Karlsruhe Institute of Technology

NNLO QCD computations for the LHC

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Outline

1) Perturbation theory for collider physics observables

2) Theoretical progress for NNLO computations

3) Phenomenological examples

4) Conclusion

Why NNLO QCD may play a special role

1) Perturbative approximation (LO, NLO, NNLO) is an expansion of the leading twist contribution to proton proton scattering in the strong coupling constant. The twist expansion is quite separate and rarely discussed approximation. It is conceivable that NNLO QCD is the last perturbative contribution that is still parametrically larger than higher-twist corrections, for a generic observable.

$$N_c^2 \left(\frac{\alpha_s}{\pi}\right)^2 L_Q^2 \sim \frac{\Lambda_{\rm QCD}}{Q}$$

2) Experimental precision for a number of interesting processes (Higgs production/ couplings, electroweak boson production with extra QCD radiation, top quark physics) may reach (or have reached already !) a few percent. Matching it on the theory side requires NNLO computations.

3) Proximity of resummed and fixed order computations for realistic selection criteria at the LHC. We are in the ``grey” region where both approaches may be used for reasonable estimates of radiative corrections provided that we can reach sufficiently high orders the strong coupling expansion. In practice, NNLO seems to be sufficient.

The main advantage of fixed-order computations is the possibility to compute fiducial cross sections for realistic selection criteria.

How far in the perturbative expansion should one go?

1) Perturbative approximation (LO, NLO, NNLO) is an expansion of the leading twist contribution to proton proton scattering in the strong coupling constant. The twist expansion is quite separate and rarely discussed approximation.

$$N_c^2 \left(\frac{\alpha_s}{\pi}\right)^2 L_Q^2 \sim \frac{\Lambda_{\rm QCD}}{Q}$$

2) Existence of a NNLO calculation for a process does not imply that any observable computed using a particular ``NNLO" code has the NNLO accuracy (pt of the Z in NNLO Drell-Yan, pt of the top pair in NNLO tT production etc.). Sometimes NLO to a higher multiplicity process is more useful than NNLO to a lower multiplicity process.

3) NNLO computations are fairly insensitive to scale choices, at least in the region where NNLO is at work. Too much of a scale choice game can be counter-productive since scale variation uncertainty is one of the few handles we have to understand how relevant of higher order corrections.

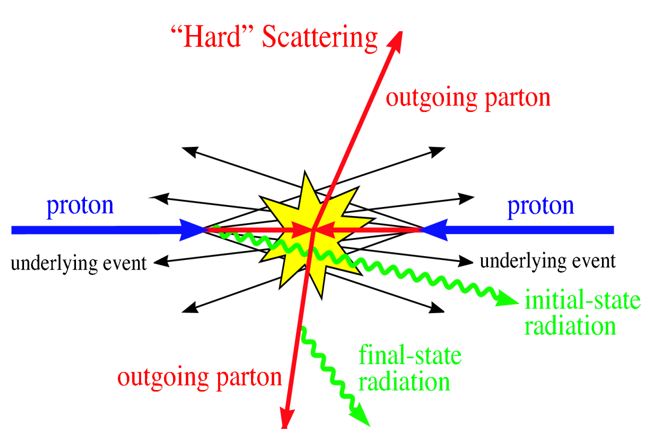
4) Proximity of resummed and fixed order computations for realistic selection criteria at the LHC. We are in the ``grey" region where both approaches provide reasonable estimates of radiative corrections provided that we can reach sufficiently high orders the strong coupling expansion. The main advantage of fixed-order computations is the possibility to compute fiducial cross sections for realistic selection criteria.

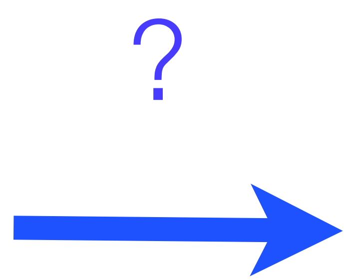
pQCD approximation for collider physics observables

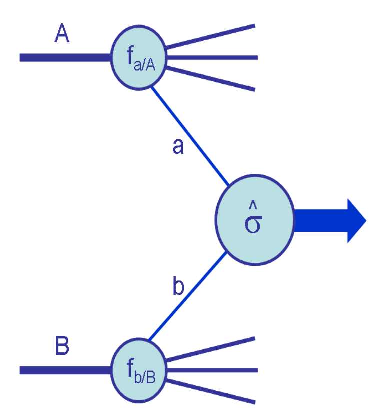
When hadrons collide with each other, many things can happen; most of these things can not be described in perturbation theory using quark and gluon degrees of freedom.

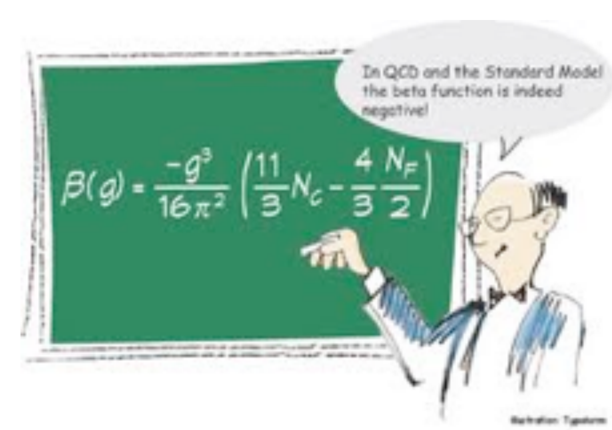
A very small fraction of hadron collisions occurs ``head on” and leads to a complete disintegration of the colliding protons. These events may have rather large energy density and, thanks to E=Mc^2, can lead to production of new, yet unknown, heavy particles.

Since such processes occur at very small distances, x ~ 1/M, where quarks and gluons behave as, essentially, free particles, they can be described in perturbation theory of QCD. The same also applies to SM processes that lead to final states similar to the ones expected in the production and decay of new heavy particles; proper description of these SM ``backgrounds” is essential for finding (small) BSM signals and elucidating their nature.









$$d\sigma = \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{\rm part}(x_1 x_2 s_{\rm hadr})$$

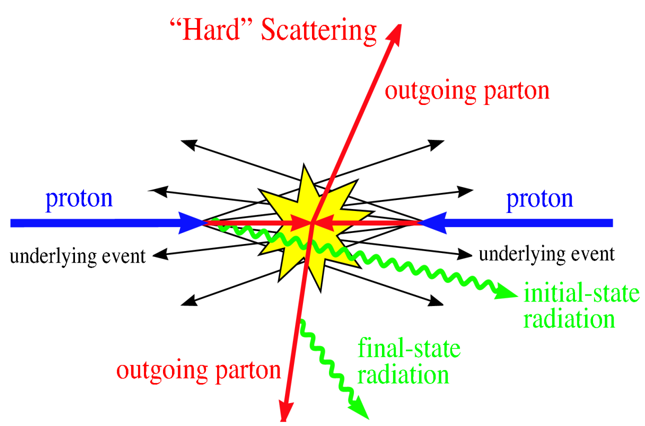
Remarks on NNLO QCD approximation

For the purposes of describing hard processes at colliders, NNLO is better than NLO that is better than LO that is better than a parton shower.

The NNLO QCD approximation is an expansion of the leading twist contribution to proton proton scattering in the strong coupling constant. The twist expansion itself is quite separate and rarely discussed approximation.

Continuous increase in the ``number of N's" is not possible without hitting a non-perturbative boundary. I do not know where this boundary is and what to do about it, but an idea that one can measure the W mass to 10 MeV (0.01 percent) or the top quark mass to better than 500 MeV (0.3 percent) without addressing non-perturbative effects theoretically from first principles seems disturbing to me.

$$N_c^2 \left(\frac{\alpha_s}{\pi}\right)^2 L_Q^2 \sim \frac{\Lambda_{\rm QCD}}{Q}$$



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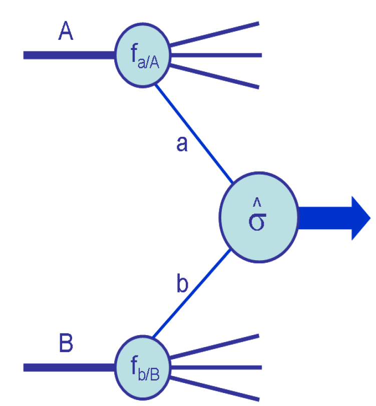
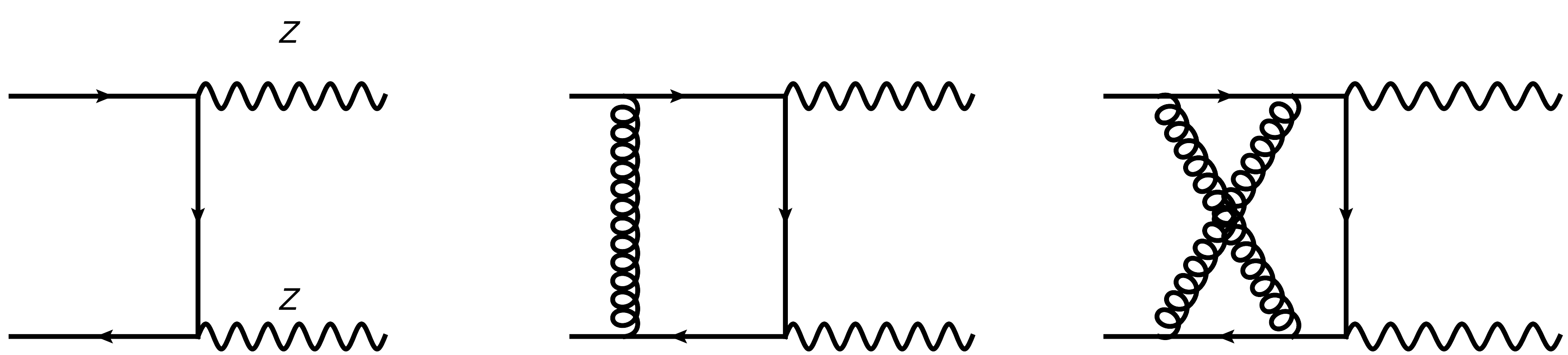


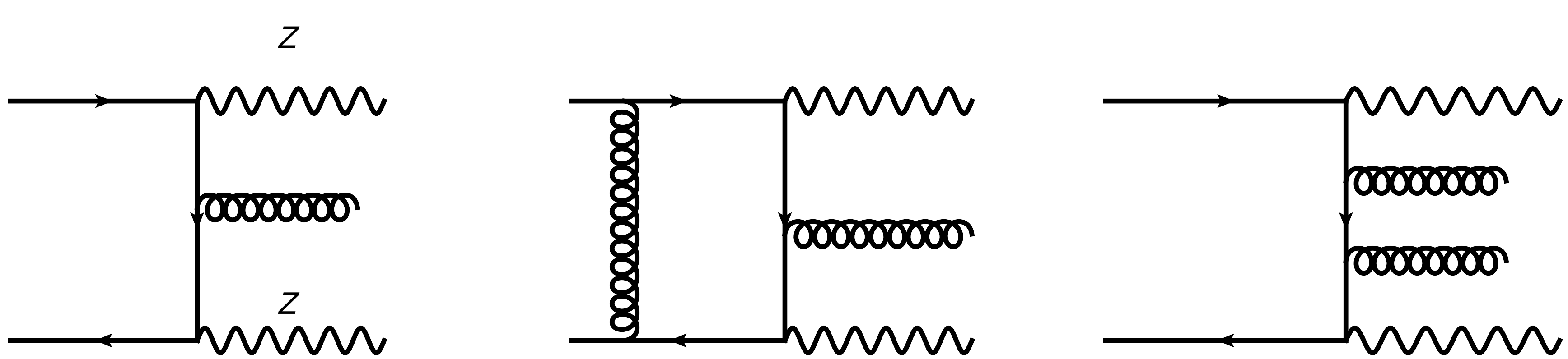
$$d\sigma = \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{\rm part}(x_1 x_2 s_{\rm hadr})$$

Perturbation theory: what needs to be done

QCD perturbation theory is, first and foremost, an expansion in the strong coupling constant. For fixed initial and final states, expansion in the strong coupling constant leads to an increased number of loops. For this reason, understanding how to deal with multi-loop diagrams is a very important aspect of perturbation theory computations.



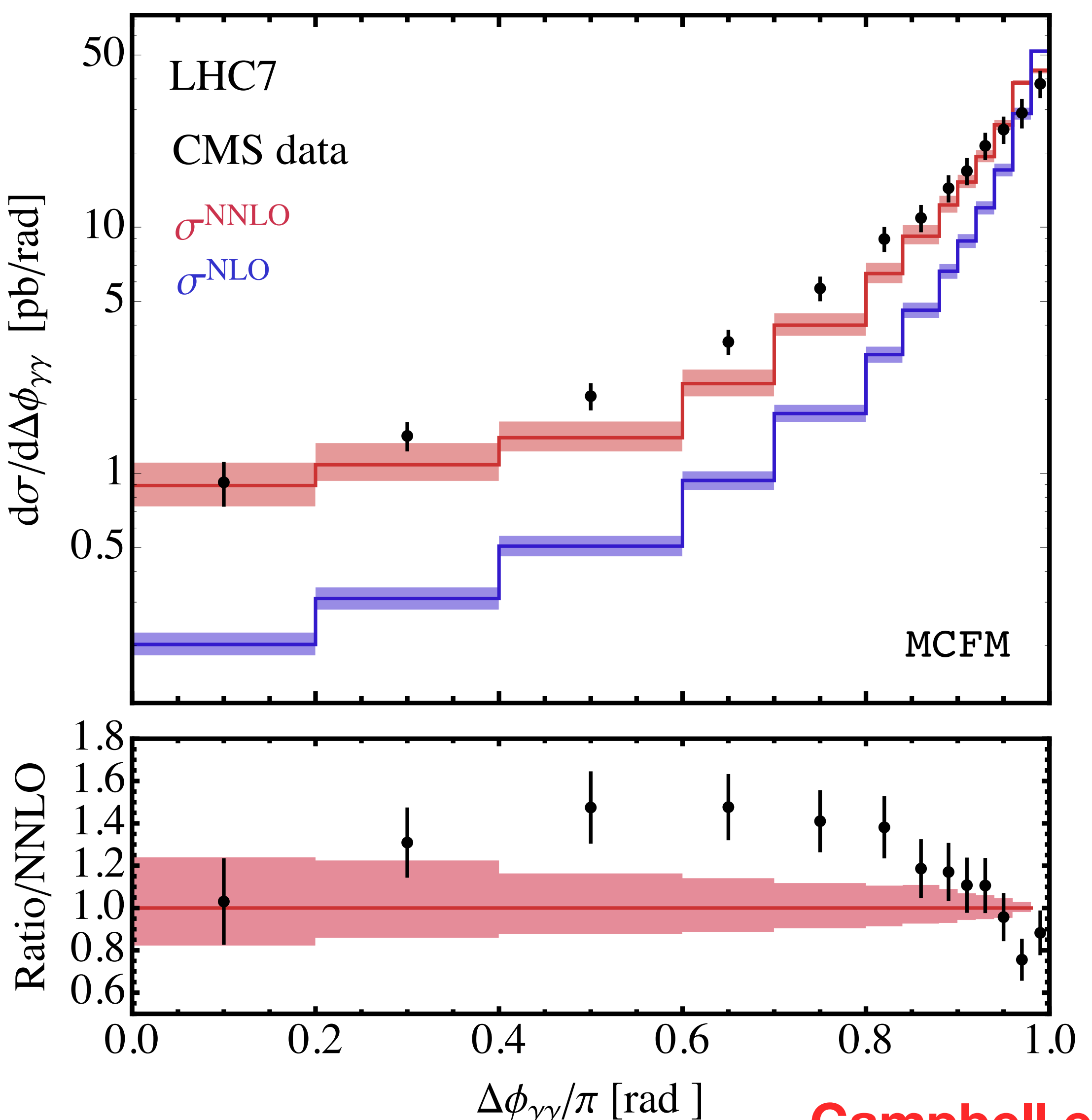
However, since quarks and gluons are massless, final states with fixed partonic multiplicities are unphysical, we need to add higher multiplicity contributions to cross sections, to obtain infra-red insensitive (or short distance) results.



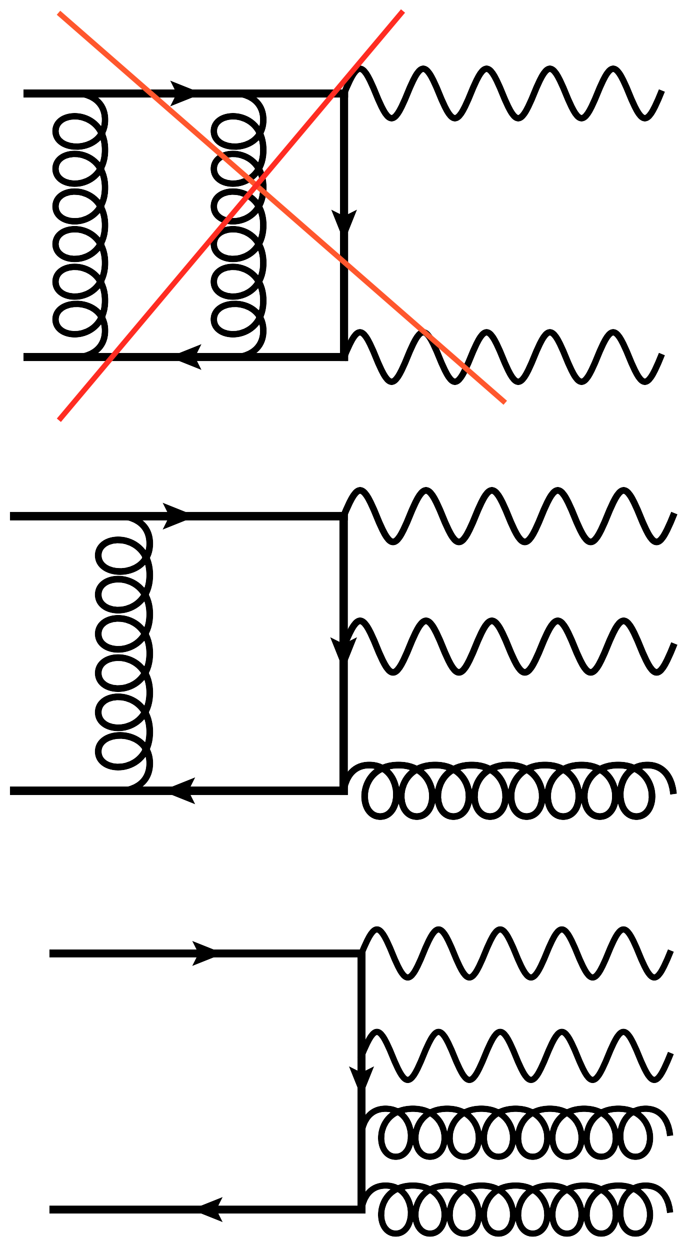
Understanding how virtual and real contributions can be combined in an efficient way, to obtain infra-red safe, fully-differential cross sections is another non-trivial aspect of perturbative computations.

NNLO >= 2 loops !

Existence of a NNLO calculation for a process does not imply that any observable computed using a particular ``NNLO” code has the NNLO accuracy (pt of the Z in NNLO Drell-Yan, pt of the top pair in NNLO tT production etc.). Sometimes NLO calculations to higher multiplicity processes are more useful than NNLO calculations for lower multiplicity process.

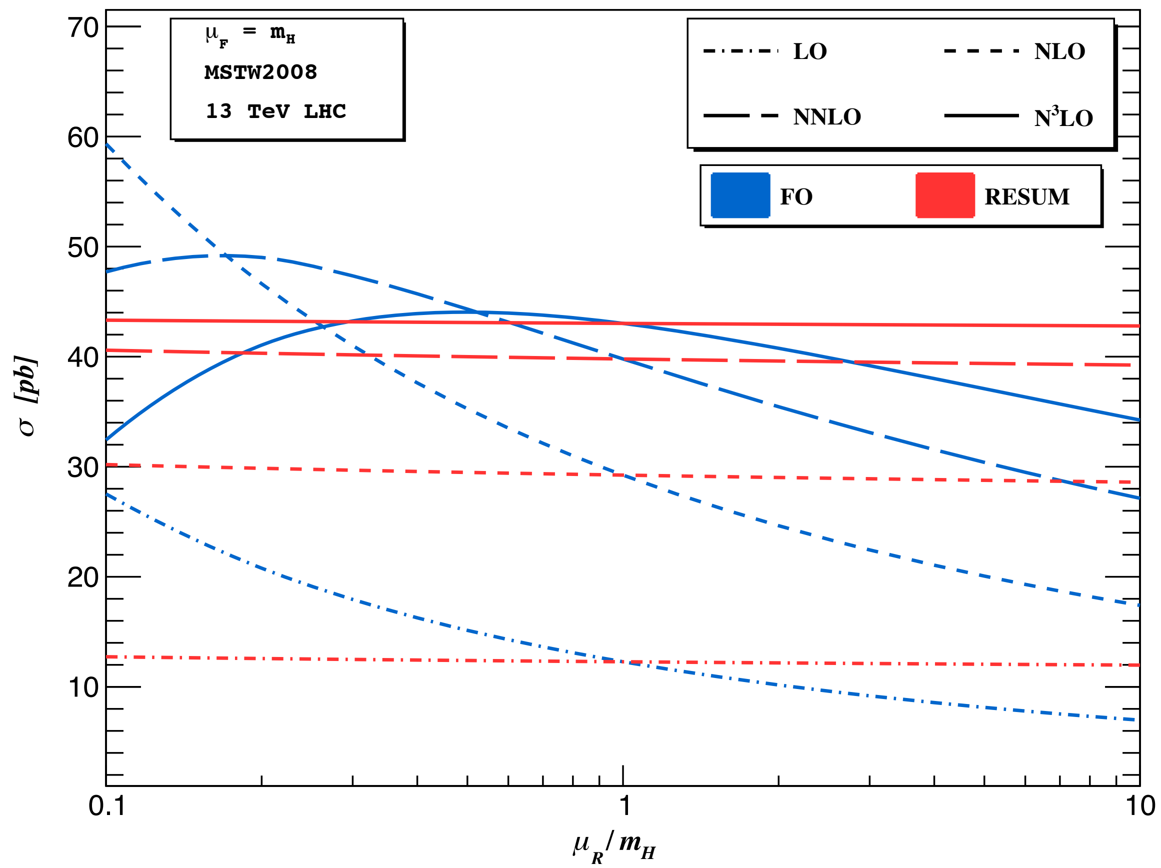


Campbell et al.



Scales

NNLO computations are fairly insensitive to scale choices, at least in the region where NNLO is at work. Too much of a scale choice game, however, can be counter-productive since scale variation uncertainty is one of a very few handles we have to estimate uncertainties in predictions caused by the truncation of perturbative expansion.



	LO	NLO	NNLO	N^3LO
FO (%)	167.26	143.40	54.99	27.01
RESUM (%)	6.11	5.47	3.39	1.23

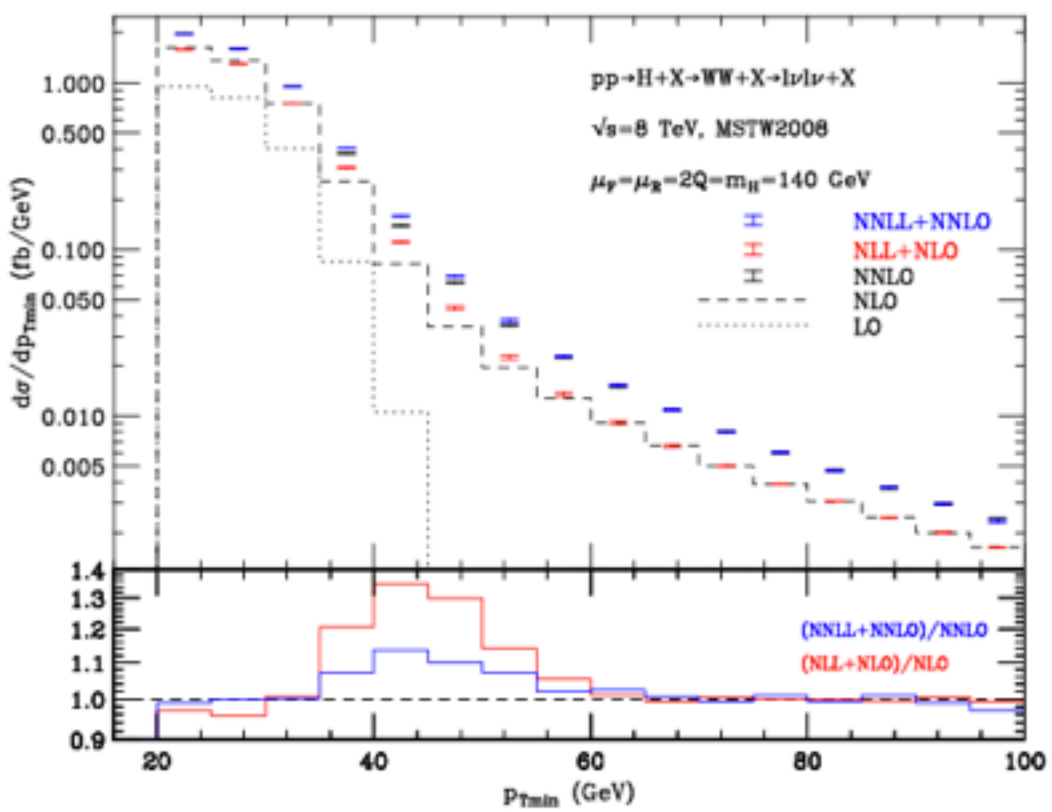
Ahmed et al.

Remarks on NNLO QCD approximation

There seems to be a close proximity of resummed and fixed order computations for realistic selection criteria at the LHC. We are in the ``grey" region where both approaches can be used for reasonable estimates of radiative corrections, provided that we can reach sufficiently high orders in the strong coupling expansion.

The main advantage of fixed-order computations is the possibility to compute fiducial cross sections for realistic selection criteria.

The main advantage of resummed computations is that they can be continued to regions where fixed order computations fail. This is good but we rarely need those regions for anything but the consistency checks of the SM.



De Florian et al.

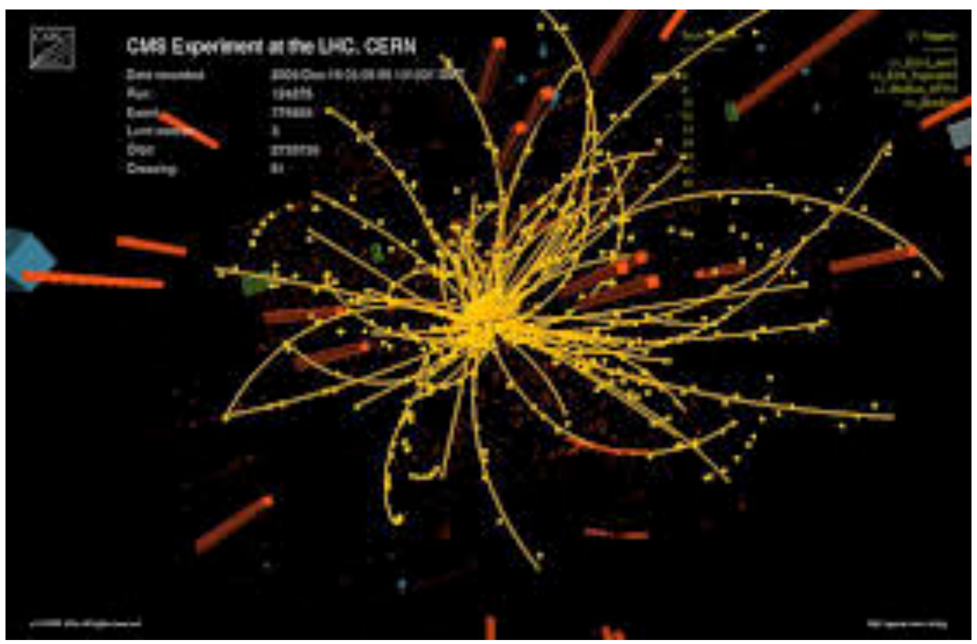
PQCD approximation for collider physics observables

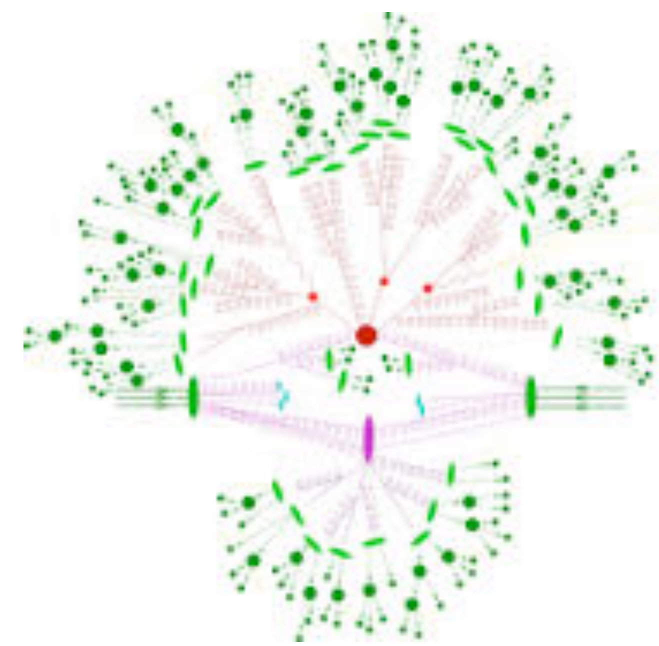
Perturbative QCD is a systematic improvable framework to describe hard scattering processes at the LHC that requires three ingredients:

1) parton distribution functions;

2) partonic scattering cross sections computed to a particular order in perturbation theory;

3) parton shower event generators, to describe multiple emissions and detector responses.





$$\mathrm{d}\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2 f_i(x_1) f_j(x_2) \mathrm{d}\sigma_{\mathrm{part}}(x_1 x_2 s_{\mathrm{hadr}})$$

For the type of physics that we are interested in, the most important ingredient is partonic cross sections; everything else depends, either directly or indirectly, on our ability to compute them.

Two-loop calculations in QCD

Traditionally, calculations of two-loop integrals rely on a large number of methods (Feynman parameter integration, Mellin-Barns, differential equations). The method of differential equations has been used to compute master integrals since quite some time already, starting from papers by Kotikov and Remiddi in the early 1990s, however it was never considered to be ``the" method.

An interesting recent development in this field is the suggestion by J. Henn to streamline the application of differential equations in external kinematic variables to compute master integrals

$$\partial_x \vec{f} = \epsilon \hat{A}_x(x,y,z\ldots)\vec{f} \qquad\qquad \vec{f} = \sum_{n=0}^{\infty} \epsilon^n \vec{f}^{(n)}$$

The important point is that on the right-hand side, the dimensional regularization parameter appears explicitly, and only as a multiplicative pre-factor. It is then possible to solve these equations iteratively order-by-order in (d-4) since in each order of this expansion the above equation contains no homogeneous terms (so that in each order in epsilon, the right-hand side is the source for the left-hand side).

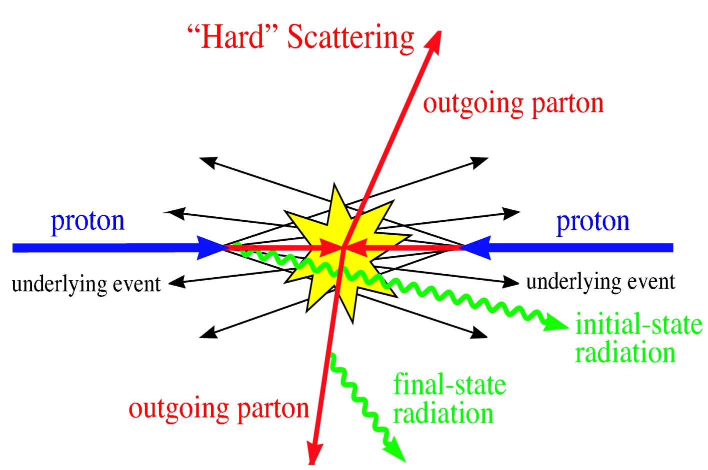
The idea by Henn streamlines and simplifies such computations significantly. This already lead to very impressive advances (e.g. master integrals for Bhabha, V1 V2 production) that will have interesting consequences for phenomenology.

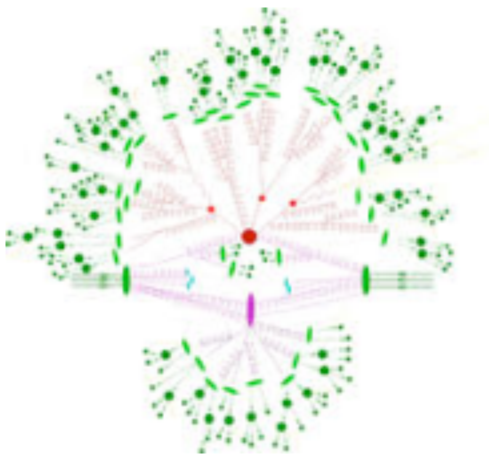
Introduction

Since the theory of non-perturbative corrections does not exist, their magnitude provides an ultimate precision target on the theory side: going beyond it does not make sense unless the theory of non-perturbative corrections is established, but reaching this (few percent) precision is justified. To get there, one needs the NNLO QCD predictions; this is a simple consequence of the numerical value of the strong coupling constant at 100 GeV.

There are many non-trivial issues (mostly of experimental nature) that have to be understood if one wants to benefit from such a high precision but this is a separate issue. On the other hand, to provide maximal benefit for theory/experiment cross-talk, such predictions should be realistic, i.e. they should be performed at a fully differential level and applied to realistic final states.

In recent years, progress towards reaching the NNLO accuracy for large number of LHC processes was very impressive. Paraphrasing what has been said about NLO computations just a few years ago, we are living through the NNLO QCD revolution. This implies that we have large and constantly increasing number of processes that are known to the NNLO QCD accuracy.





Processes currently known through NNLO

dijets	O(3%)	gluon-gluon, gluon-quark	PDFs, strong couplings, BSM
H+0 jet	O(3-5 %)	fully inclusive (N3LO)	Higgs couplings
H+1 jet	O(7%)	fully exclusive; Higgs decays, infinite mass tops	Higgs couplings, Higgs $p_{t,}$ structure for the ggH vertex.
tT pair	O(4%)	fully exclusive, stable tops	top cross section, mass, p_t, FB asymmetry, PDFs, BSM
single top	O(1%)	fully exclusive, stable tops, t-channel	V_{tb}, width, PDFs
WBF	O(1%)	exclusive, VBF cuts	Higgs couplings
W+j	O(1%)	fully exclusive, decays	PDFs
Z+j	O(1-3%)	decays, off-shell effects	PDFs
ZH	O(3-5 %)	decays to bb at NLO	Higgs couplings (H-> bb)
ZZ	O(4%)	fully exclusive	Trilinear gauge couplings, BSM
WW	O(3%)	fully exclusive	Trilinear gauge couplings, BSM
top decay	O(1-2 %)	exclusive	Top couplings
H -> bb	O(1-2 %)	exclusive, massless	Higgs couplings, boosted

Phenomenology and NNLO computations

Master integrals

Different methods are used for computing two-loop integrals (direct integration, Mellin-Barns, differential equations). The method of differential equations has been used to find master integrals for a long time, starting from papers by Kotikov and Remiddi in the early 1990s, however it was never considered to be ``the'' method.

$$\partial_{p_i \cdot p_j} I_\alpha = \sum c_{\alpha,ij,\beta}(\{p\},\epsilon) I_\beta$$

An interesting recent development in this field is the suggestion by J. Henn to streamline the application of differential equations in external kinematic variables to compute master integrals. Imagine that it is possible to write the differential equations in the following form

$$\partial_x \vec{f} = \epsilon \hat{A}_x(x,y,z,..)\vec{f}, \qquad \vec{f} = \sum_{n=0}^{\infty} \epsilon^n \vec{f}^{(n)}$$

The important point is that on the right-hand side, the dimensional regularization parameter appears explicitly (and only) as a multiplicative pre-factor. It is then possible to solve these equations iteratively order-by-order in (d-4) since in each order of this expansion the above equation contains no homogeneous terms (so that in each order in epsilon, the right-hand side is the source for the left-hand side).

The idea by Henn streamlines and simplifies such computations significantly, making bookkeeping particularly straightforward.

Ingredients for NNLO computations

A NNLO QCD computation is, essentially, a two-loop computation. However, in theories with massless particles, two-loop computations are insufficient for obtaining a physical answer: two-loop computations need to be combined with contributions of higher-multiplicity processes to physical observables.

Suppose we want to compute the NNLO QCD correction to a process pp -> X . To do this, we need:

a) two-loop scattering amplitudes for a process X ;

b) one-loop amplitudes for a process X+g;

d) tree-level amplitudes for a process X+gg, X+qQ etc.

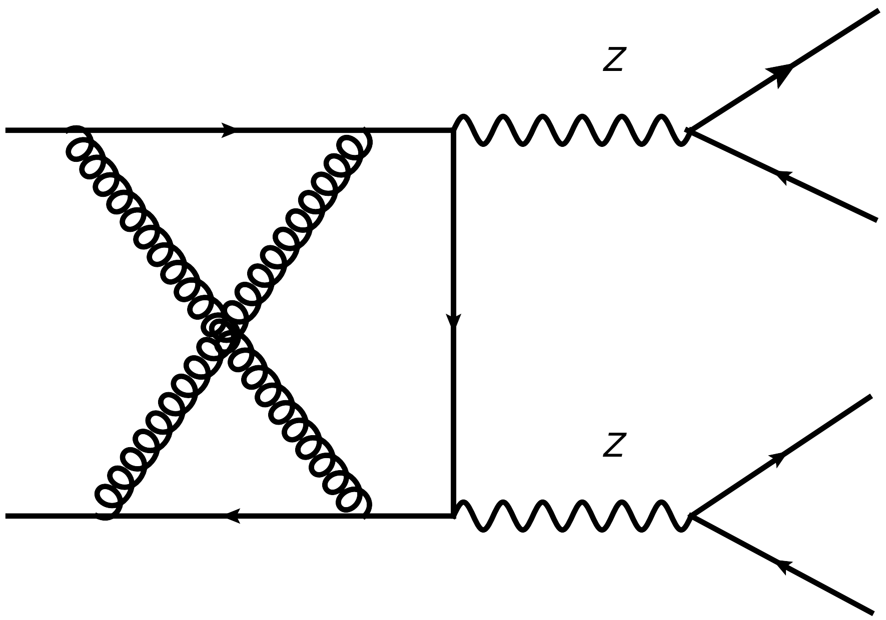
Computation of two-loop scattering amplitudes is a significant challenge;

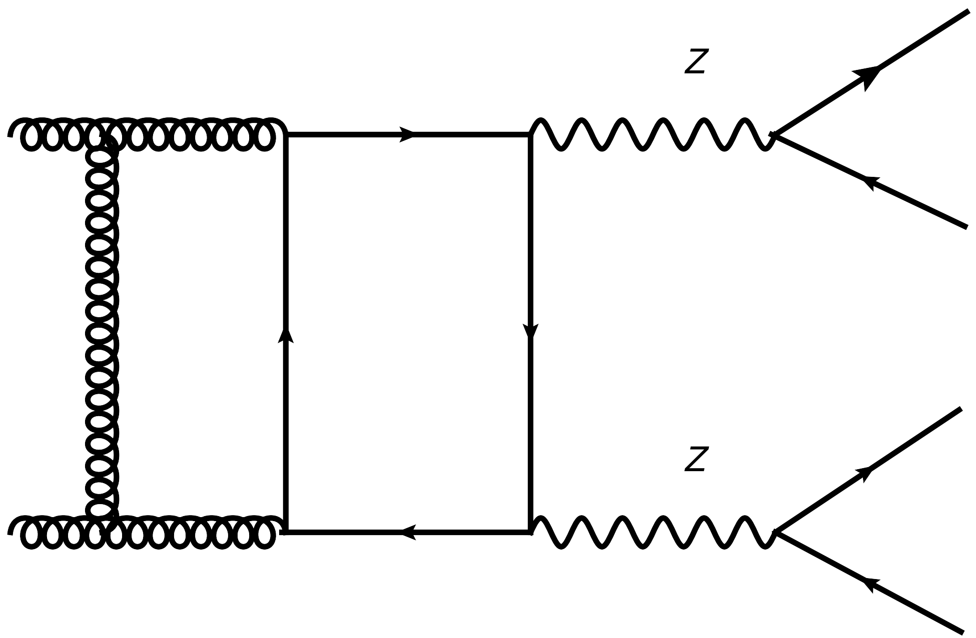
Integration of tree-level amplitudes over available phase-space requires some procedure that allows an extraction of infra-red divergences (subtraction/slicing techniques).

One-looop amplitudes need to be known in an unresolved region; although one loop computations are ``standard" by now, they are not easy especially in unresolved regions.

NNLO QCD predictions for the background

NNLO QCD predictions for ZZ production require computation of complicated two-loop scattering amplitudes.





These are computed using the standard steps that include: parametrization of amplitudes in terms of Lorentz-invariant form factors; reduction to master integrals followed by the calculation of master integrals.

Interestingly, with these standard procedures, we are getting to the point were these computations become hardly manageable (the amplitude depends on four kinematic invariants).

Concluding remarks

Consolidating precision physics at the LHC

The very rapid progress with NNLO QCD computations strongly suggests that there is a realistic opportunity to perform precision studies at the LHC. This opportunity is new and somewhat unexpected; it arises because of spectacular progress in theory and expreriment in recent years. Taking up this opportunity may also become necessary because of no clear BSM signals at the LHC.

To fully benefit from these theoretical developments, we will need (in the long run)

1) to better understand inputs for cross section calculations (PDFs, masses, couplings, etc.)

2) to include electroweak corrections;

3) to work with realistic final states and fiducial cross sections (how does ``experimental acceptance" fit together with ``precision physics" anyway?);

4) to understand the limitations of the various approximations that we currently use in the most advanced computations (finite = infinite, large N_c arguments etc.).

Two-loop calculations: amplitudes and integrals

Here are a few general remarks about two-loop computations:

1) Calculation of master integrals using differential equations in kinematic variables is now a method of choice. It has benefited from an understanding of how the bookkeeping in such calculations can be streamlined by choosing appropriate master integrals and working with particular special functions.

Remiddi, Kotikov, Henn, Papadopoulos

2) We are able to compute master integrals with up to 4 kinematic invariants and there are indications that even larger number of kinematic invariants can be handled.

Gehrmann, Henn, Tancredi, Caola, Smirnov(s), Papadopoulos, Tommasini, Wever

3) Internal masses is a big challenge since they introduce new special functions whose properties are currently being explored. Very recently, an interesting development related to direct numerical evaluation of two loop Feynman integrals with internal masses.

Weinzierl, Tancredi, Remiddi; Czakon, Heinrich et al.

4) There are interesting attempts to understand if two-loop computations can be done using unitarity techniques, that turned out to be so powerful at one-loop. While there was an impressive progress in this field related to classification of integrand residuals based on techniques from algebraic geometry, there are still many outstanding issues.

Badger, Frellesvig, Zhang, Mastrolia, Ita

NNLO calculations: loops and real emissions

An important achievement of the past few years was the development of theoretical methods that allow us to perform NNLO QCD computations for hard hadron collider processes of a sufficiently general nature.

Consider NNLO QCD corrections to a tree process pp -> X. There are three sources of infra-red divergencies that must be considered:

1) two-loop virtual corrections to pp -> X, where all infrared singularities are explicit;

2) one-loop virtual corrections to pp -> X+g, where some infrared singularities are explicit and some appear only after the integration of the final state gluon;

3) process pp -> X+ g+ g where all infra-red singularities appear only after integration over final state gluon(s) is carried out.

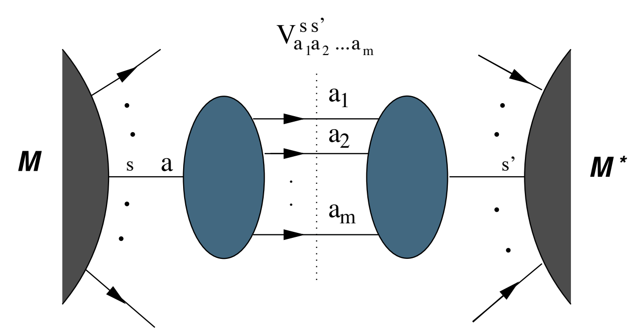
The key problem here is that we would like to achieve the cancellation of infra-red singularities at NNLO without integrating over kinematic variables of those final state particles that are accessible in experiment; but this seems to be impossible given that in real emission processes singularities are produced only after the phase-space integration...

NNLO calculations: loops and real emissions

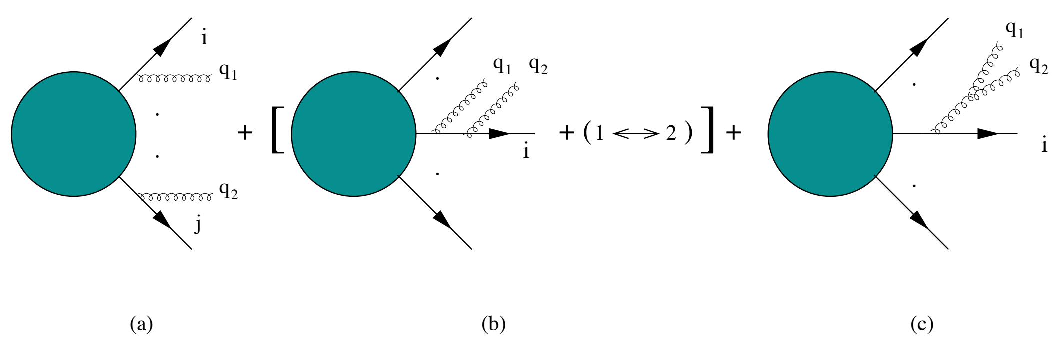
It is easy to recognize that for achieving the cancellation of infra-red and collinear divergences, we only need to integrate over phase-space regions which can generate the singularities.

These are the regions where external particles can become soft and/or collinear to each other and where measurable differences between final states with different multiplicities become unobservable. In these regions, ``singular” matrix elements factorize into universal singular functions and non-singular matrix element of lower multiplicity.

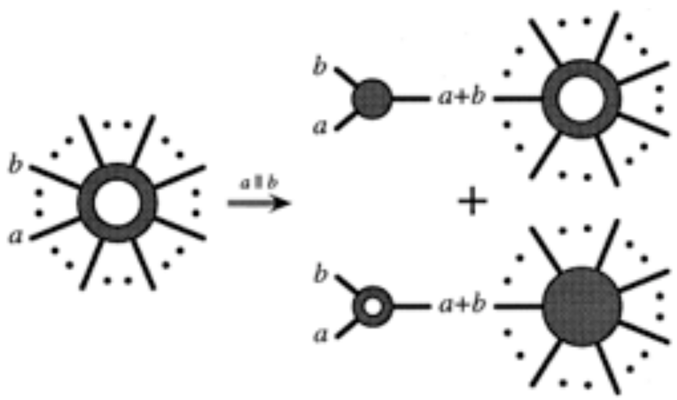
$$\mathcal{M}_{n+i+j} = F_{ij}\mathcal{M}_n$$



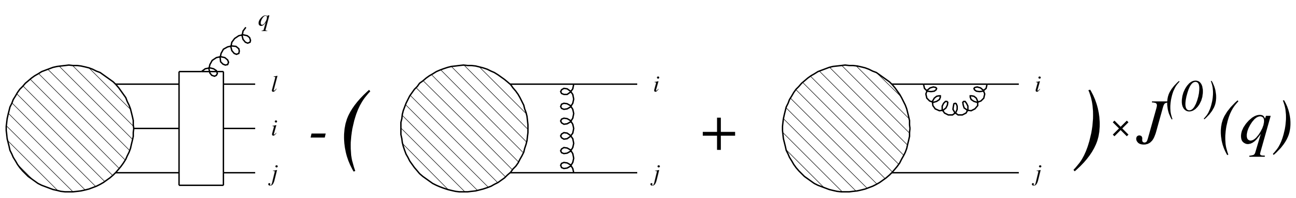
Collinear factorization (Catani, Grazzini)



Soft factorization (Catani, Grazzini)



Collinear factorization at one-loop (Kosower, Uwer)



Soft factorization at one-loop (Catani, Grazzini)

NNLO calculations: loops and real emissions

A universal, simplified form of scattering amplitudes in kinematic regions responsible for the appearance of singularities, together with factorization of multi-particle phase-space, allows us to extract the singularities and, cancel them in a generic, process-independent way.

There are two basic methods familiar from NLO computations: slicing and subtraction.

Slicing methods (q_t-subtraction and N-jettiness) are based on splitting the phase-space into regular and singular parts.

$$\int \mathrm{d}\Phi_n |\mathcal{M}|^2 F_J = \int_{\text{regular}} \mathrm{d}\Phi_n |\mathcal{M}|^2 F_J + \int_{\text{singular}} \mathrm{d}\Phi_n |\mathcal{M}|^2_{\text{approx}} \tilde{F}_J$$

Catani, Grazzini; Bougezhal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh.

Subtraction methods (antenna, improved sector decomposition and projection to Born) are based on subtracting approximate expressions for the amplitude squared from the integrand to make the difference integrable.

$$\int \mathrm{d}\Phi_n |\mathcal{M}|^2 F_J = \int \mathrm{d}\Phi_n \left(|\mathcal{M}|^2 F_J - |\mathcal{M}|^2_{\text{approx}} \tilde{F}_J \right) + \int \mathrm{d}\Phi_n |\mathcal{M}|^2_{\text{approx}} \tilde{F}_J$$

Gehrmann-de Ridder, Gehrmann, Glover; Czakon; Bougezhal, Petriello, K.M.
Cacciari, Dreyer, Kalberg, Salam, Zanderighi

All these methods work and have been used in a large number of recent NNLO QCD computations.

NNLO calculations: real emissions

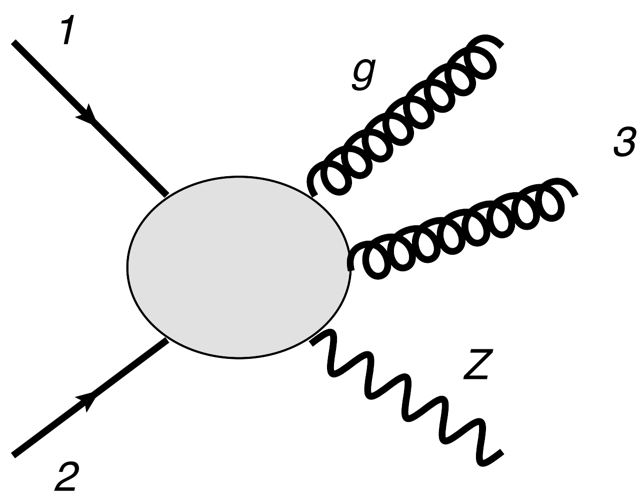
A few issues arise when we think about constructing subtraction terms :

1) subtractions need to be local (i.e. make integrands finite point-by-point in the phase-space);

2) one should avoid over-subtraction;

3) subtraction terms should be integrable, either analytically or numerically. Analytic integration is difficult (antenna). Numerical integration is possible if one partitions the phase-space.

Frixione, Kunszt, Signer



$$F(1,2,3;g) \approx 4\pi\alpha_s \sum_{i,j} \vec{T}_i \cdot \vec{T}_j \frac{p_i \cdot p_j}{(p_i \cdot g)(p_j \cdot g)} F(1,2,3)$$

$$1 = \frac{\rho_{g1}\rho_{g2}}{d_{12}} + \frac{\rho_{g1}\rho_{g3}}{d_{13}} + \frac{\rho_{g2}\rho_{g3}}{d_{23}} \qquad \rho_{ij} = 1 - \vec{n}_i \vec{n}_j$$

$$\int [\mathrm{d}g] F_(1,2,3,g) = \sum_{\{ij\}} \int [\mathrm{d}g] w^{ij} F(1,2,3,g) \qquad w^{ij} = \frac{\rho_{gi}\rho_{gj}}{d_{ij}}$$

Each of the contributing terms has one and only one collinear singularity. Each singularity can be easily extracted by choosing a reference frame where the z-axis is aligned with the (only) collinear direction in each of the contributing sectors; the remaining integrations even in the subtraction term can be done numerically.

NNLO calculations: real emissions summary

The above discussion summarizes the recent techniques for NNLO QCD computations that combine sector decomposition and phase-space partitioning. There are a few other things that are worth mentioning:

1) Within this framework, the necessary local subtraction terms are generated automatically; similar to the original FKS, the new framework is very robust.

2) All of the subtraction terms are related to universal limits of scattering amplitudes making the whole procedure scalable in the right way (need no diagrams, need amplitudes, all limits are hard-coded once and for all);

3) Can work with helicity states for external resolved particles;

4) All spin-correlations in amplitudes are subtracted locally;

5) No need for (d-4) terms in amplitudes squared, except in their collinear limits;

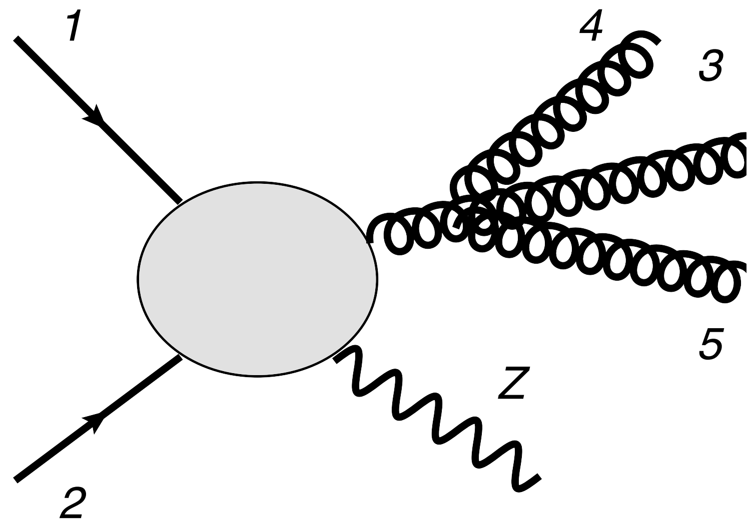
6) Massive particles aren't a problem;

7) Decay kinematics is not a problem;

8) Important to have ``good" (fast and stable) NLO amplitudes.

NNLO calculations: real emissions

At NNLO, there are singularities when two gluons are collinear to the direction of a harder gluon or quark. There are also singularities when two unresolved gluons become collinear to two different collinear hard directions. The soft singularities are always present. A partitioning is more complex but it exists.



The trademark of NNLO computations are triple-collinear singularities; they appear when

$$p_4||p_3, \quad p_5||p_3, \quad p_4||p_5.$$

The collinear/soft singular limits of the matrix elements factorize if we choose particular parametrization for energies and angles in the reference frame where the z-axis is aligned with the direction of the gluon 3.

$$2\eta_4 = 1 - \vec{n}_4 \cdot \vec{n}_3,$$
$$2\eta_5 = 1 - \vec{n}_4 \cdot \vec{n}_5,$$

Czakon

1) $E_4 = x_1, \quad E_5 = x_1 x_2, \quad \eta_4 = x_3, \quad \eta_5 = x_3 x_4/2$

2) $E_4 = x_1, \quad E_5 = x_1 x_2, \quad \eta_4 = x_3, \quad \eta_5 = x_3(1 - x_4/2)$

3) $E_4 = x_1, \quad E_5 = x_1 x_2 x_4, \quad \eta_4 = x_3 x_4/2, \quad \eta_5 = x_3$

4) $E_4 = x_1, \quad E_5 = x_1 x_2, \quad \eta_4 = x_3 x_4 x_2/2, \quad \eta_5 = x_3$

5) $E_4 = x_1, \quad E_5 = x_1 x_2, \quad \eta_4 = x_3(1 - x_4/2), \quad \eta_5 = x_3$

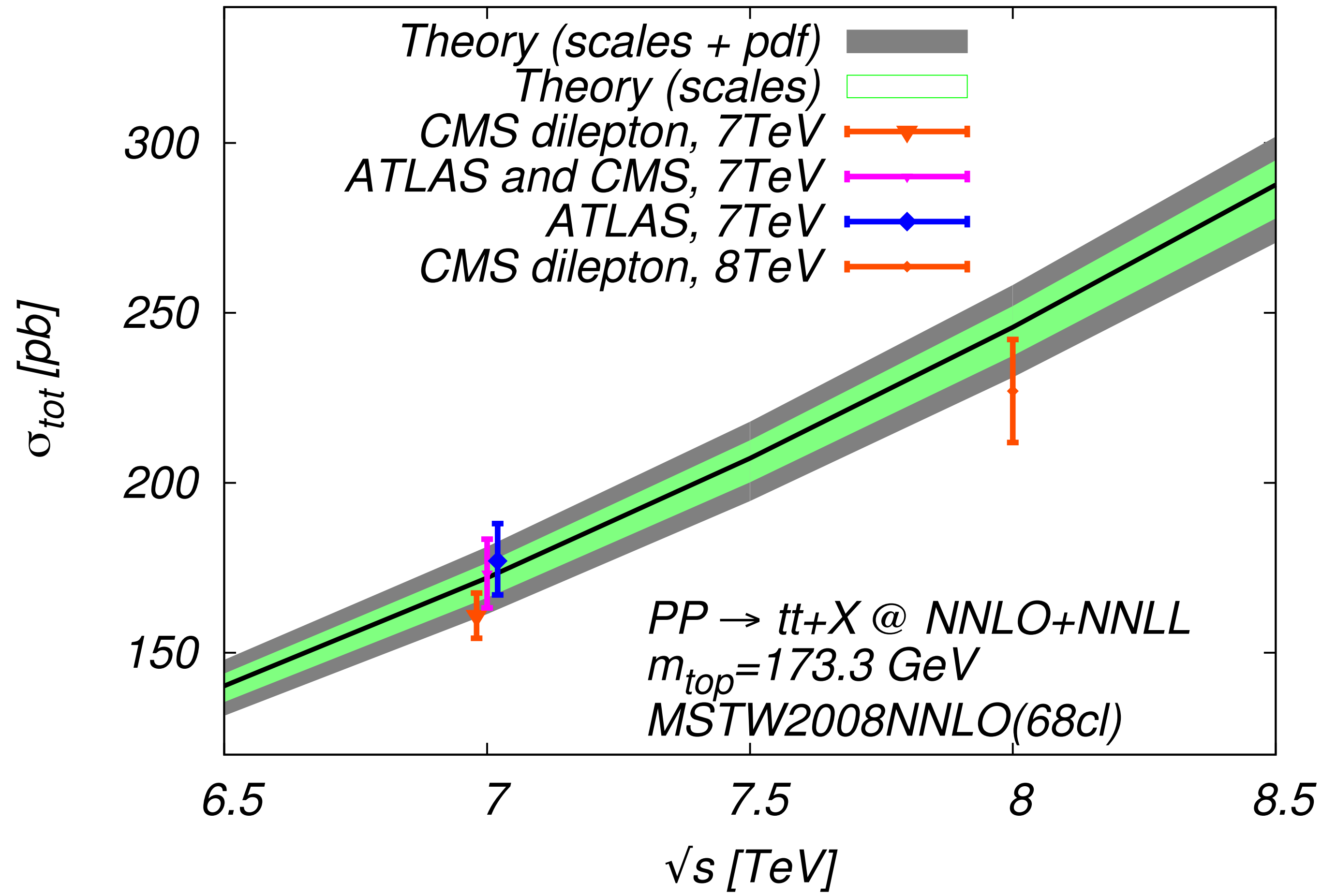
$$\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon}\delta(x) + \left[\frac{1}{x}\right]_+ + \ldots$$

$$\sum \int \frac{\mathrm{d}x_1}{x_1^{1+e_1\epsilon}} \frac{\mathrm{d}x_2}{x_2^{1+a_2\epsilon}} \frac{\mathrm{d}x_3}{x_3^{1+a_3\epsilon}} \ldots F(x_1, x_2, x_3, \ldots), \qquad F(x_1, x_2, x_3, \ldots) = x_1^{b_1} x_2^{b_2} \ldots |\mathcal{M}|^2.$$

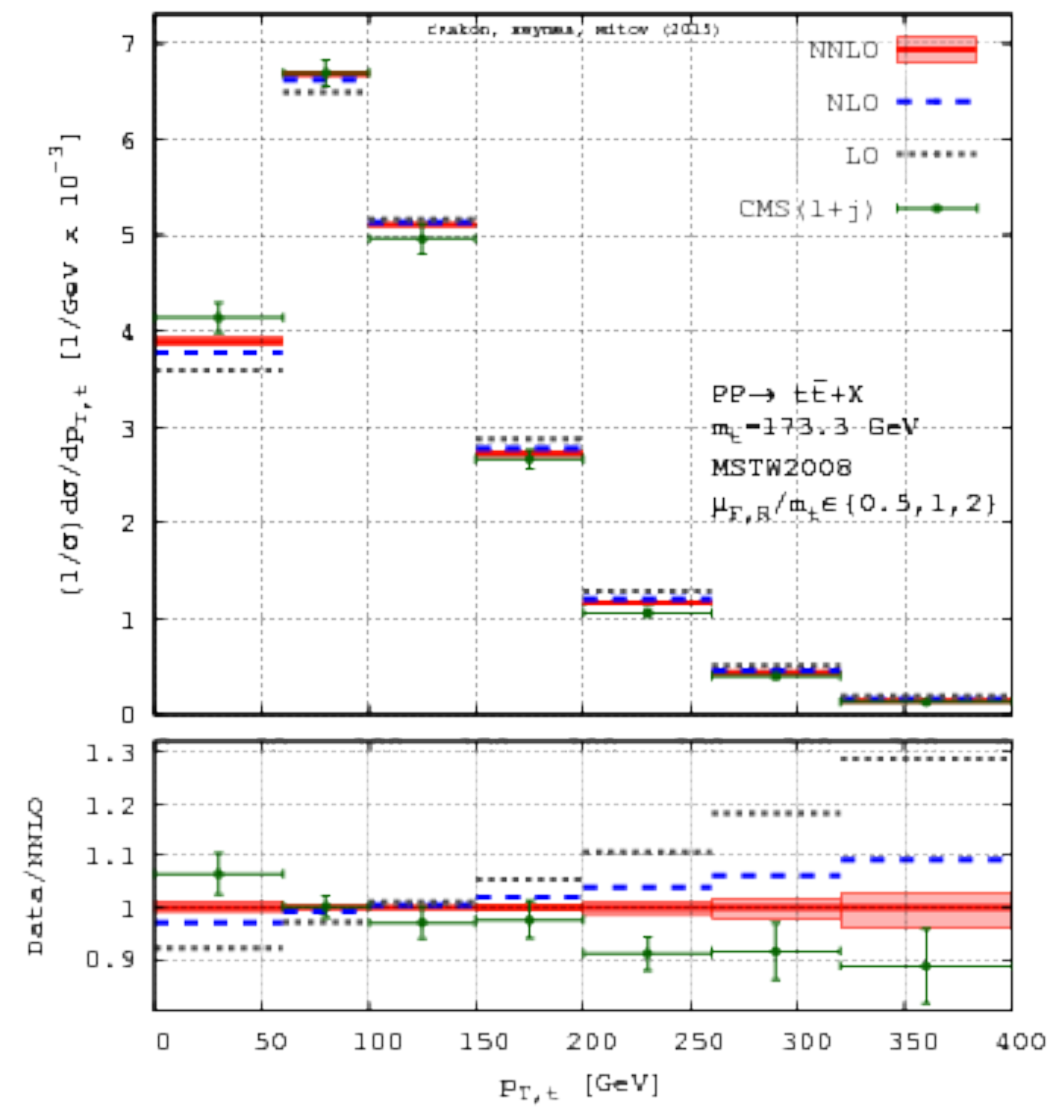
Example: the Higgs boson width

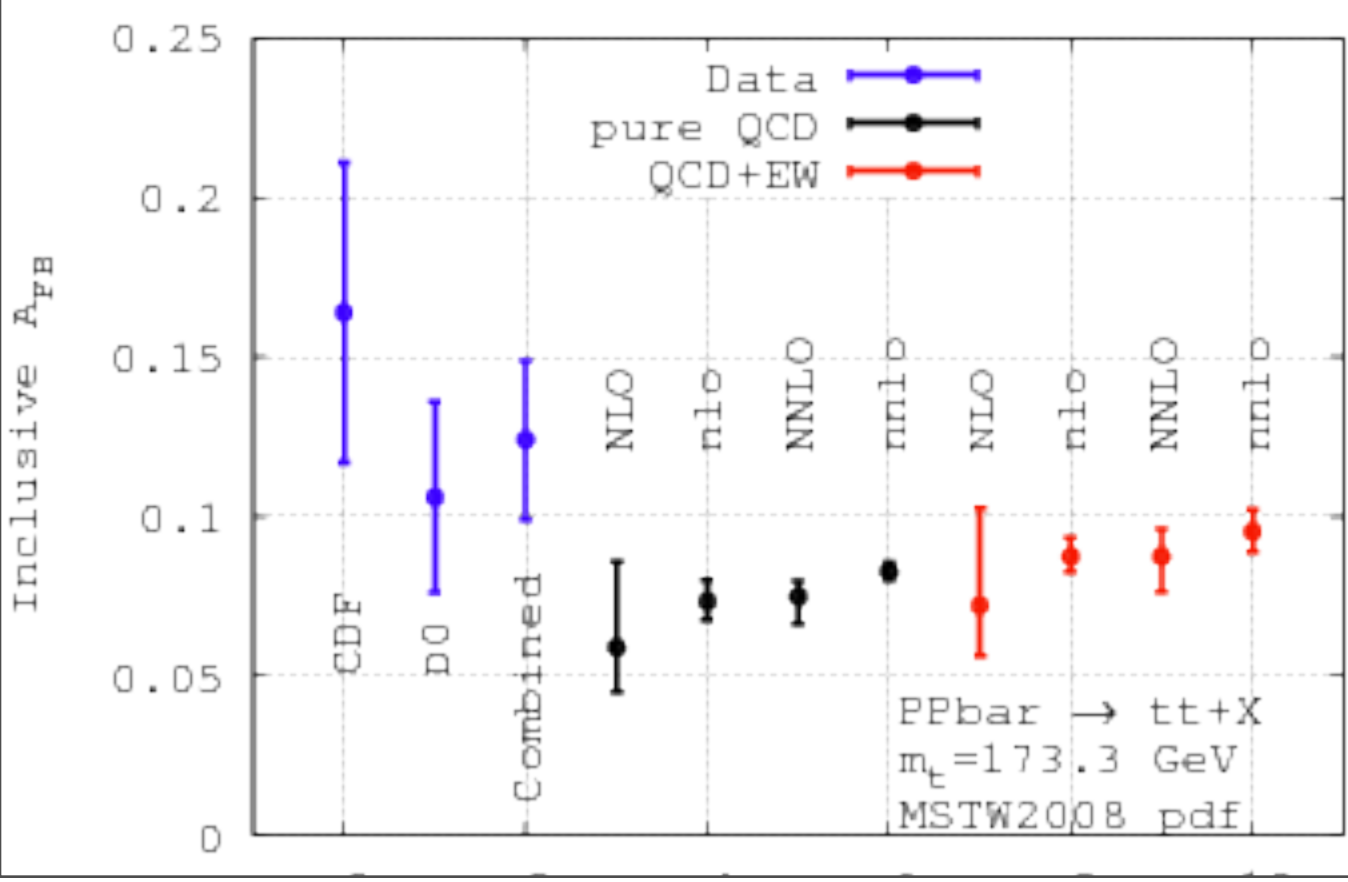
Example: exclusive/fiducial Higgs cross sections

Top pair production



Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	+0.259(3.7%) −0.374(5.3%)	+0.169(2.4%) −0.121(1.7%)
LHC 7 TeV	167.0	+6.7(4.0%) −10.7(6.4%)	+4.6(2.8%) −4.7(2.8%)
LHC 8 TeV	239.1	+9.2(3.9%) −14.8(6.2%)	+6.1(2.5%) −6.2(2.6%)
LHC 14 TeV	933.0	+31.8(3.4%) −51.0(5.5%)	+16.1(1.7%) −17.6(1.9%)

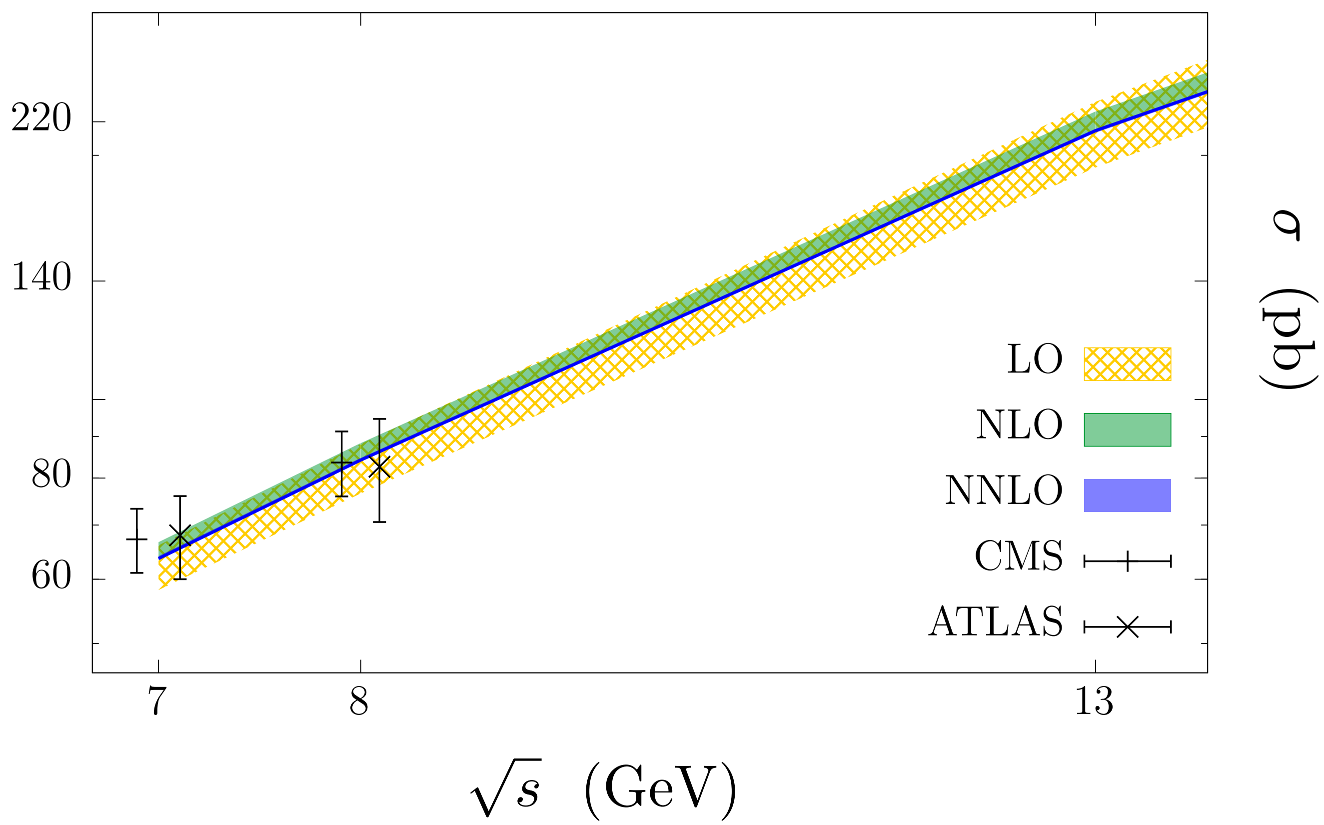


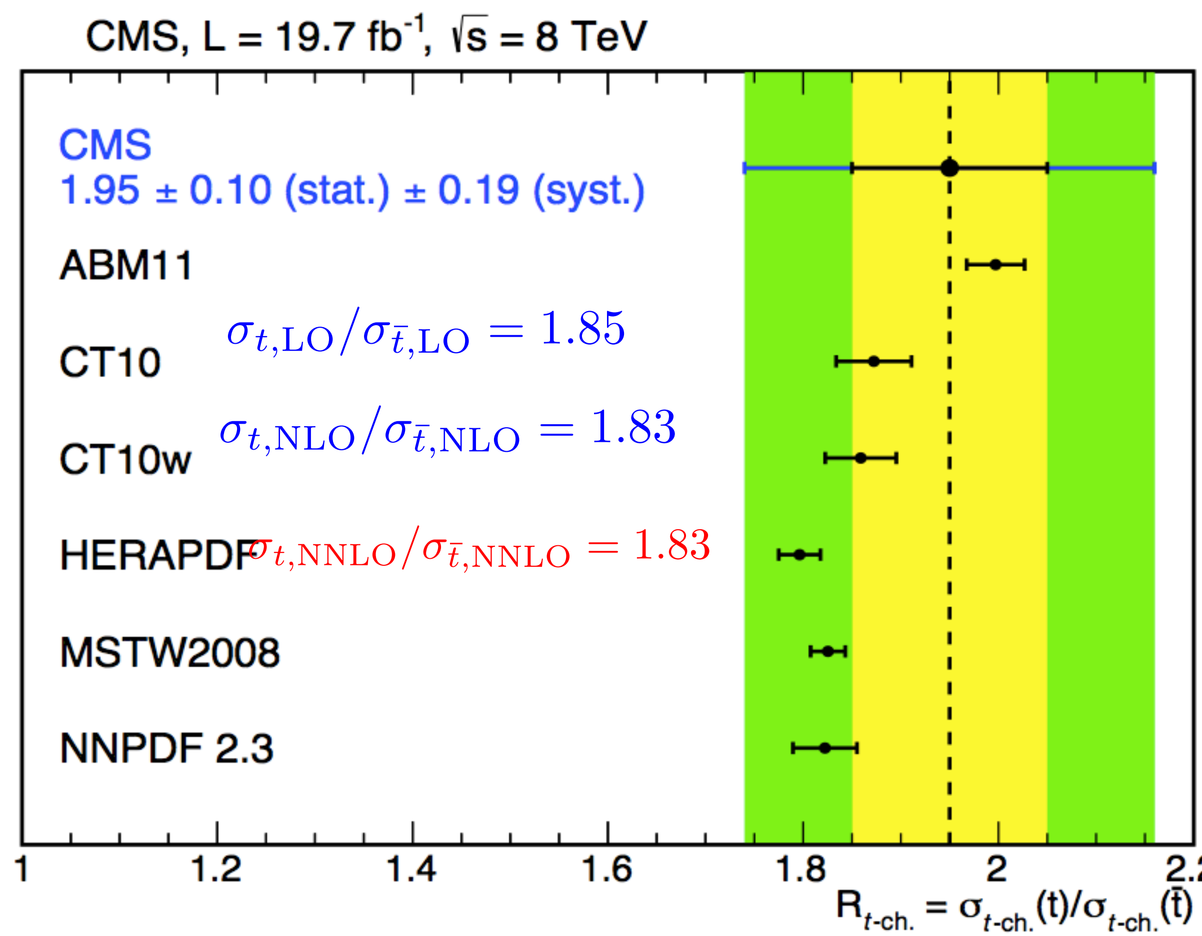


Czakon, Mitov, Fiedler, Heymes

An ongoing effort by Abelof, Gehrmann de Ridder , Pozzorini

Single top production (t-channel)



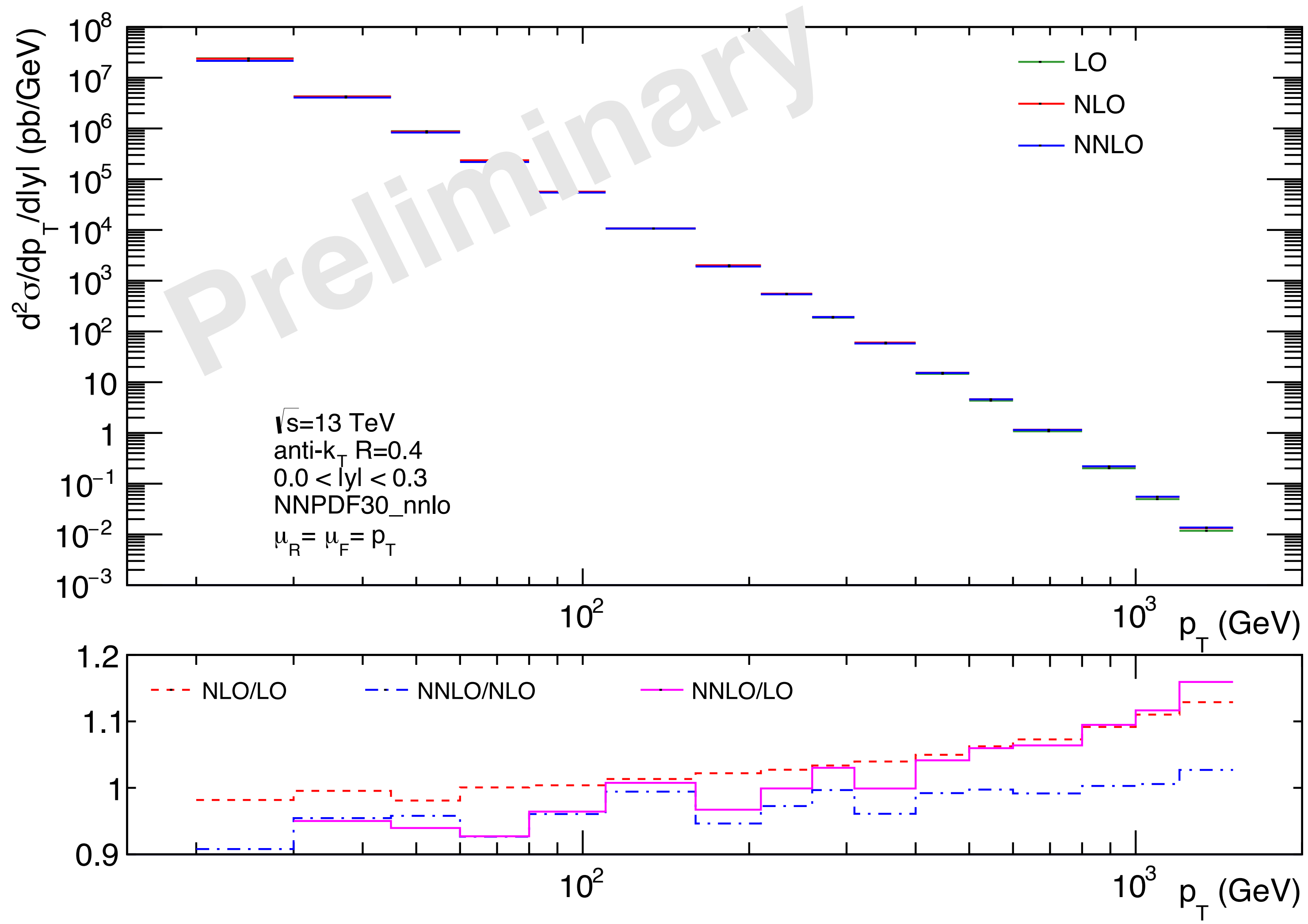


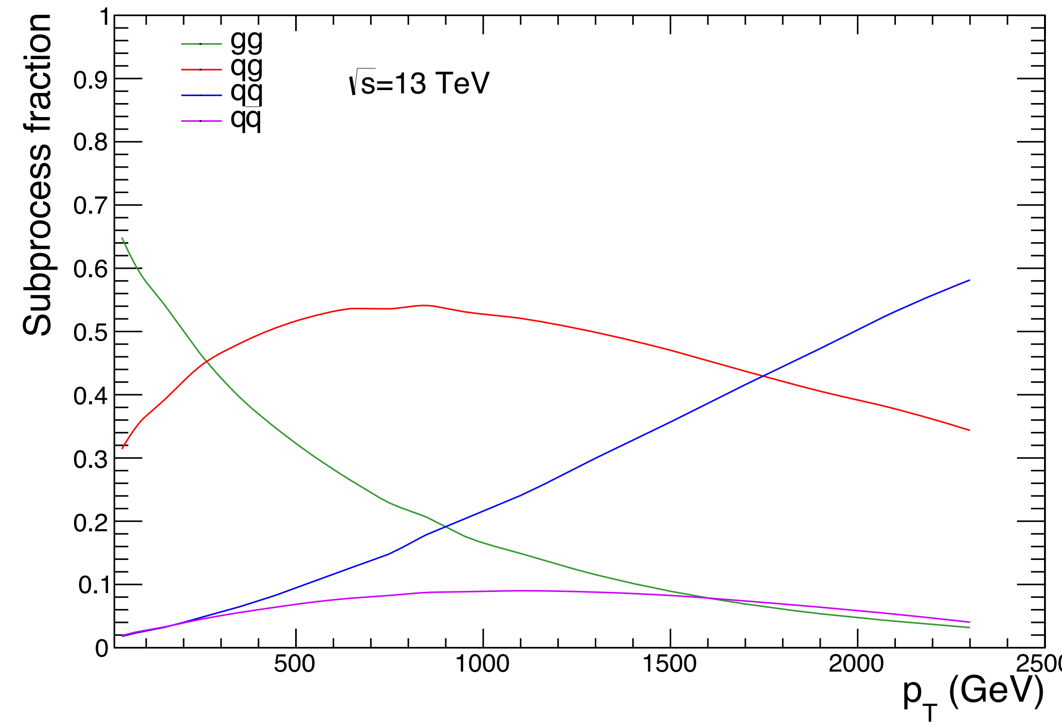
Burcherseifer, Caola, K.M.

$p_\perp$	σ_{LO}, pb	σ_{NLO}, pb	δ_{NLO}	σ_{NNLO}, pb	δ_{NNLO}
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	$+2.4\%$	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	$+4.9\%$	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	$+9.3\%$	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	$+13.6\%$	$25.4^{-0.1}_{+0.2}$	$+1.6\%$

The precision on the inclusive cross section is about one percent. Ratio of top and anti-top cross sections is sensitive to parton distribution functions at relatively large values of x and should be used as one of the standard candles for PDF determinations.

Di-jet production





Results are for gluon-gluon and quark-gluon (preliminary) initial states. Not all color factors included for quark-gluon channel. Flat NNLO/NLO K-factors; small corrections (may change if other channels included). Results for various orders obtained with NNLO PDFs.

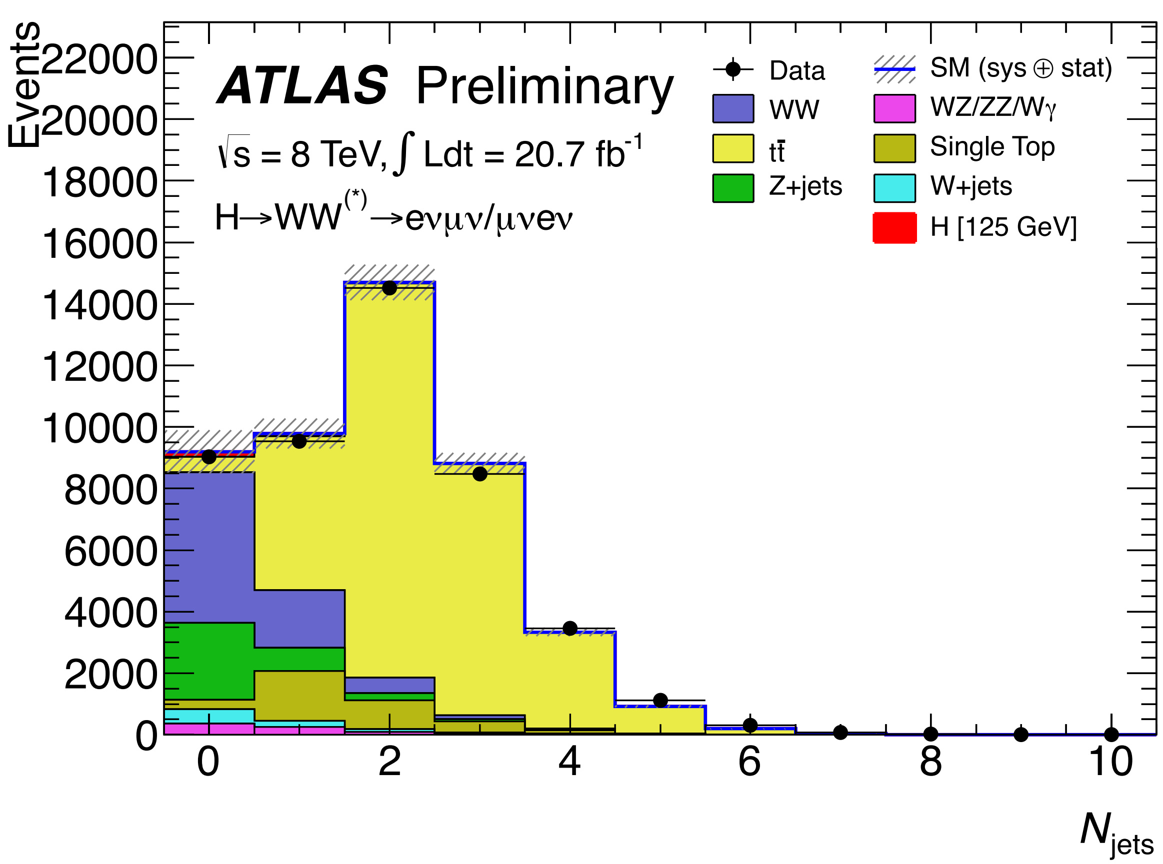
Currie, Gehrmann-de Ridder, Gehrmann, Glover, Pires

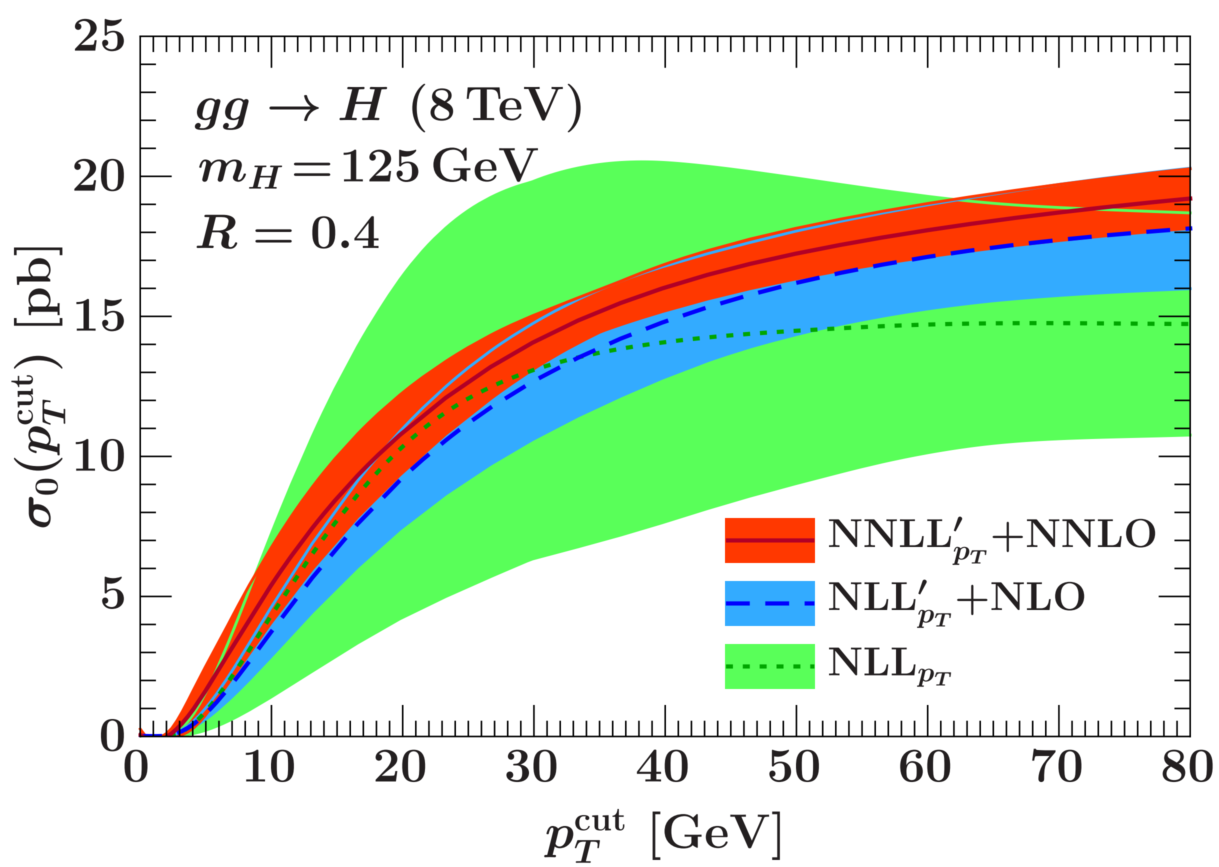
Realistic cross sections

The Higgs boson couplings are extracted from cross sections that are subject to kinematic constraints on the final states. This happens because detectors have only restricted angular coverage and because by selecting final states with particular kinematic properties, certain backgrounds can be significantly reduced.

This, however, requires precision predictions for exclusive/fiducial cross sections, including jet-binning, Higgs boson decays etc, making them highly non-trivial. Without such predictions, the Higgs couplings can not be extracted from the LHC data with the ultimate precision.

Jet binning requires jet identification; this may introduce perturbative computations unstable; attempts to resum logarithmically enhanced terms.



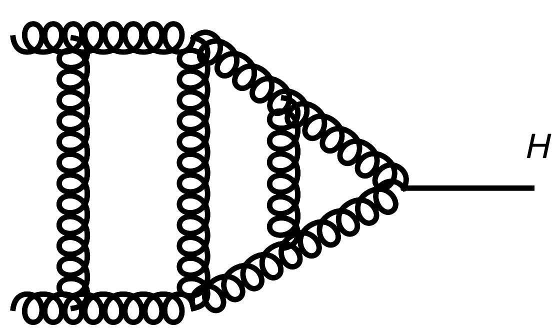


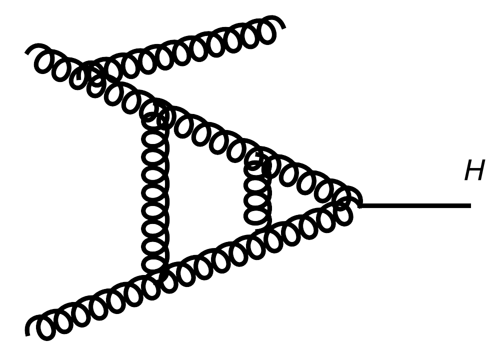
Banfi, Zanderighi, Salam; Tackmann, Zuberi, Walsh; Becher, Neubert

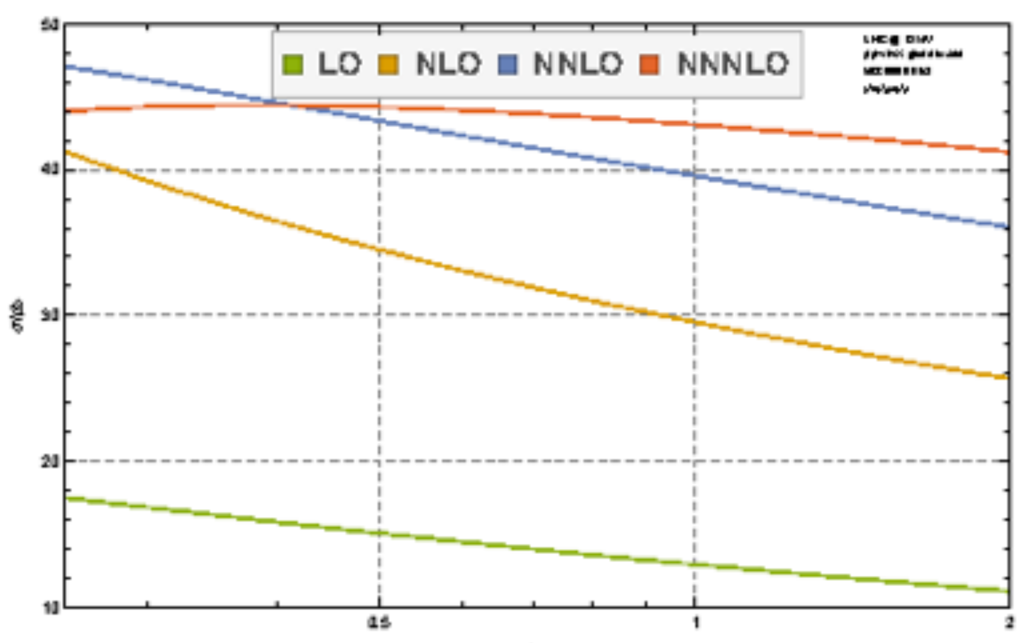
Higgs production: jet-binned cross sections

To obtain the zero-jet cross section for the Higgs production, we subtract the one-jet inclusive cross section from the total inclusive cross section, at matching orders in pQCD.

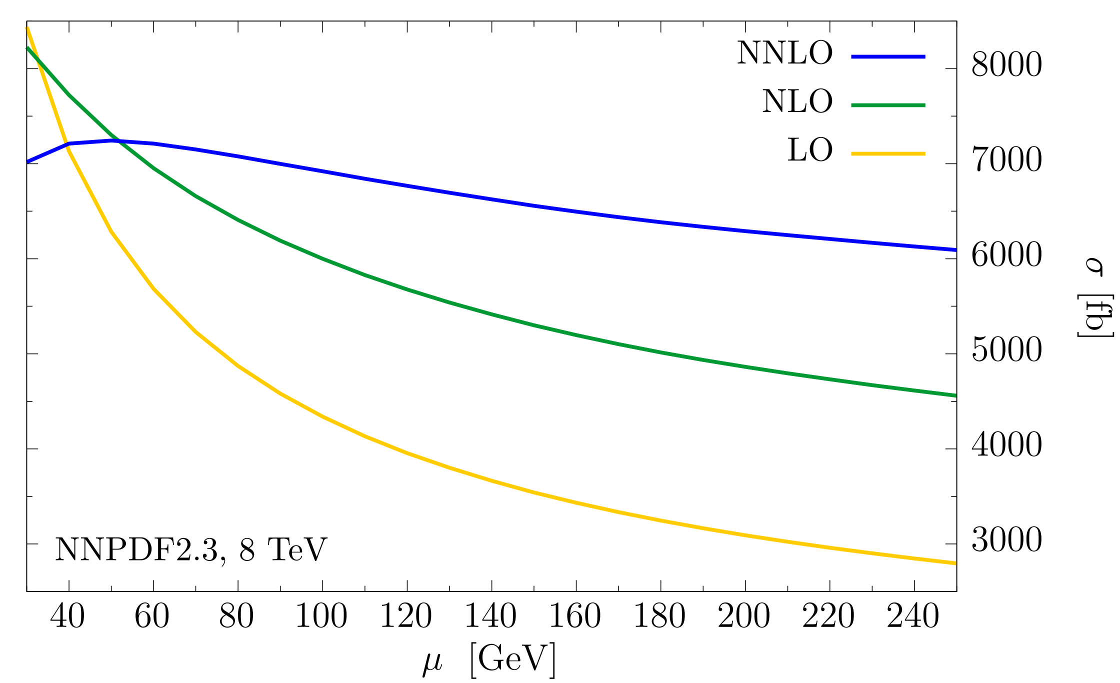
The inclusive Higgs production was computed recently through N^3LO and the H+jet production was computed through NNLO QCD; these are same orders in perturbation theory. Using these results, one can improve on predictions for jet-binned cross sections.







Anastasiou, Duhr, Dulat, Furlan, Herzog, Gehrmann, Mistlberger etc.



R. Boughezal, F. Caola, K.M., F. Petriello, M. Schulze

Fiducial cross sections

The results of N^3LO computation for inclusive Higgs production, NNLO for the H+j production as well as advances with re-summations of jet-radius logarithms allow one to improve on existing predictions for 0-jet and 1-jet bin cross sections.

For the 13 TeV LHC, using NNPDF2.3, anti-k_T, R=0.5, $\mu_0=m_H/2$, $Q_{res} = m_H/2$ and accounting for top and bottom mass effects, one finds the following results:

0-jet bin

LHC 13 TeV	$\epsilon^{\mathrm{N^3LO+NNLL+LL_R}}$	$\Sigma_{\text{0-jet}}^{\mathrm{N^3LO+NNLL+LL_R}}$ [pb]	$\Sigma_{\text{0-jet}}^{\mathrm{N^3LO}}$	$\Sigma_{\text{0-jet}}^{\mathrm{NNLO+NNLL}}$
$p_{t,\mathrm{veto}} = 25\,\mathrm{GeV}$	$0.539^{+0.017}_{-0.008}$	$24.7^{+0.8}_{-1.0}$	$24.3^{+0.5}_{-1.0}$	$24.6^{+2.6}_{-3.8}$
$p_{t,\mathrm{veto}} = 30\,\mathrm{GeV}$	$0.608^{+0.016}_{-0.007}$	$27.9^{+0.7}_{-1.1}$	$27.5^{+0.5}_{-1.1}$	$27.7^{+2.9}_{-4.0}$

≥1-jet bin

LHC 13 TeV	$\Sigma_{\geq\text{1-jet}}^{\mathrm{NNLO+NNLL+LL_R}}$ [pb]	$\Sigma_{\geq\text{ 1-jet}}^{\mathrm{NNLO}}$ [pb]
$p_{t,\mathrm{min}} = 25\,\mathrm{GeV}$	$21.2^{+0.4}_{-1.1}$	$21.6^{+0.5}_{-1.0}$
$p_{t,\mathrm{min}} = 30\,\mathrm{GeV}$	$18.0^{+0.3}_{-1.0}$	$18.4^{+0.4}_{-0.8}$

- No breakdown of fixed order perturbation theory for $p_T \sim$ 25- 30 GeV ;
- Reliable error estimate from lower orders ; residual errors O(3-5) percent for the two jet bins;
- Re-summed results change fixed-order results within the error bars of the former/ latter. There seems to be little difference between re-summed and fixed order results.

A. Banfi, F. Caola, F. Dreyer, P. Monni, G.Salam, G. Zanderighi, F. Dulat

Higgs cross sections: even more fiducial

To go even more fiducial (i.e. realistic), one can let the Higgs decay and compare results with measured cross sections / distributions of the ATLAS collaboration.

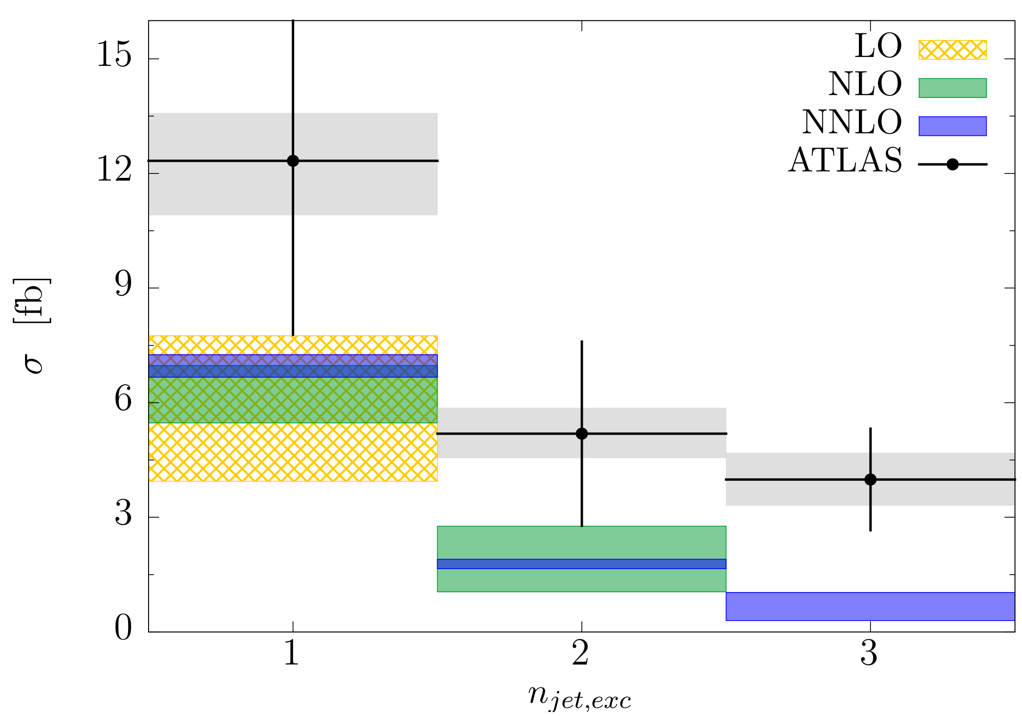
Atlas cuts on photons and jets

$$\mathrm{anti}-\mathrm{k}_t,\quad \Delta R = 0.4,\quad p_{j\perp} = 30\ \mathrm{GeV}, \mathrm{abs}(y_j) < 4.4$$
$$p_{\perp,\gamma_1} > 43.75\ \mathrm{GeV},\quad p_{\perp,\gamma_2} = 31.25\ \mathrm{GeV},\quad \Delta R_{\gamma j} > 0.4$$

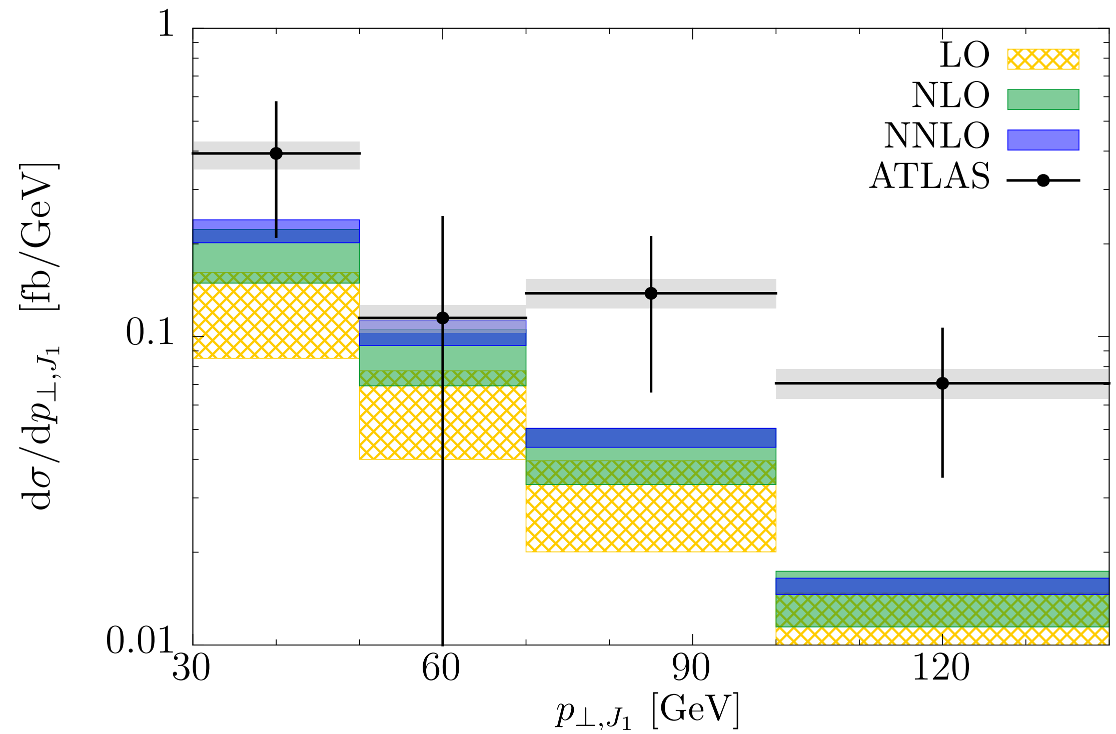
$$\sigma^{\mathrm{fid}}_{1j,\mathrm{ATLAS}} = 21.5 \pm 5.3(\mathrm{stat}) \pm 2.3(\mathrm{syst}) \pm 0.6\ \mathrm{lum\ fb}$$

$$\sigma^{\mathrm{fid}}_{\mathrm{NNLO}} = 9.46^{+0.56}_{-0.84}\ \mathrm{fb}$$

F. Caola, K.M., M. Schulze



Exclusive jet cross sections



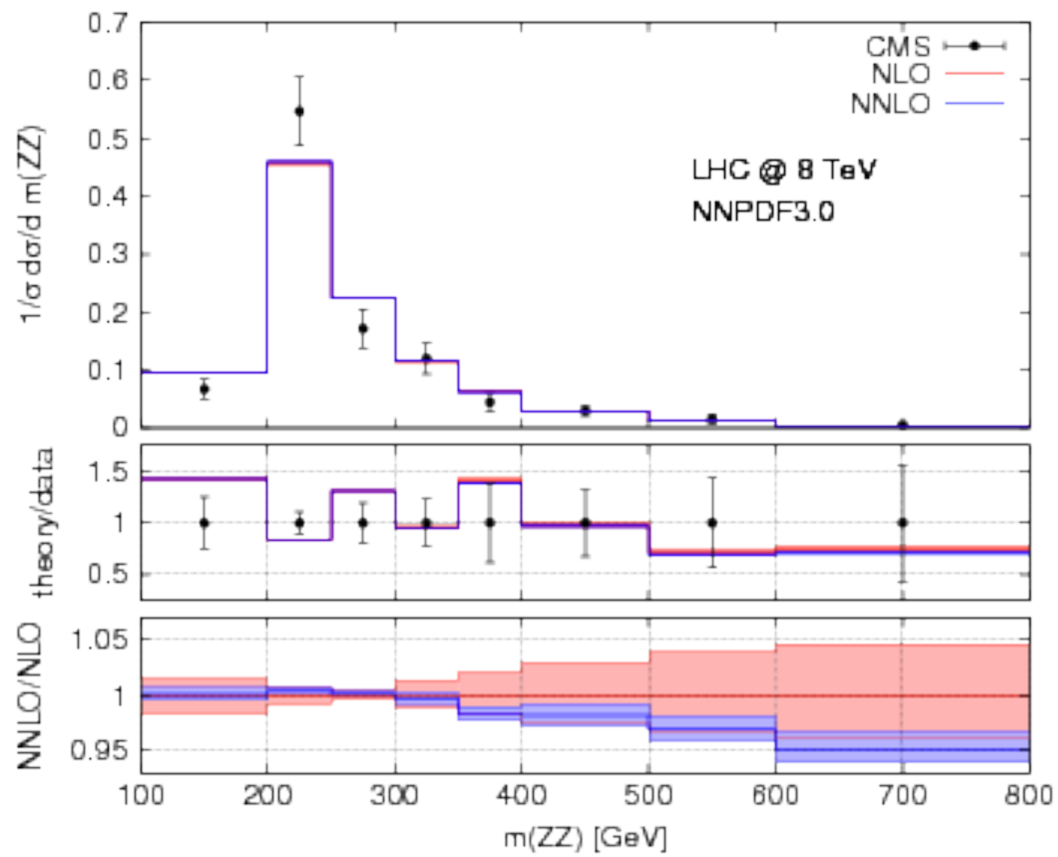
Transverse momentum distribution of a leading jet

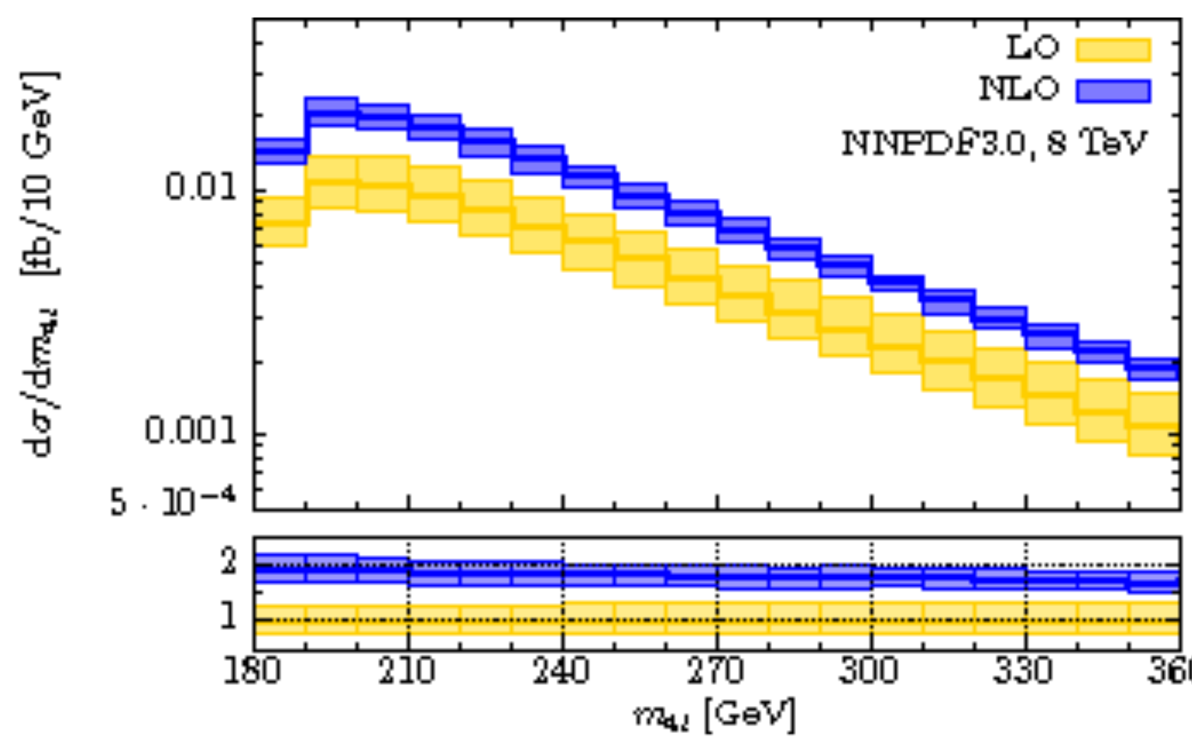
ZZ pair production at the LHC

By the end of the Run I, we had an interesting problem with the production of two W-bosons; the measured cross -sections came out too high. By now, the issue seems to be understood; it appears that it was caused by improper extrapolation.

Monni, Zanderighi

ZZ cross sections are smaller but cleaner. The NNLO fully-differential predictions for ZZ final state are available. NNLO corrections are dominated by (LO) gg -> ZZ which is subject to large QCD corrections (N^3LO formally).





$$\sigma_{\rm NNLO}^{q\bar{q}\to 2e2\mu} = 19.6(6)\ \text{fb}$$

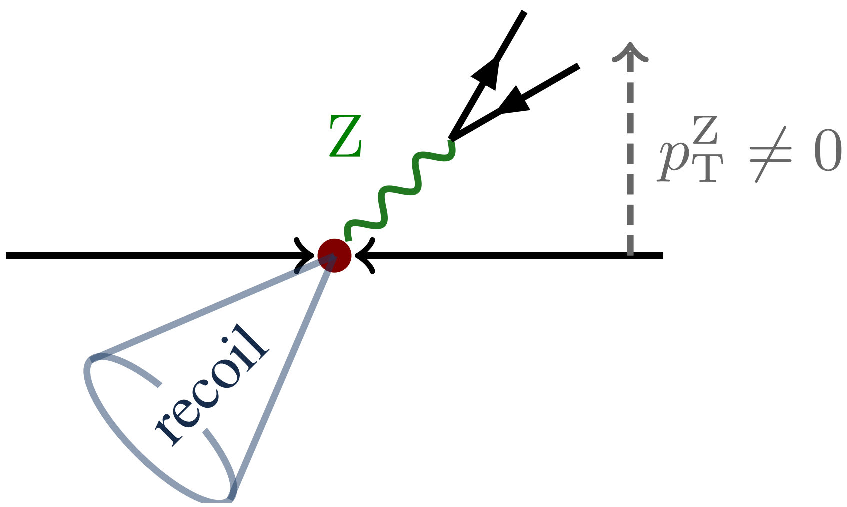
T. Gehrmann, M. Grazzini, S. Kallweit, P. Maierhoefer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi

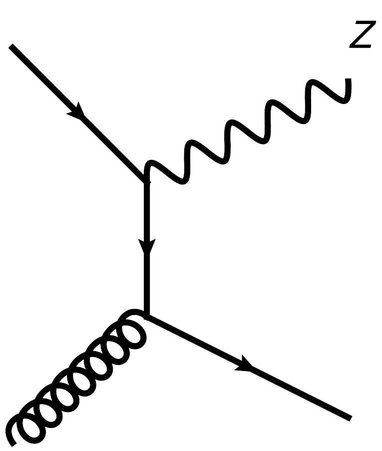
$$\sigma_{\rm NNLO}^{gg\to 2e2\mu} = 2.0(2)\ \text{fb}$$

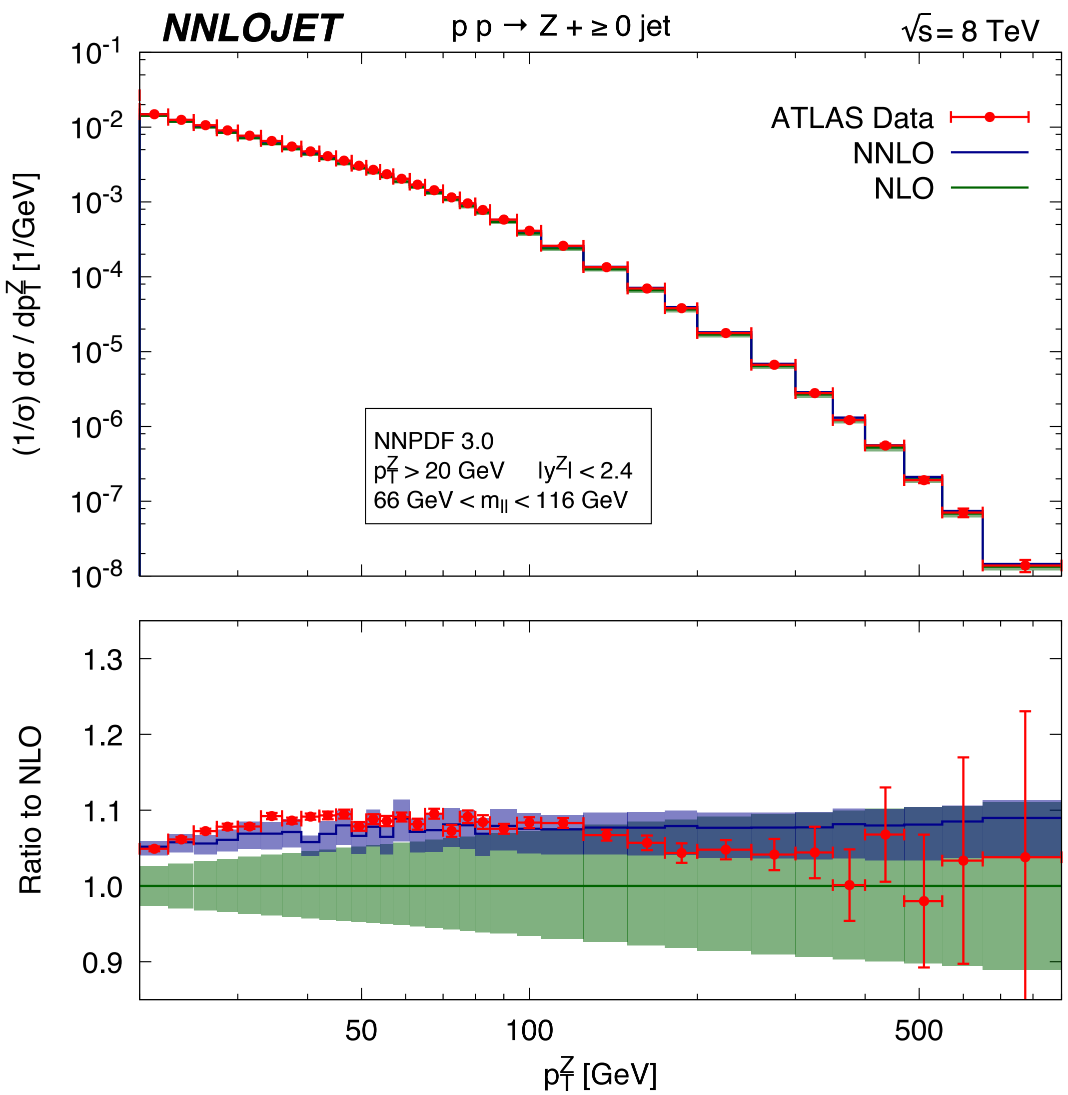
F. Caola, K. Melnikov, R. Rontsch, L. Tancredi

Z+jet production at the LHC

The transverse momentum distribution of the Z boson is measured with a very high (few percent) precision. An important observable for constraining gluon PDF. The NNLO QCD computation of Z+j production at the LHC leads to a precise description of the Z transverse momentum distribution and improves agreement between theory and experiment.



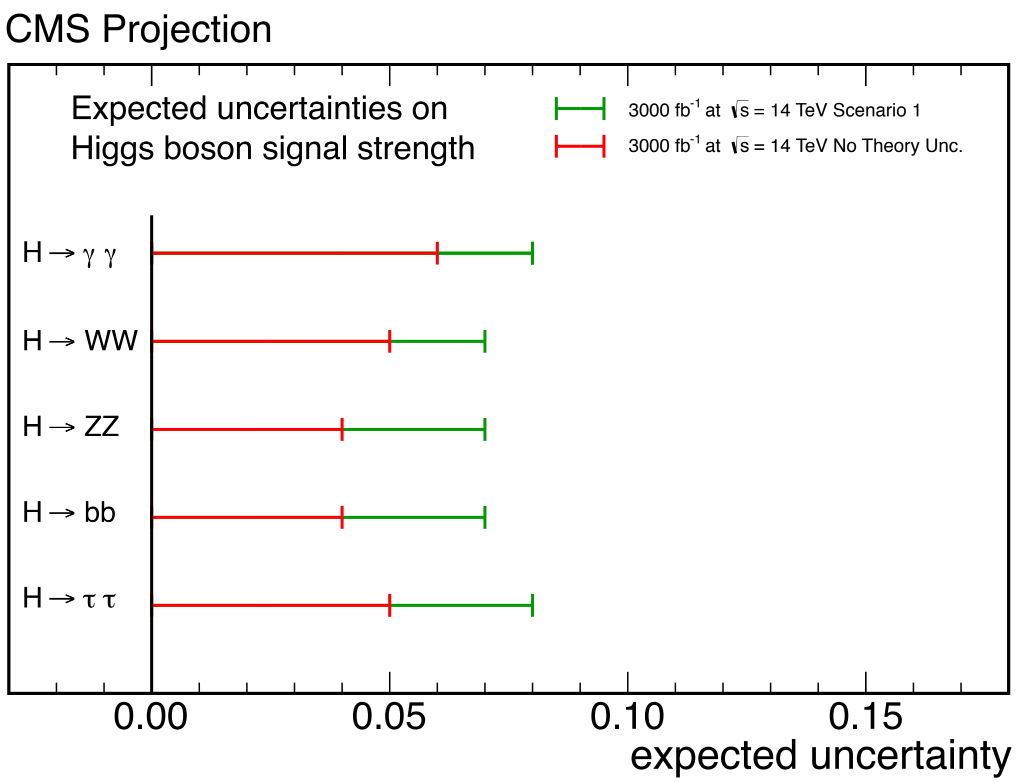


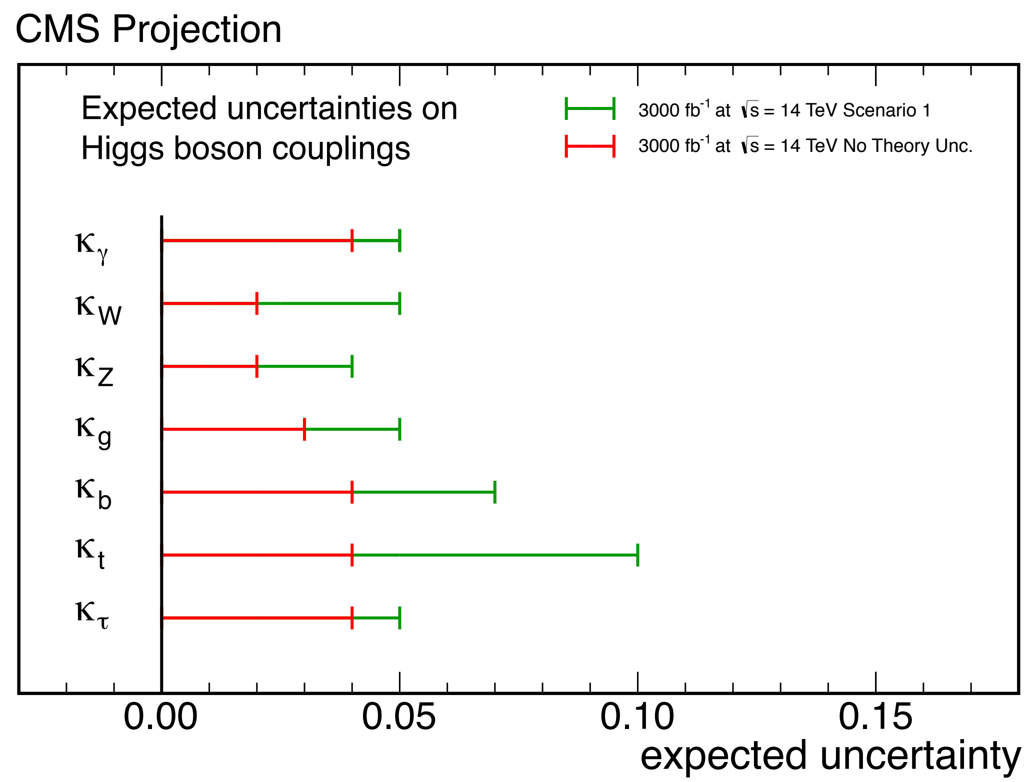


Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan

Theoretical precision and 3000/fb expectations

H+0 jet	N^3LO	O(3-5 %)	10 pb	fully inclusive
H+1 jet	N^2LO	O(7%)	7 pb	fully exclusive; Higgs decays, infinite mass limit
H+2 jet	NLO	O(20%)	1.5 pb	matched/merged
H+3 jet	NLO	O(20%)	0.4 pb	matched/merged/almost
WBF	N^2LO	O(1%)	1.5 pb	exclusive, no VBF cuts
WBF	N^2LO	O(5%)	0.2 pb	exclusive, VBF cuts
ZH, WH	N^2LO	O(2-3%)	O(1) pb	decays to bottom quarks
ttH	NLO	O(5%)	0.2pb	decays, off-shell effects





Theoretical precision on major Higgs production cross sections, that we already have, seems to match the experimental precision achievable with 3000/fb. A new situation, thanks to the recent theoretical results.

Consolidating precision

Progress with perturbative QCD computations at the LHC strongly suggests that there is a real chance to perform precision studies at the LHC.

To fully benefit from these developments, we will need

1) to better understand parameters that enter calculation of cross sections (PDFs, masses, couplings, etc.) ;

2) to include electroweak corrections;

3) to work with realistic final states and fiducial cross sections ;

4) to understand the limitations of various approximations that we currently use in theoretical computations (finite = infinite, parton showers, etc.)

A hadron collider as a machine for precision studies?

Traditionally, hadron colliders played a role of the discovery machines but, given spectacular theoretical advances of recent years, it may be possible to do precision physics at those machines. A new situation, right in time for the beginning of the Run II.

As an illustration, compare theoretical precision on major Higgs production cross sections, that we already have, with experimental precision expected with 3000/fb.

H+0 jet	N^3LO	O(3-5 %)	10 pb	fully inclusive
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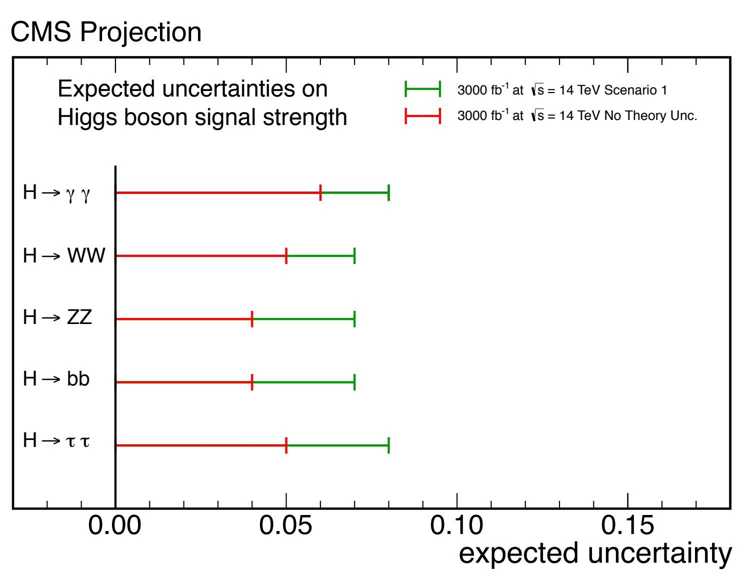
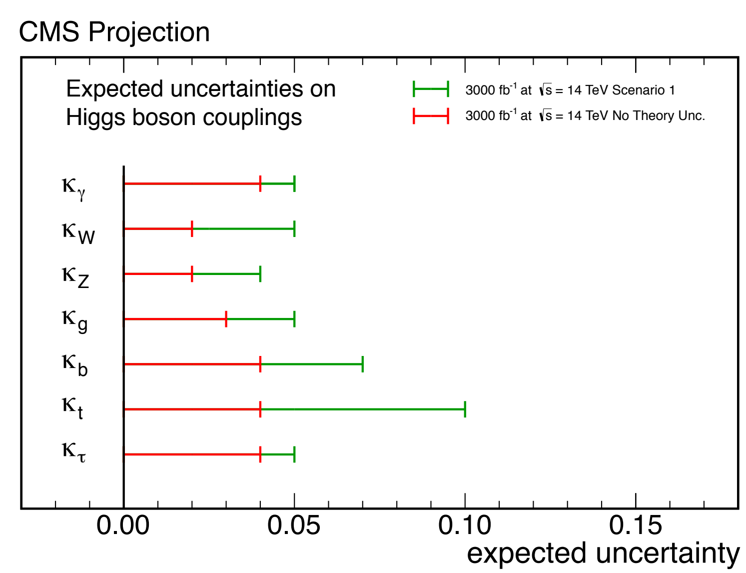
ttH	NLO	O(5%)	0.2pb	decays, off-shell effects





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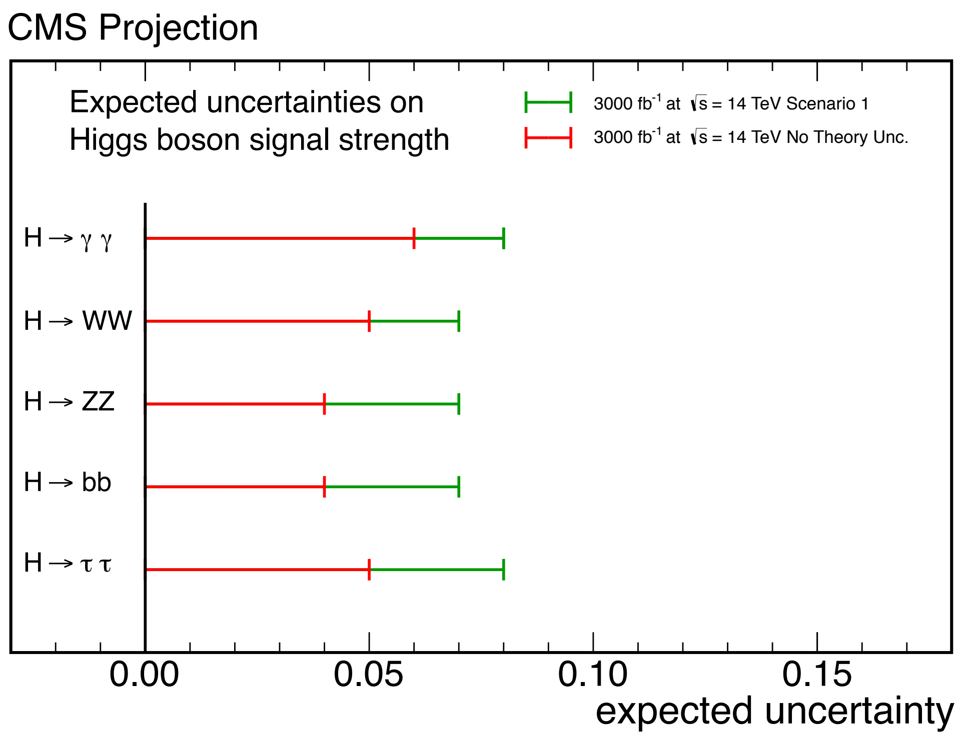
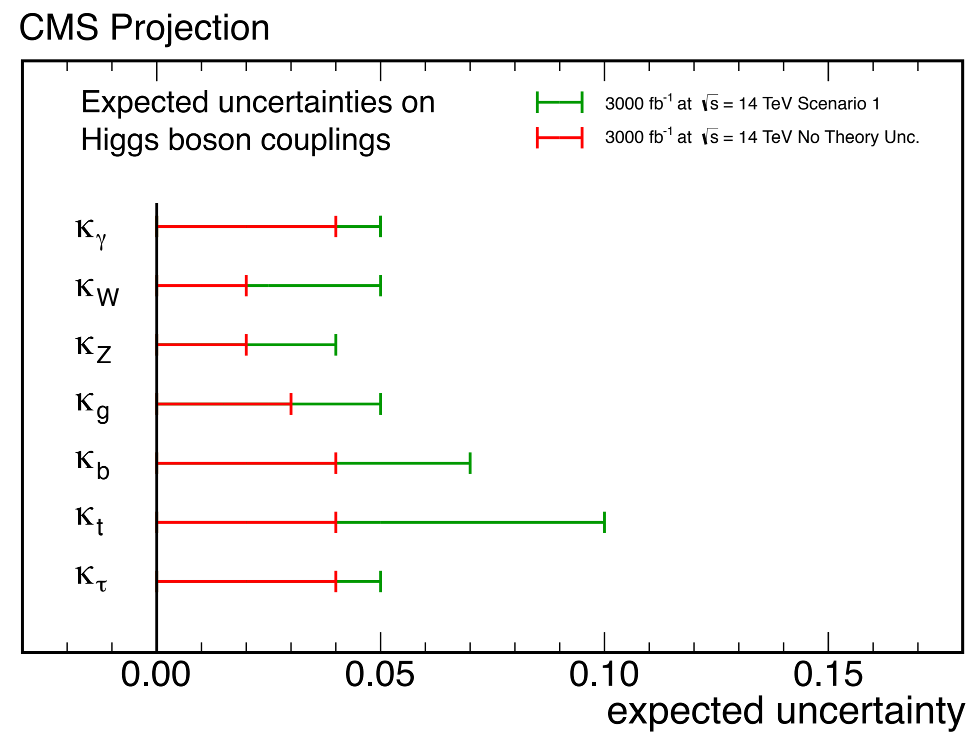
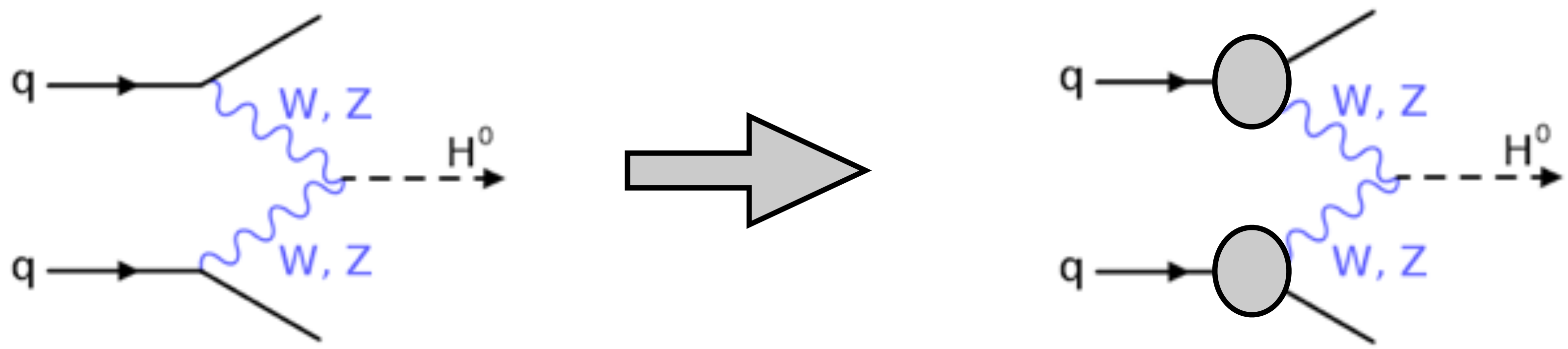




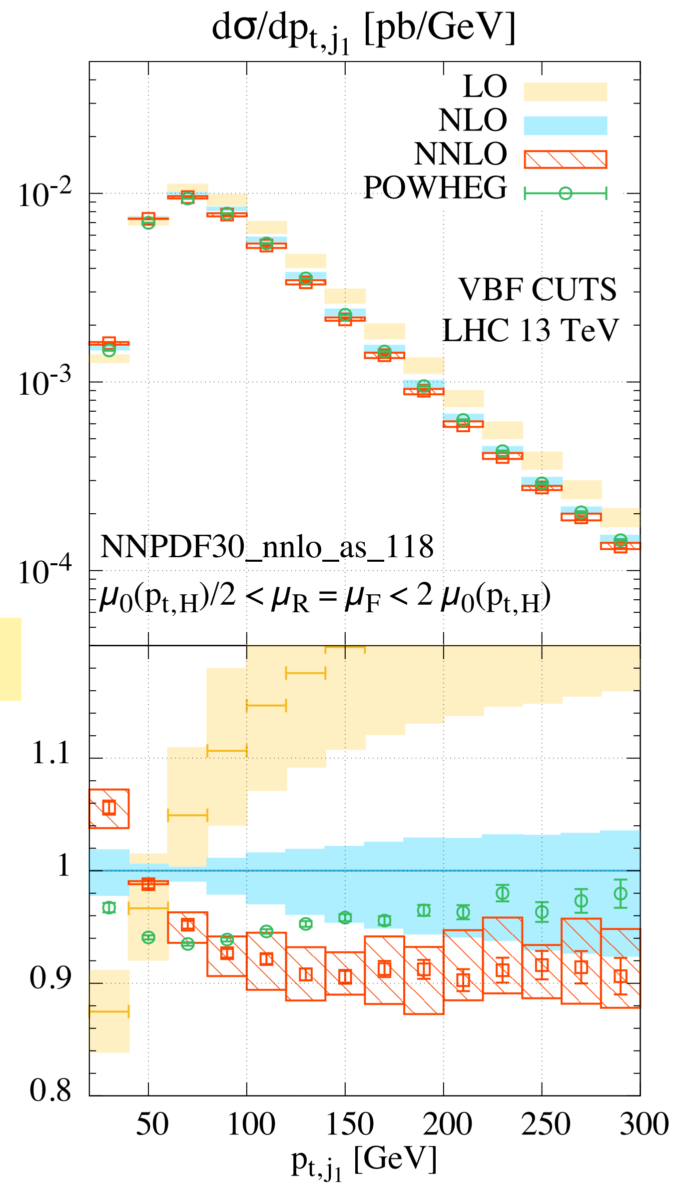
Higgs boson production in weak boson fusion

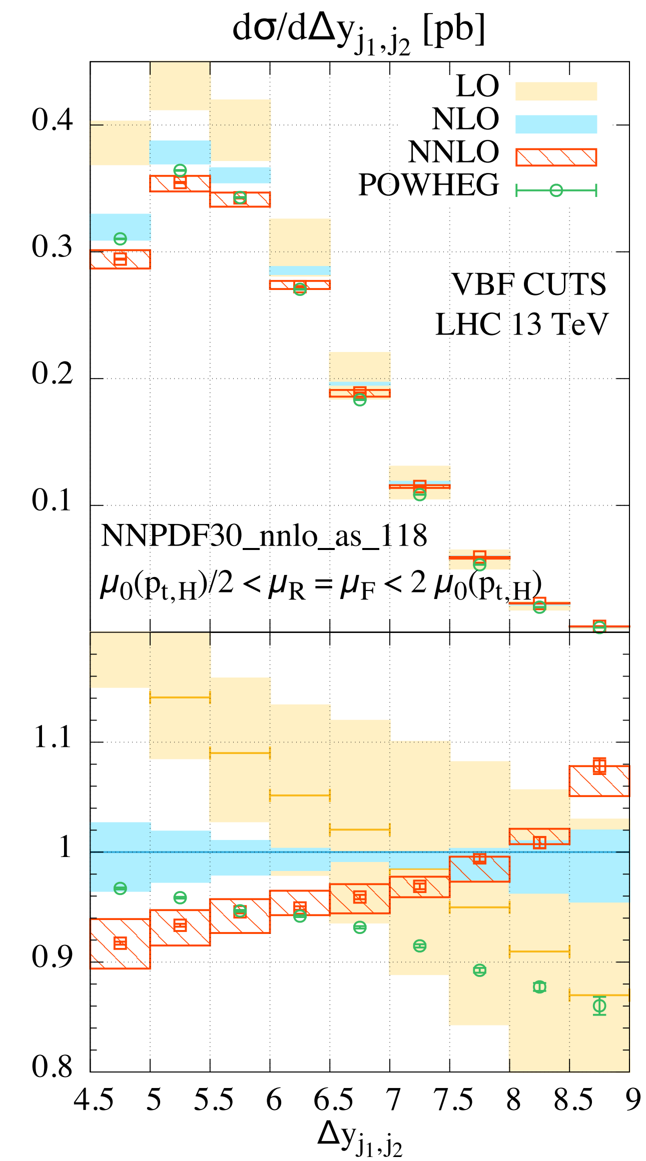
Estimating NNLO QCD corrections to WBF fusion by mapping the problem on the inclusive DIS apparently does not work. QCD corrections are different.



WBF cuts

$$p_{\perp}^{j_{1,2}} > 25 \text{ GeV}, \quad |y_{j_{1,2}}| < 4.5,$$
$$\Delta y_{j_1,j_2} = 4.5, \quad m_{j_1,j_2} > 600 \text{ GeV},$$
$$y_{j_1} y_{j_2} < 0, \quad \Delta R > 0.4$$





Cross sections with and without WBF cuts

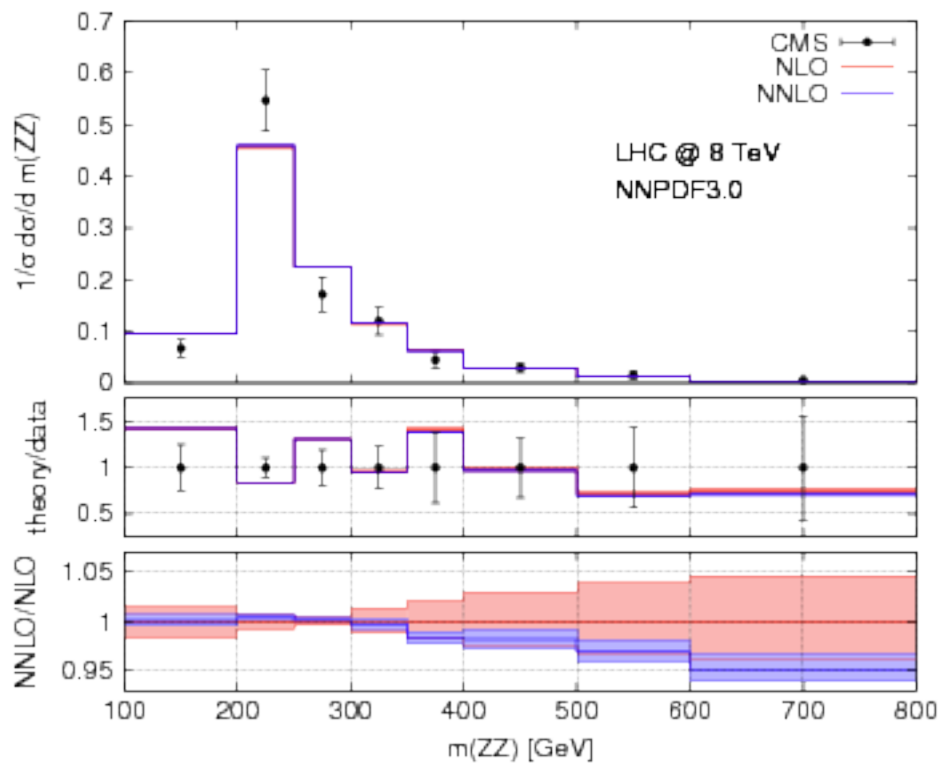
	σ^{nocuts}[pb]	$\sigma^{\text{VBF cuts}}$[pb]
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.826^{+0.013}_{-0.014}$

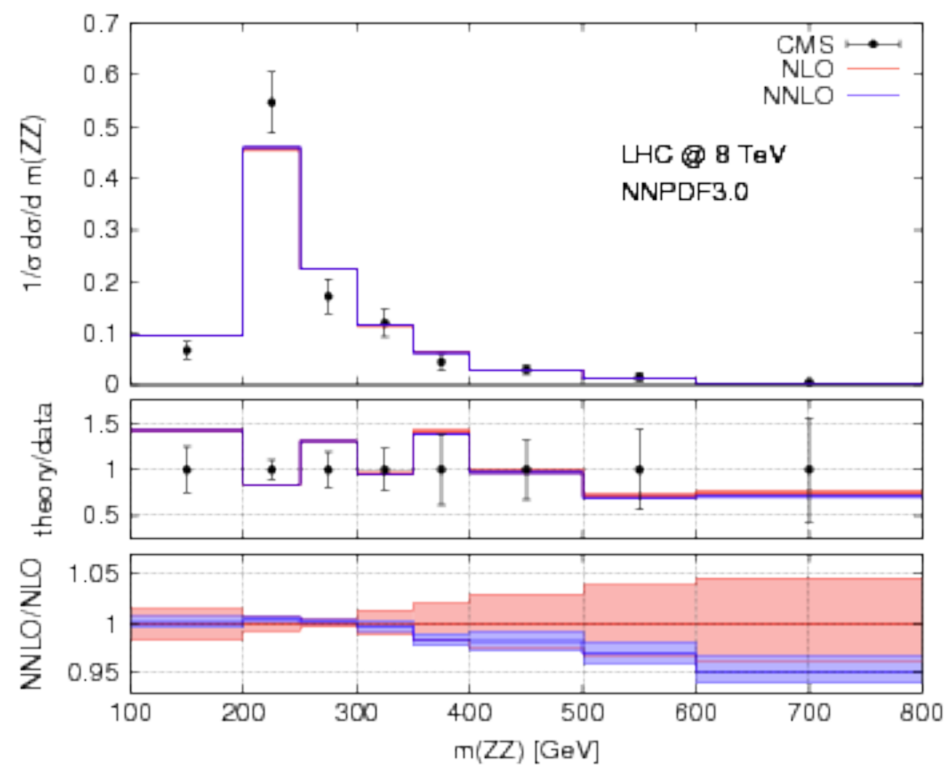
Cacciari, Dreyer, Kalberg, Salam, Zanderighi

Z-boson pair production: quark annihilation

The fully-differential production of two Z-bosons in quark-anti-quark annihilation was computed through NNLO QCD, including off-shell effects and decays of the Z-bosons.

The residual uncertainty on the cross section is estimated to be of the order of 3%; this should enable precise predictions for the ``background" for the determination of the Higgs boson width. Note that this calculation relies on the two-loop amplitudes for qq->V_1V_2 and uses the q_t-subtraction scheme, to combine real and virtual corrections.

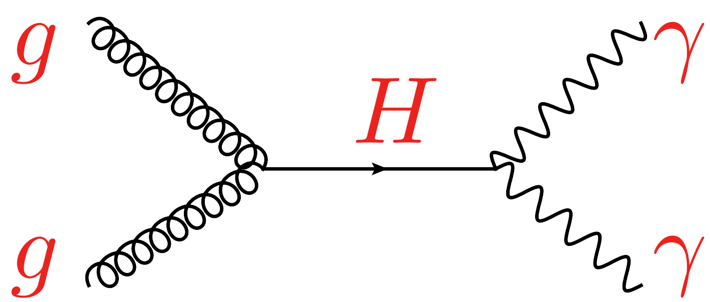




M. Grazzini, S. Kallweit,   D. Rathlev

What is the width of the Higgs boson?

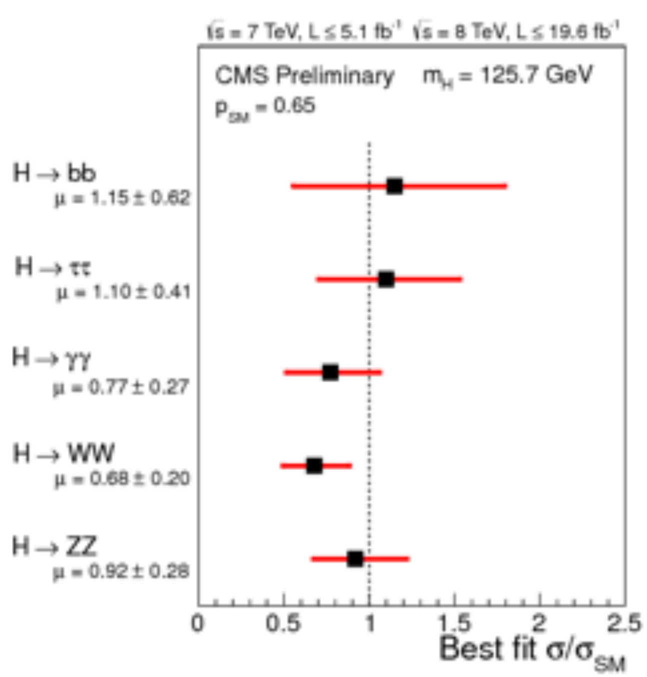
Although many properties of the Higgs bosons appear to be consistent with the Standard Model, reaching this conclusion requires hidden assumptions. One of such assumptions is the Standard Model value of the Higgs boson width.

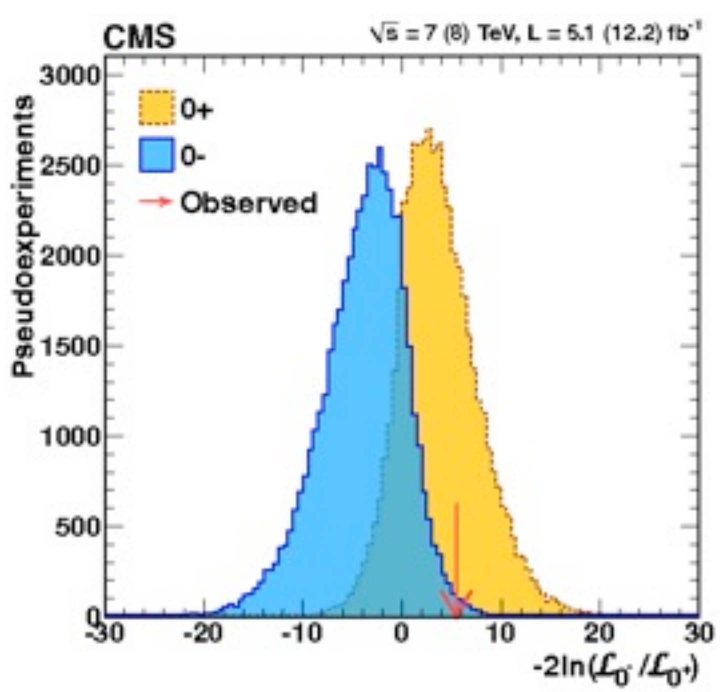


$$\sigma_{i\to H\to f} \sim \frac{g_i^2 g_f^2}{\Gamma_H}$$

The on-shell production cross section is invariant under a simultaneous change of the couplings and the width, resulting in infinitely many solutions. To break the degeneracy, one should find the way to measure the couplings and the width independently of each other.

$$g \to \xi g, \quad \Gamma_H \to \xi^4 \Gamma_H \quad \Rightarrow \quad \sigma_H \to \sigma_H$$



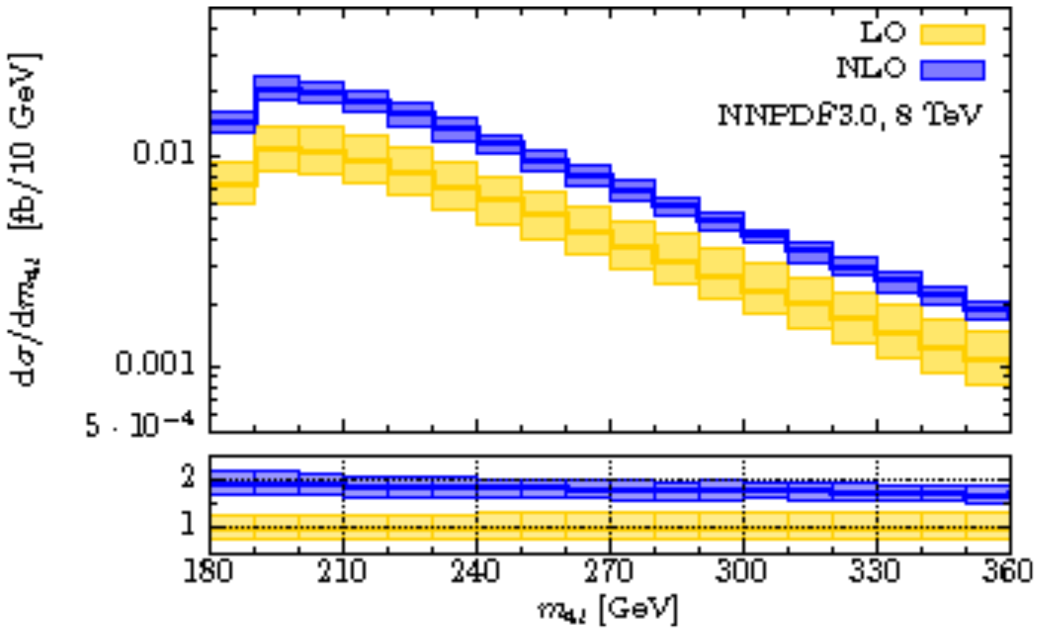


Z-boson pair production: gluon annihilation

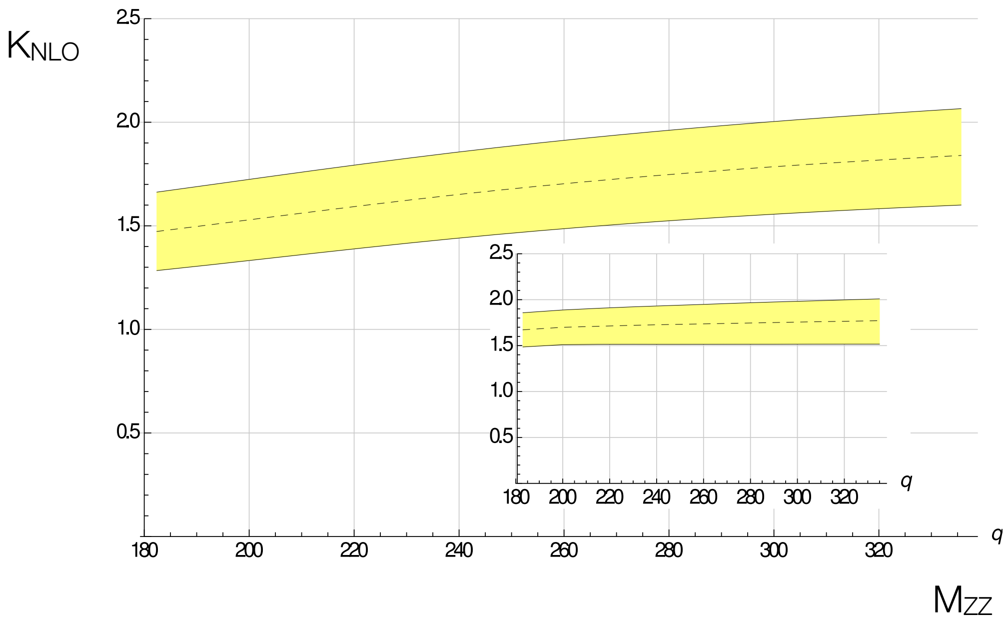
Gluon fusion into a pair of Z-bosons is an irreducible background to Higgs production (the amplitudes interfere). It starts at one-loop, so calculation of even NLO QCD corrections to it is highly non-trivial.

Nevertheless, the NLO QCD corrections to gg -> ZZ production through massless quark loops were computed; large perturbative corrections (70-90%) were found and the residual uncertainty was estimated to be close to 10 percent.

Top quark loops perhaps are not important for the cross-section but are likely to be relevant for the interference with the Higgs. Recent results for gg ->ZZ cross-section in the approximation of the infinitely heavy top quark indicate large (1.8) K-factor.



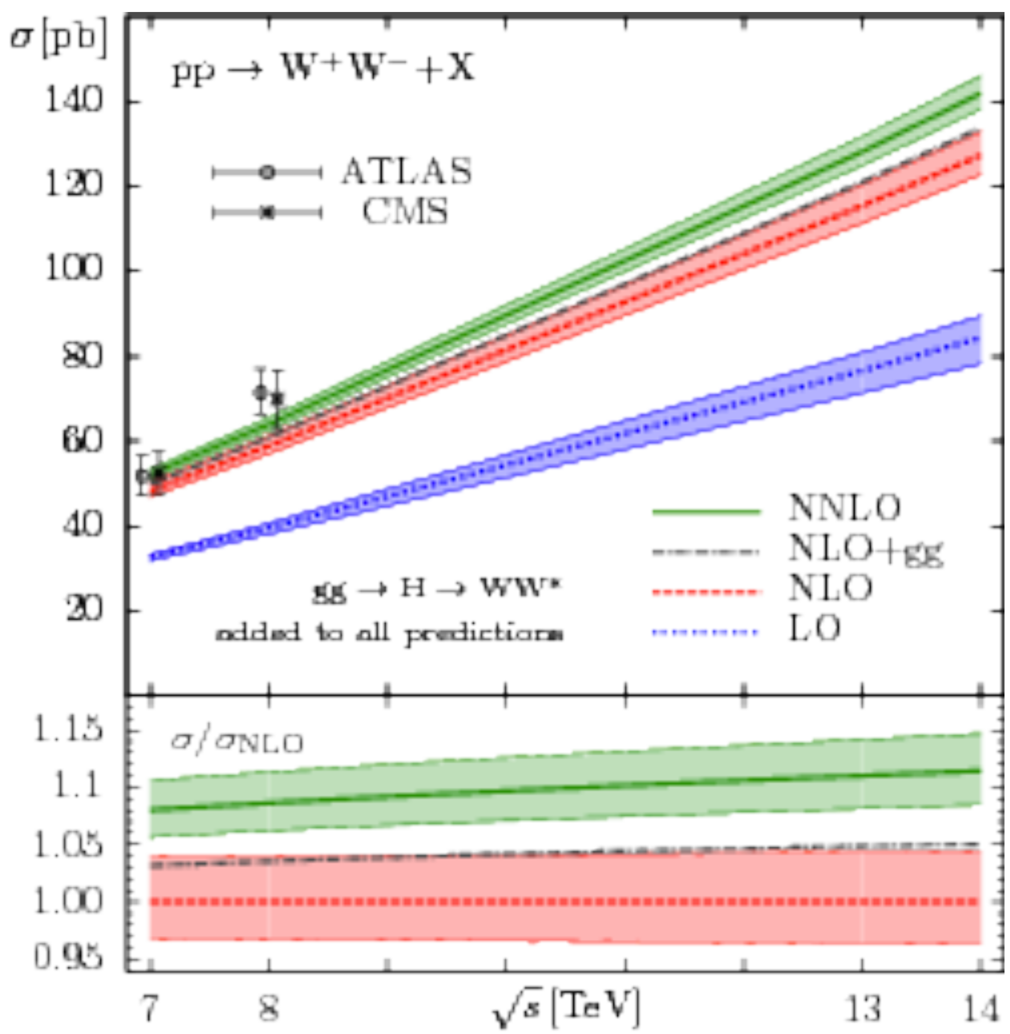
F. Caola, K. Melnikov, R. Rontsch, L. Tancredi



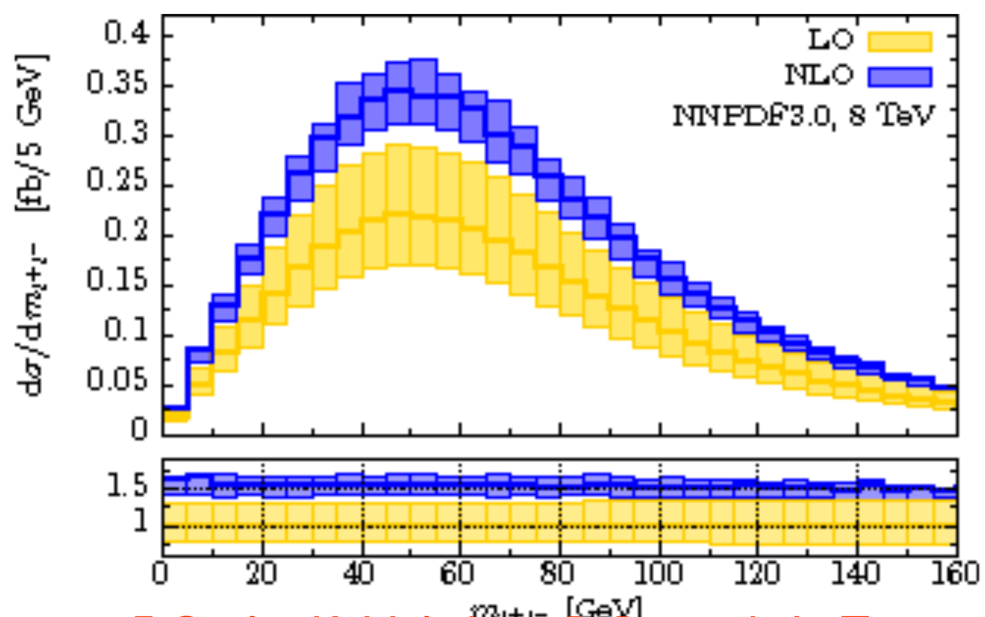
Dowling, Melnikov

W-boson pair production

Interest in this process is related to a two-sigma excess that was observed by both ATLAS and CMS in 7 TeV and 8 TeV data. The NNLO QCD corrections to quark-anti-quark annihilation as well as the NLO QCD corrections to gluon fusion push the theory prediction much closer to experiment.



T. Gehrmann, M. Grazzini, S. Kallweit, P. Maierhoefer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi



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$$\sigma^{q\bar{q}+H+gg,\mathrm{NLO}}_{\mu\mu,ee,e\mu+\mu e} = (72.0^{+1.3}_{-2.1},\ 66.3^{+1.2}_{-1.7},\ 337.3^{+6.3}_{-4.5}).$$

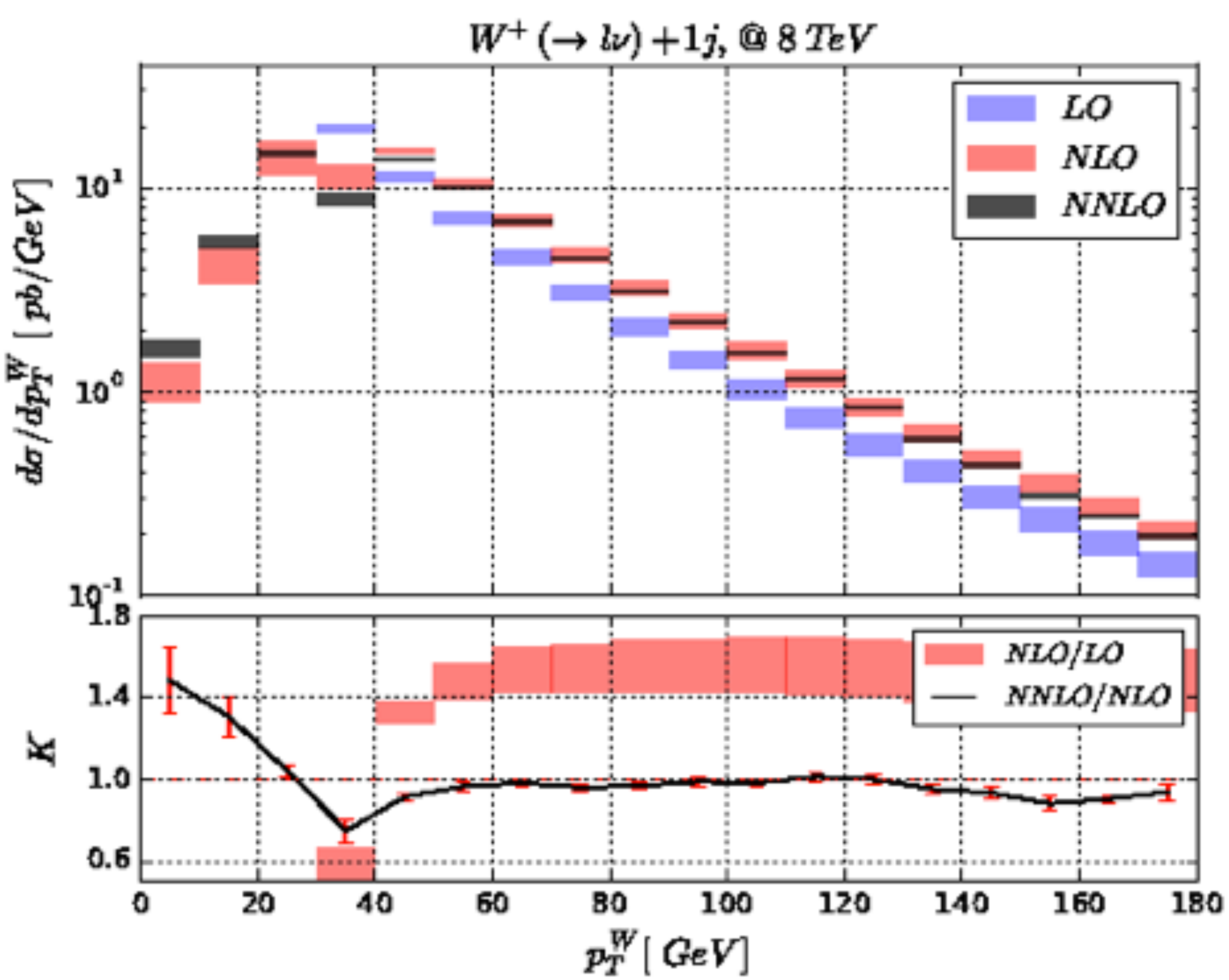
$$\sigma_{\mu\mu,ee,e\mu+\mu e} = (74.4^{+8.1}_{-7.1},\ 68.5^{+9.0}_{-8.0},\ 377.8^{+28.4}_{-25.6})$$

Estimating the NNLO QCD corrections by re-scaling inclusive ones, we find that they can add additional 4-20 fb, for ee and electron-muon channels, respectively. This will make theory and experiment agree to within one sigma.

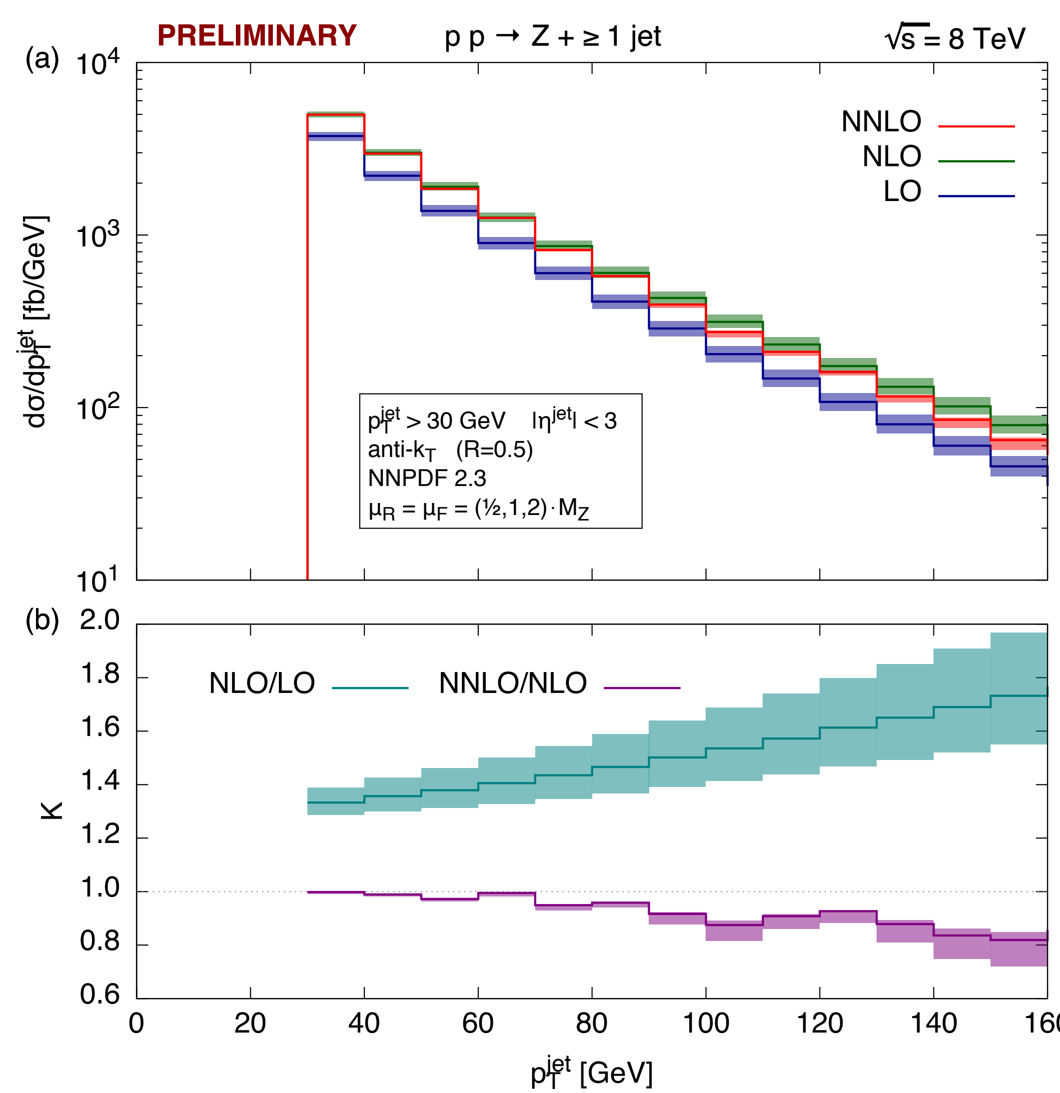
Vector bosons plus jet

NNLO QCD computations for W+j and Z+j are now available. Corrections are found to be quite small.

These results can be used for better background modeling, for improved understanding of the W and Z bosons transverse momentum distribution and for constraining the gluon PDF.



Bougezhal, Focke, Liu, Petriello



Gehrmann-de Ridder, Gehrmann, Glower, Huss, Morgan

Also studied by Bougezhal, Campbell, Ellis, Focke, Giele, Liu, Petriello

Conclusion

Our ability to perform NNLO QCD computations increased dramatically during the past year. Development of robust theoretical methods finally paid off and allowed us to compute large number of 2 -> 2 processes through NNLO QCD in a fully exclusive manner.

NNLO QCD is the ``last perturbative order" that is possible to study without understanding non-perturbative effects at colliders (exceptions are processes with very large NLO QCD corrections).

NNLO is a high enough perturbative order to provide both correct physics and high precision. Use of NNLO should naturally reduce the reliance on resummations and parton showers outside of their applicability region.

NNLO QCD predictions show that after a certain level of precision, it is not possible to rely on the approximate ways of computing radiative corrections; full fixed order calculations are needed. This is especially true for hard fiducial cross sections that, in fixed order calculations, can be computed for the same sets of cuts that are used in the measurement.

Phenomenological reach of these computations is very broad and impacts studies of top quark properties, understanding the Higgs boson couplings, extraction of parton distribution functions, measurements of the strong coupling constant and refined modeling of backgrounds.

Further developments of theoretical methods for these computations will involve massive loops, higher multiplicity final states, unitarity and improvements in the efficiency of subtraction methods.

Conclusion

The LHC is the first hadron collider where outcomes of hard proton collisions can be predicted with a few percent precision for a large number of diverse final states. The possibility to do that is the result of spectacular progress in technology of perturbative QCD that occurred in recent years.

Precision studies at the LHC will allow determination of Higgs couplings with a few percent precision or perhaps even better if theoretical and experimental progress continues at a pace. that we have seen in recent years.

Equally important, progress with precision predictions for complex multi-particle final states should allow for broad-band searches for (correlated) deviations in multitude of kinematic distributions that can be measured for various final states at the LHC. Such correlated deviations -- if discovered -- will signal the presence of physics beyond the SM which is too heavy to be observed at the LHC and, in this way, will allow us to determine the energy scale where the Standard Model breaks down.

Further improvements of theoretical methods are required to pursue this research program. They include understanding massive loops, development of two-loop unitarity and improvements in the efficiency of subtraction methods.

Moreover, to fully benefit from these theoretical developments, we will need to better understand parameters that enter calculation of cross sections (PDFs, masses, couplings, etc.), to include electroweak corrections, to work only with realistic final states and fiducial cross sections and to understand the limitations of various approximations that we currently use in theoretical computations.

Conclusion

With data taken in coming years at or near to the design energy of 14 TeV, a broader picture for physics at the TeV scale will emerge with implications for the future of the energy frontier program. Amongst the essential inputs will be precision measurements of the properties of the Higgs boson and direct (as well as indirect (K.M.)) searches for new physics that will make significant inroads into new territory.

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