order good for jet rates, not for jet substructure

- LO: 1 jet = 1 parton



- NLO: 1 jet = 1 or 2 partons



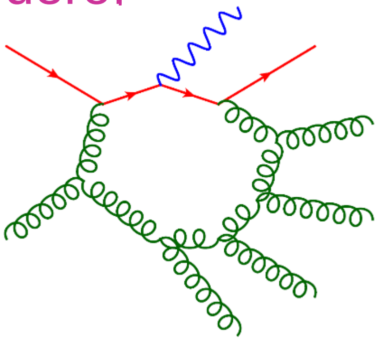
- NNLO: 1 jet = 1,2 or 3 partons

- Better to use Monte Carlo simulations or resummation methosd for jet substructure

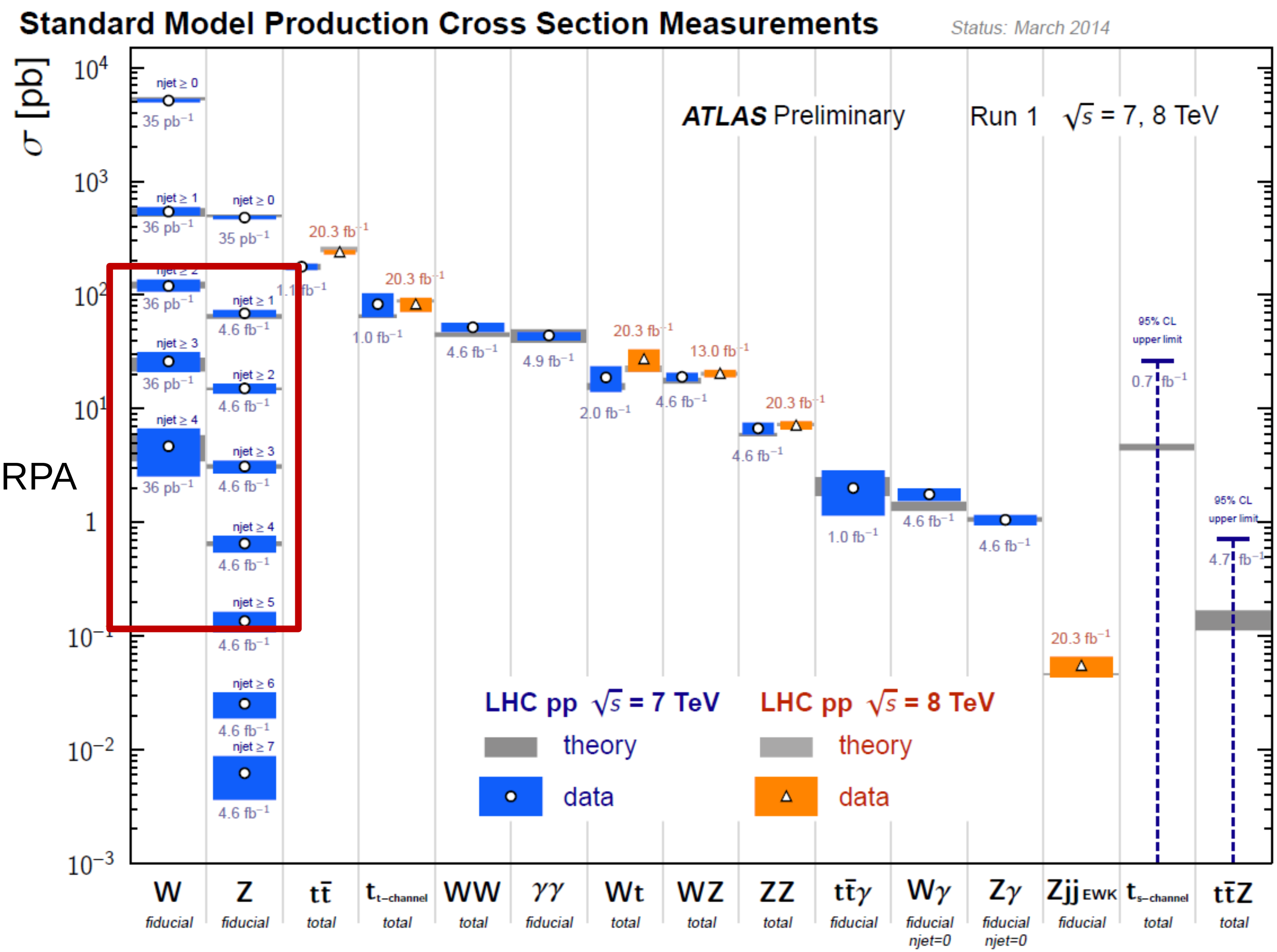
0803.4180

BlackHat automates one-loop SHERPA automates real subtractions

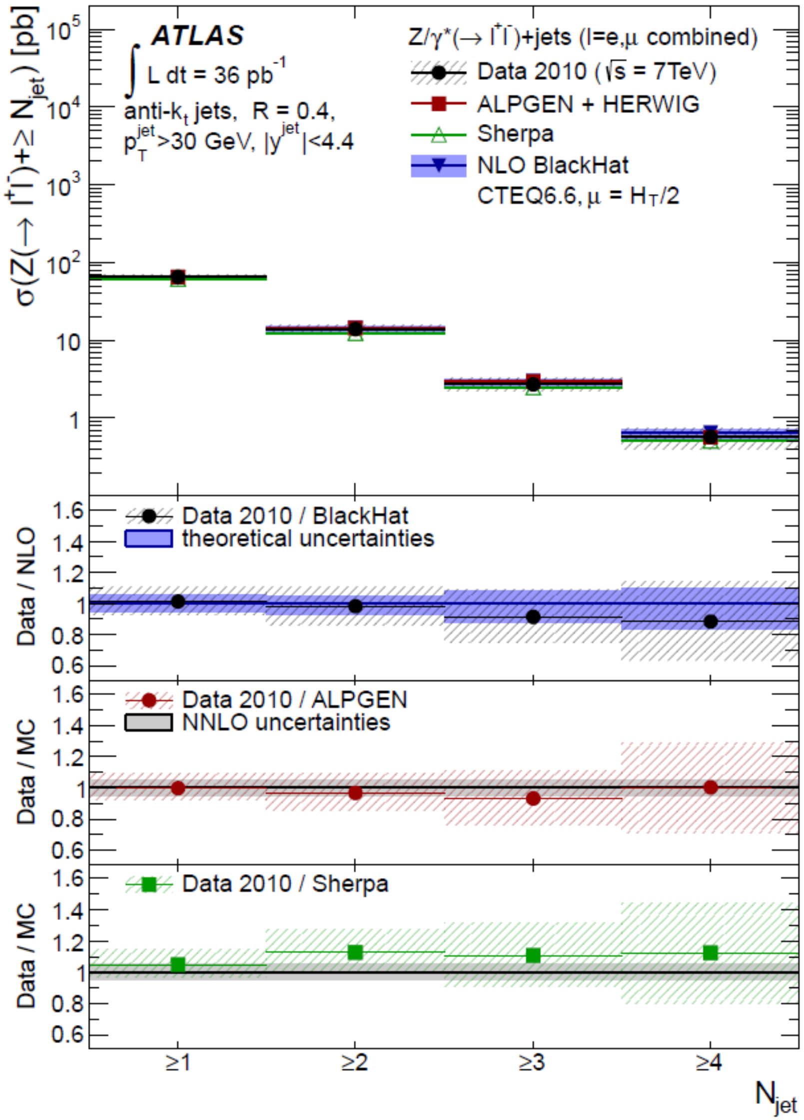
- 4 jets [1112.3940, Z. Bern, G. Diana, LD, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren]
- Z+4 jets [1108.2229, H. Ita, Z. Bern, LD, F. Febres Cordero, D. Kosower, D. Maitre]
- W+5 jets [1304.1253, Z. Bern, LD, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren]
- Ntuple framework for varying pdfs, scales, cuts efficiently [1310.7439, Z. Bern, F. Febres Cordero, LD, S. Höche, H. Ita, D. Kosower, D. Maitre]



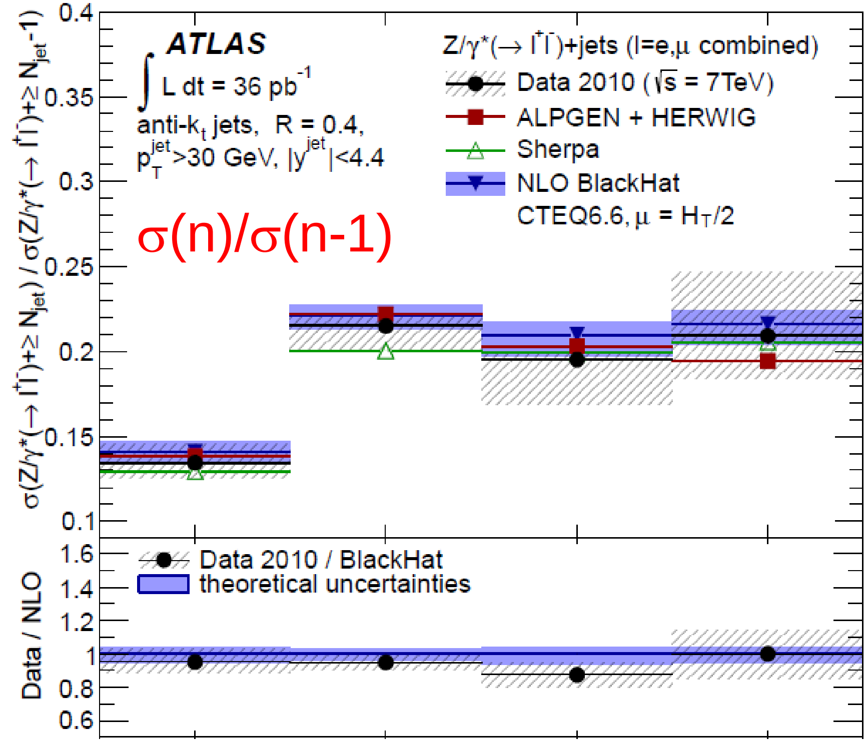
theory: BlackHat+SHERPA



NLO pp → Z + 1,2,3,4 jets vs. 2010 ATLAS data

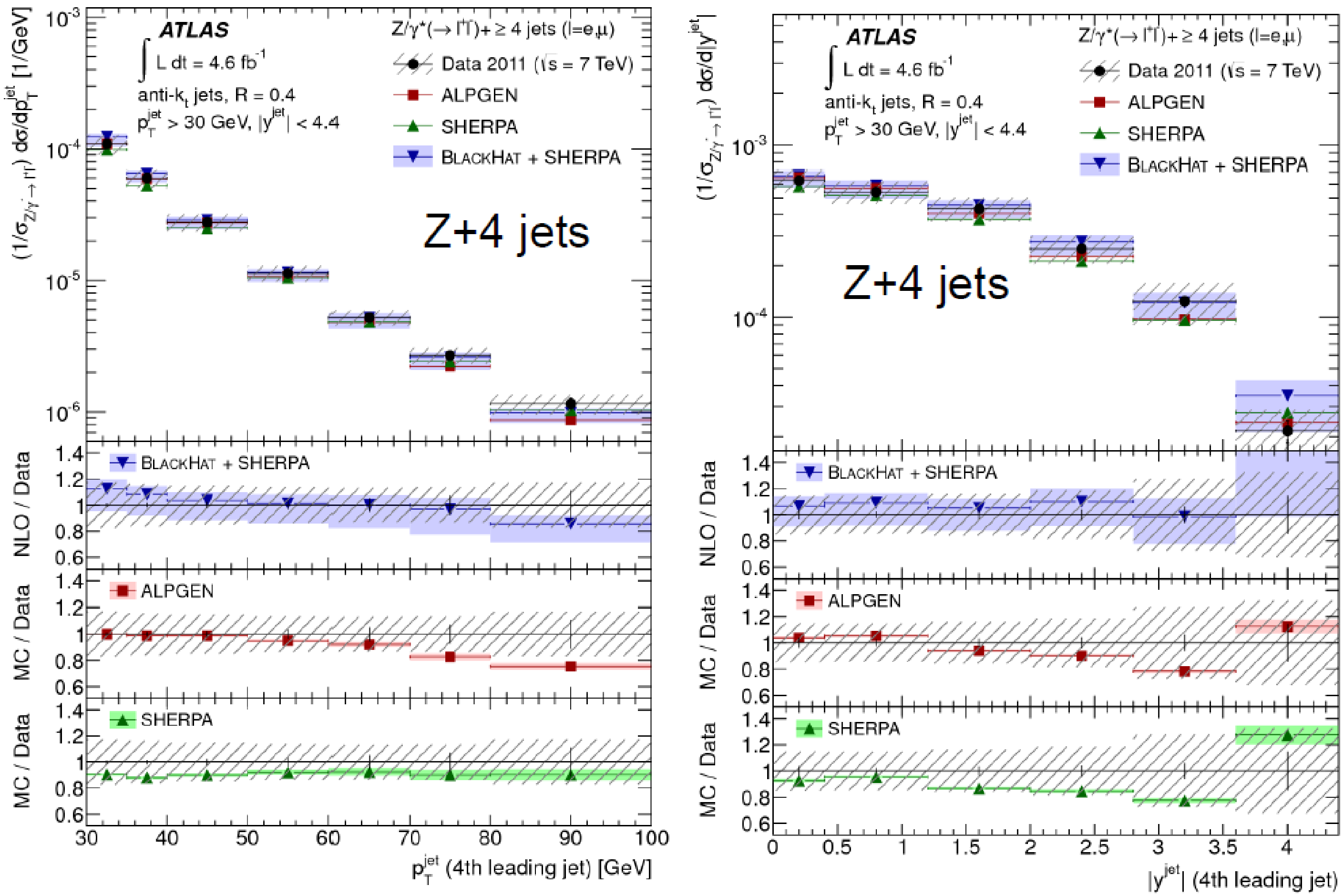


arXiv:1111.2690 [hep-ex]



n = 1 2 3 4

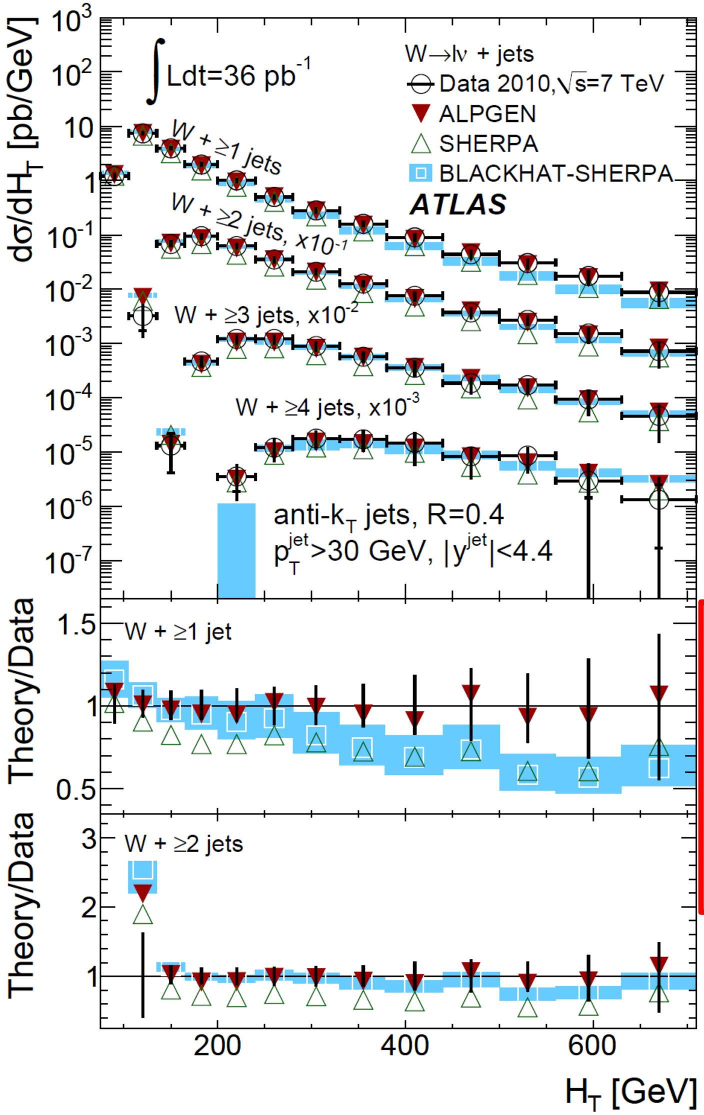
pp → Z + 4 jets



ATLAS
arXiv:1304.7098

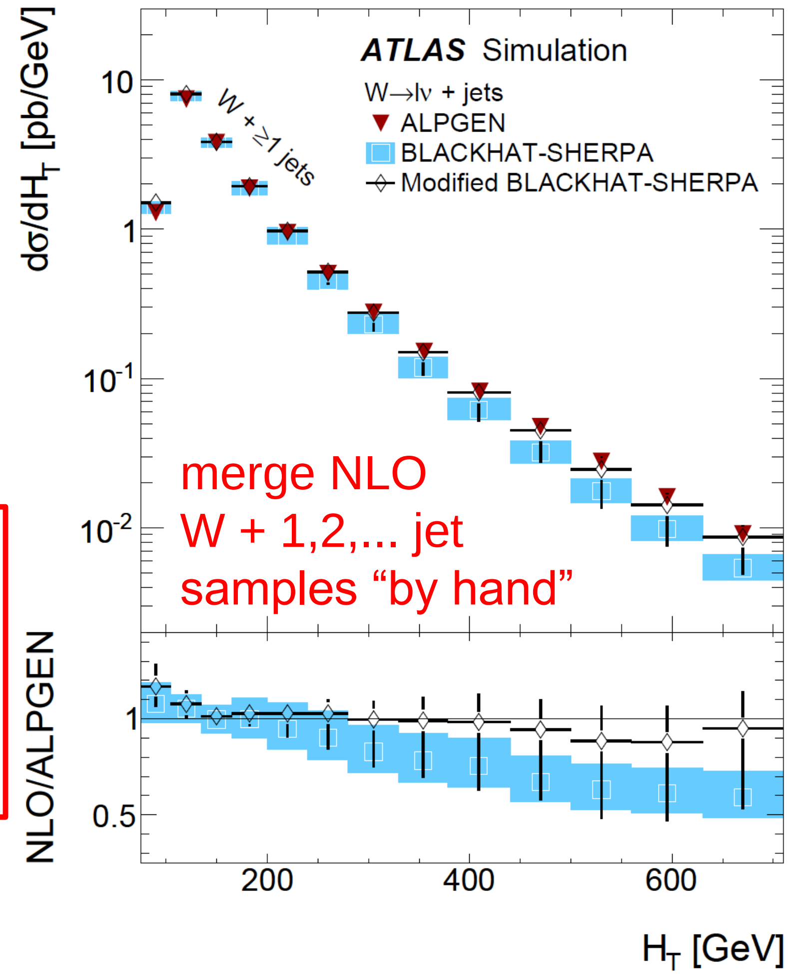
- NLO Z+≥ 4 jets and ME+PS consistent with data

NLO pp → W + 1,2,3,4 jets vs. 2010 ATLAS data

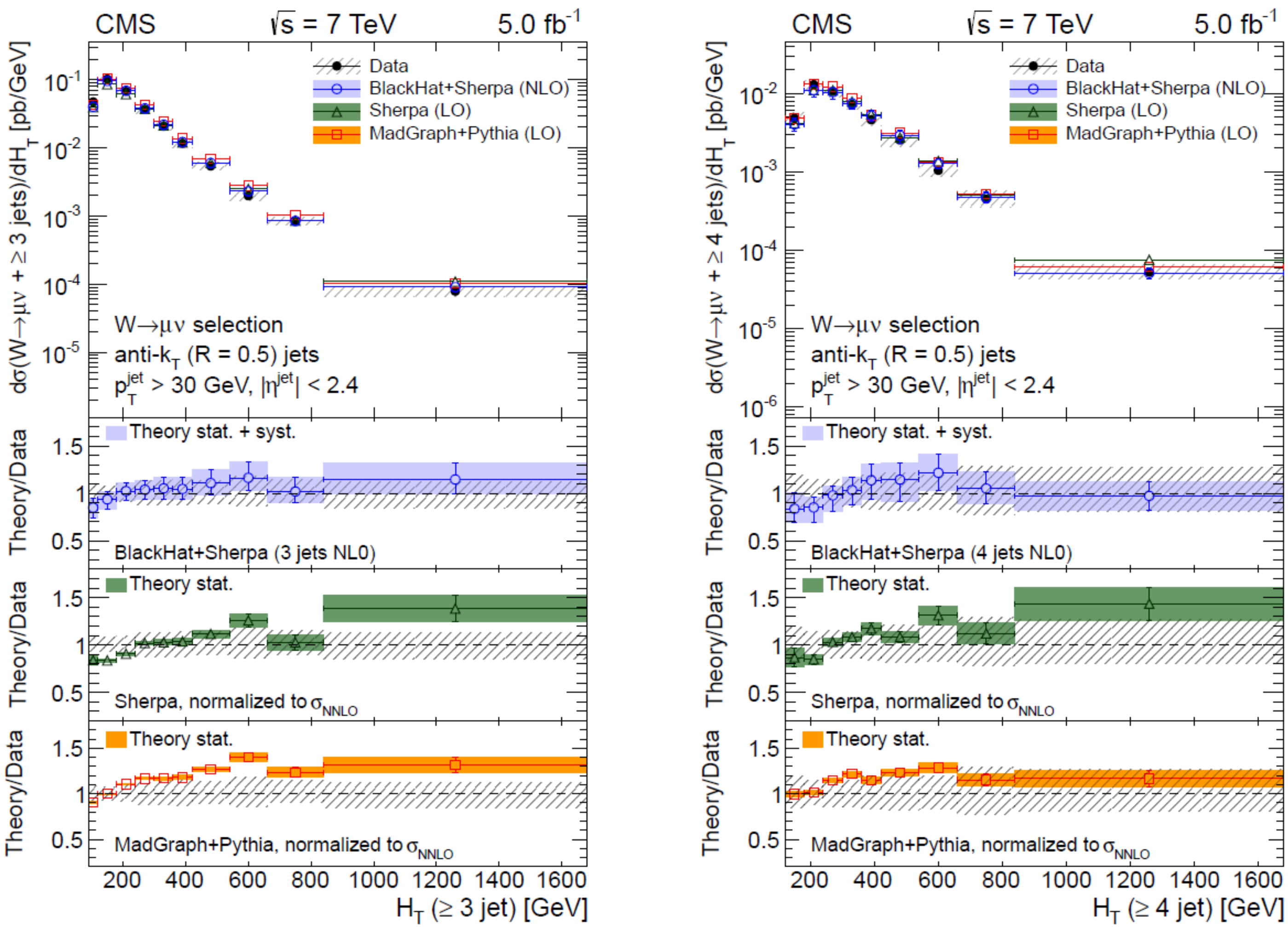


difficult observable for fixed order
Rubin, Sapeta, Salam, 1006.2144

arXiv:1201.1276 [hep-ex]



pp → W + 3,4 jets

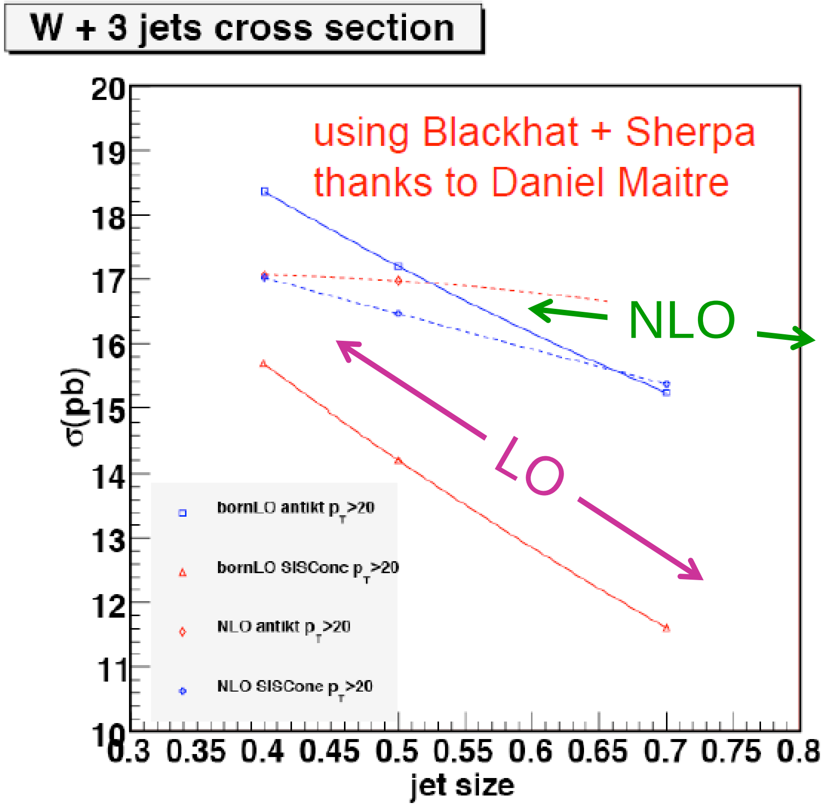


CMS
arXiv:1406.7533

Jet size dependence

J. Huston

- Let's look at predictions for W + 3 jets for two different jet algorithms as a function of jet size at the LHC (7 TeV)
- At LO, both antikT and SISCone show a marked decrease in cross section as the jet size increases
  - because of the log(1/ΔR) effect
- But at NLO, the two cross sections show little dependence on the jet size, and are similar to each other
- You'll see the same thing in ATLAS Monte Carlo



note NLO~LO because a scale of H_T has been used

Conclusions

- The soft and collinear structure of QCD is exposed by the simplest 4 and 5 point amplitudes, computed in the spinor helicity formalism.
- Infrared safety allows for perturbative computation of short-distance quantities at colliders.
- Combined with experimentally determined parton distributions, evolved to the relevant scales using DGLAP evolution, one can make precise (NLO) predictions for processes at the LHC with many final state objects.
- NNLO results are even more precise, but currently limited to at most 2 objects in the final state.
- An ongoing challenge, being tackled here in Freiburg and elsewhere, is to extend NNLO precision to LHC processes with 3 or more objects in the final state.
- Tomorrow we'll discuss the energy-energy correlation (EEC) in electron-positron annihilation. This process is so simple that it can be computed analytically at NLO. In the small-angle limit, it can be resummed to NNLL, and it may lead to more computable jet substructure at the LHC.

Extra Slides

Soft-gluon/Sudakov resummation

- A prevalent theme in QCD whenever one is at an edge of phase space.
- Infrared-safe but sensitive to a second, smaller scale
- Same physics as in (high-energy) QED: $e^+e^- \;=\; e^+e^-(\gamma)$
- What is prob. of no γ with $E > \Delta E$, $\theta > \Delta\,\theta$?

$$P = 1 - \frac{\alpha}{\pi}\int_{\Delta E}\frac{dE}{E}\int_{\Delta\theta}\frac{d\theta}{\theta} + \cdots \;=\; 1 - \frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta + \cdots$$
$$= \exp\left(-\frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta\right) + \cdots$$

soft

collinear

leading double logarithms -- in contrast to single logs of renormalization group, DGLAP equations.

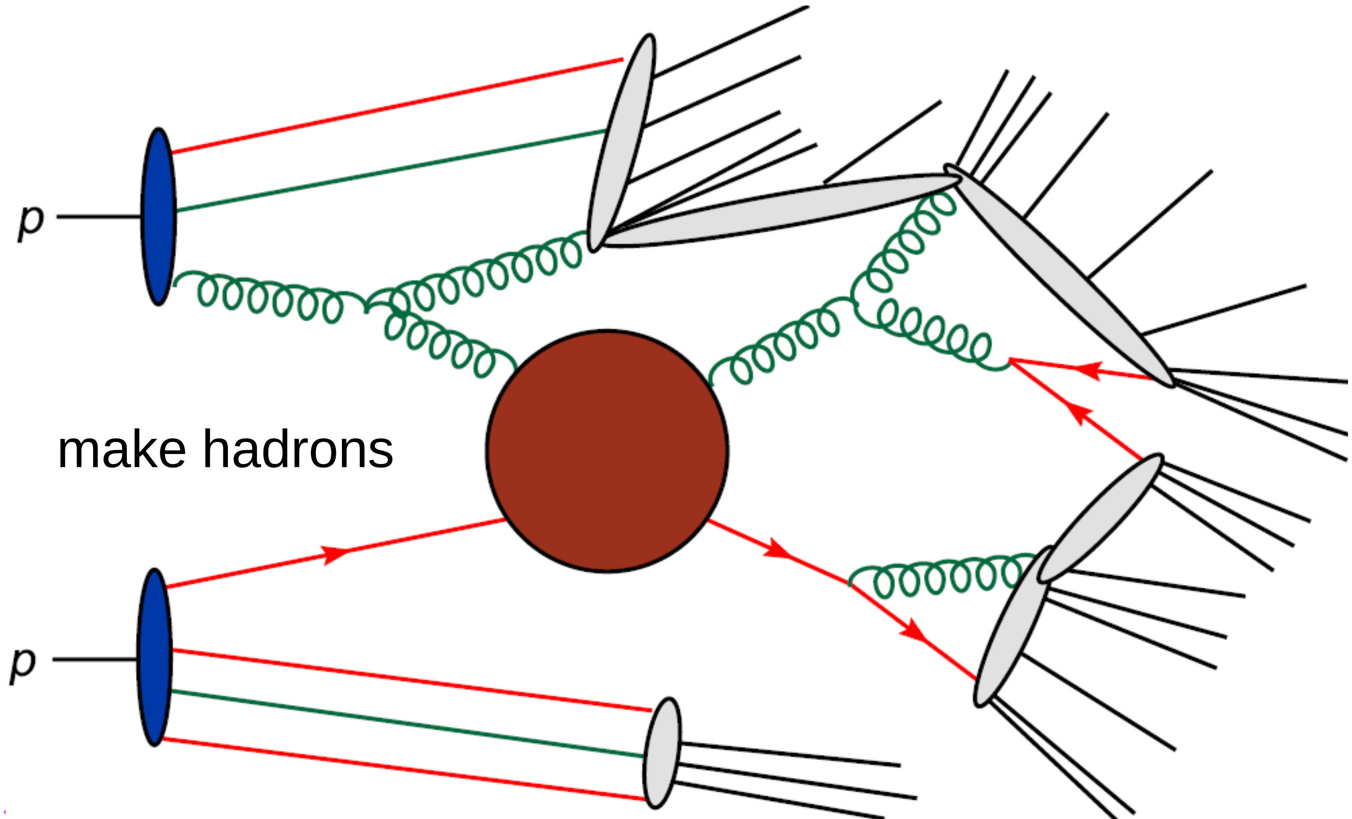
exponentiation because soft emissions are independent

Monte Carlos

- Based on properties of soft and collinear radiation in QCD
- Partons surrounded by “cloud” of soft and collinear partons
- Leading double logs of Q_{hard}/Q_{soft} exponentiate, can be generated probabilistically
- Shower starts with basic 2 → 2 parton scattering
  -- or basic production process for *W, Z, tt*, etc.
- Further radiation approximate, requires infrared cutoff
- Shower can be evolved down to very low Q_{soft} , where models for hadronization and spectator interactions can be applied
- Complete hadron-level event description attained
- Normalization of event rates unreliable
- Event “shapes” sometimes unreliable

Monte Carlos in pictures

Splitting probability: $\mathsf{P}_g(q^2) = \int_0^1 dz \frac{\alpha_s(q^2)}{2\pi} \hat{P}_{gg}(z) \Theta(q^2 - q_0^2)$



Sudakov

$\Delta_g(Q^2, q^2) = \exp\left[- \int_{q^2} \frac{}{\tilde{q}^2} \mathsf{P}_g(q^2)\right]$