Neutrino Physics

Outline

1. Neutrinos in the SM
2. Neutrino masses
3. ν oscillations
4. Experiments
5. ew ν physics

1. ν in the SM

$$\mathcal{L} \supset \frac{g}{\sqrt{2}} \overline{\nu}_{\alpha,L} \gamma^\mu e_{\alpha,L} W_\mu^+ + \frac{g}{2\cos\theta_W} \overline{\nu}_{\alpha,L} \gamma^\mu \nu_{\alpha,L} Z_\mu$$

$$-(m_{\alpha\beta} \overline{\nu}_{\alpha L} \nu_{\beta R} + h.c.)$$

$\Rightarrow$ ν state produced in weak int.:

$$|\nu_\alpha\rangle = U^*_{\alpha j} |\nu_j\rangle$$

flavor eigenstates; PMNS matrix; mass eigenstate

2. ν masses

$$\mathcal{L}_m = -m_D \overline{\nu}_L \nu_R - \frac{1}{2} m_M \overline{\nu_R^c} \nu_R + h.c.$$

$$= -\frac{1}{2}\,\overline{n^c}\, M\, n$$

$$n = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \qquad M = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}$$

Diagonalize M.

$$\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}$$

$$\Rightarrow \tan 2\theta = \frac{2m_D}{m_M}$$

$$m_{1,2} = \frac{m_M}{2} \pm \sqrt{\frac{m_M^2}{4} + m_D^2}$$

↳ for $m_M \gg m_D$:

$$m_1 \simeq m_M$$

$$\boxed{m_2 = \frac{m_D^2}{m_M}}$$

For $m_D \sim 100\,\text{GeV}$, $m_M \sim 10^{14}\,\text{GeV}$

↳ $m_2 \sim 0.1\,\text{eV}$

3. ν oscillations

Neutrino state produced in source:

$$|\nu_\alpha\rangle = U^*_{\alpha j} |\nu_j\rangle$$

After time T, distance L:

$$|\nu_\alpha(T, L)\rangle = U^*_{\alpha j} \, e^{-iE_j T + i p_j L} |\nu_j\rangle$$

Detection: Project onto

$$\langle \nu_\beta | = U_{\beta k} \langle \nu_k |$$

$$\Rightarrow \langle \nu_\beta | \nu_\alpha(T, L)\rangle = U^*_{\alpha j} U_{\beta j} \, e^{-iE_j T + i p_j L}$$

Oscillation propability:

$$\boxed{\begin{aligned} P_{\alpha\beta} &= U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \\ &\quad \cdot e^{-i(E_j - E_k)T + i(p_j - p_k)L} \end{aligned}}$$

Usually, large uncertainty on T

$\hookrightarrow$ integrate $\int dT$

$$\Longrightarrow P_{\alpha\beta}(L) \propto U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}$$

$$\cdot \underbrace{\exp\left[i\left(\sqrt{E^2-m_j^2}-\sqrt{E^2-m_k^2}\right)L\right]}$$

$$\approx \exp\left[-i\frac{\Delta m_{jk}^2 L}{2E}\right] \quad \text{for } m_{j,k} \ll E$$

with $\Delta m_{jk}^2 \equiv m_j^2 - m_k^2$.

- 2-Flavor approx.

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\Rightarrow P_{2f} = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}$$

- 3 Flavor

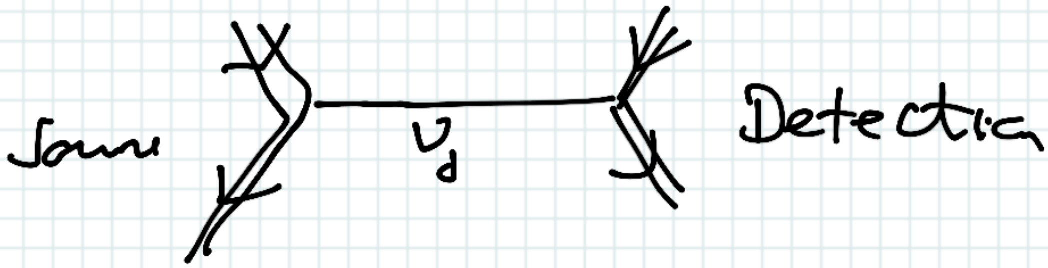
$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix}$$

$s_{ij} \equiv \sin\theta_{ij}$

$c_{ij} \equiv \cos\theta_{ij}$

$$\cdot \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

- ν oscillations in QFT



External states described as wave packets

$\Rightarrow$ compute amplitude

sum over j

expand propagator for large L

$$\Rightarrow |\mathcal{M}|^2 = (\text{prod. terms})$$
$$\cdot (\text{cross section terms})$$
$$\cdot P_{\kappa/\lambda}$$
$$\cdot \text{coherence terms}$$

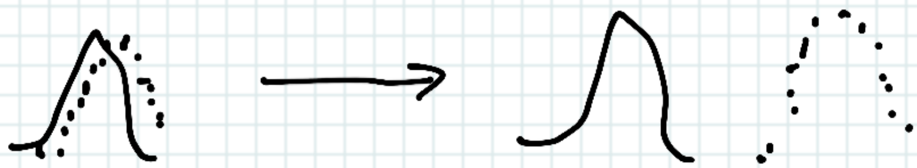
Comments on decoherence:

ν produced as wave packet

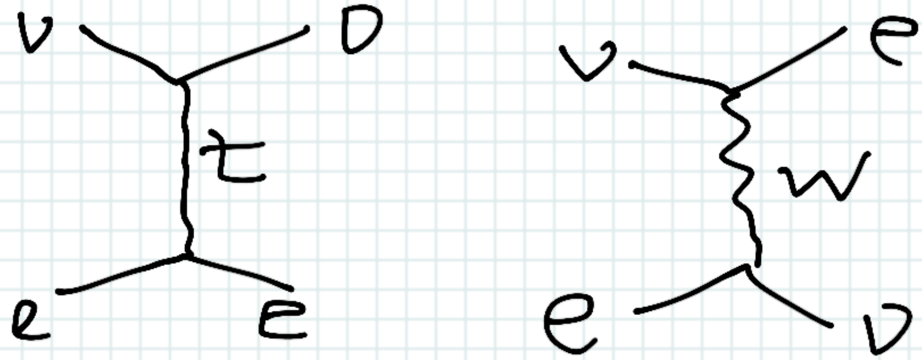
(one for each mass eigenstate)

over time, they separate because

of different group velocities



ν osc. in matter



Coherent forward scattering
(ν, e momenta do not change)
→ all e contribute coherently
↳ potentially large
⇒ ν feel an effective potential V,
which is flavor-dependent
(V is 3×3 matrix)

Phase factor $\exp[i(\phi_j - \phi_k)]$

$= p_j \cdot L$

In presence of V:

$$\hat{p} = \sqrt{(\hat{H} - \hat{V})^2 - \hat{M}^2}$$

Diagonalise $\hat{p}$ $\Rightarrow$ "Matter basis"
(states of definite p in matter)

matter basis is nearly aligned with flavour basis for $V \gg \frac{\Delta m^2}{2E}$,
with the mass basis for $V \ll \frac{\Delta m^2}{2E}$

For $\boxed{V \sim \frac{\Delta m^2}{2E}}$, possibility of

MSW resonance: small vacuum mixing angle becomes dynamically large