Neutrino Mass and Lepton Mixing

Max-Planck-Institut für Kernphysik

Werner Rodejohann

RTG Seminar

24/06/15

$m_\nu = m_L - m_D^T M_R^{-1} m_D$

MANITOP

Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

GRK 2044

Masse und Symmetrien nach der Entdeckung des Higgs-Teilchens am LHC

erc

Max-Planck-Gesellschaft

Heisenberg-Programm

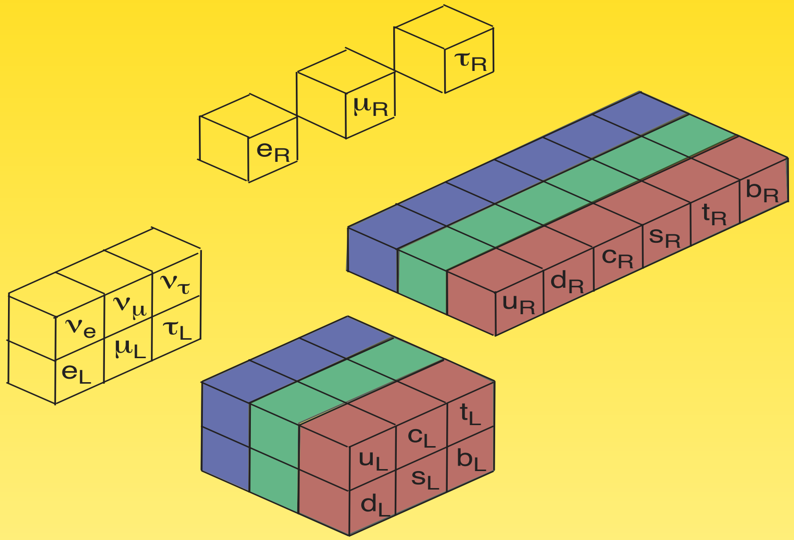
DFG

Neutrino properties in short

- massive
- mix
- probably Majorana

Neutrinos and the Standard Model

Standard Model of Elementary Particle Physics $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18
strong CP	1	19

19 free parameters...

+ Gravitation

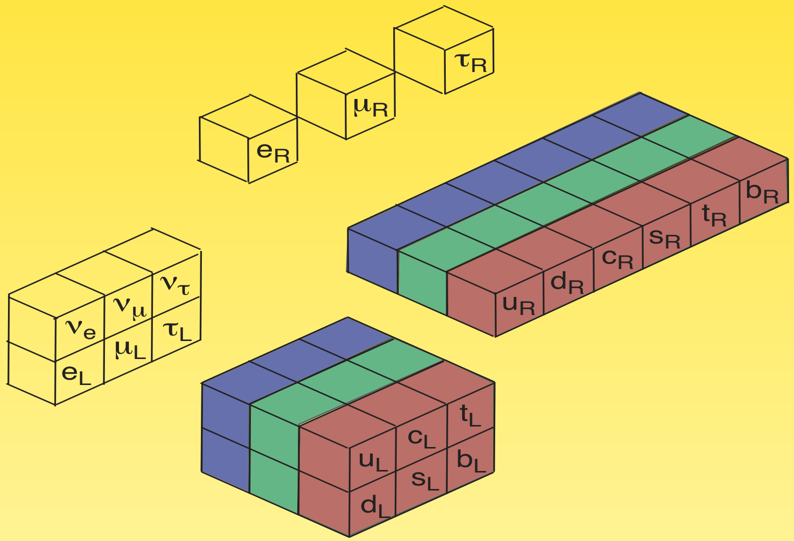
+ Dark Energy

+ Dark Matter

+ Baryon Asymmetry

Neutrinos and the Standard Model

Standard Model of Elementary Particle Physics $SU(3)_C \times SU(2)_L \times U(1)_Y$



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+ Neutrino Mass m_ν

Standard Model

add m_ν

Species	$\#$	$\sum$
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$\longrightarrow$

Species	$\#$	$\sum$
Quarks	10	10
Leptons	12 (10)	22 (20)
Charge	3	25 (23)
Higgs	2	27 (25)
strong CP	1	28 (26)

Standard Model*

add m_ν

Species	#	$\sum$
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$\longrightarrow$

Species	#	$\sum$
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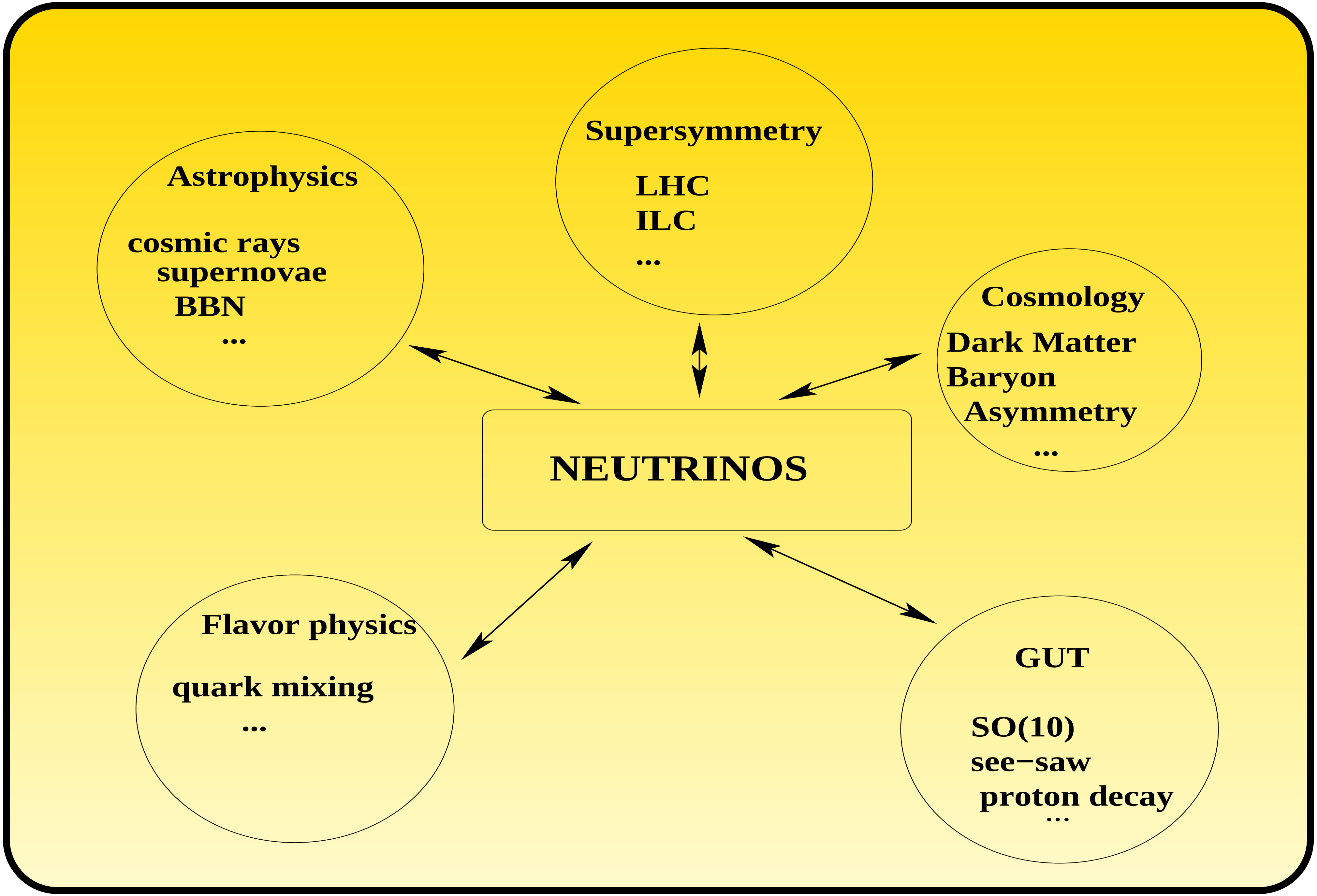
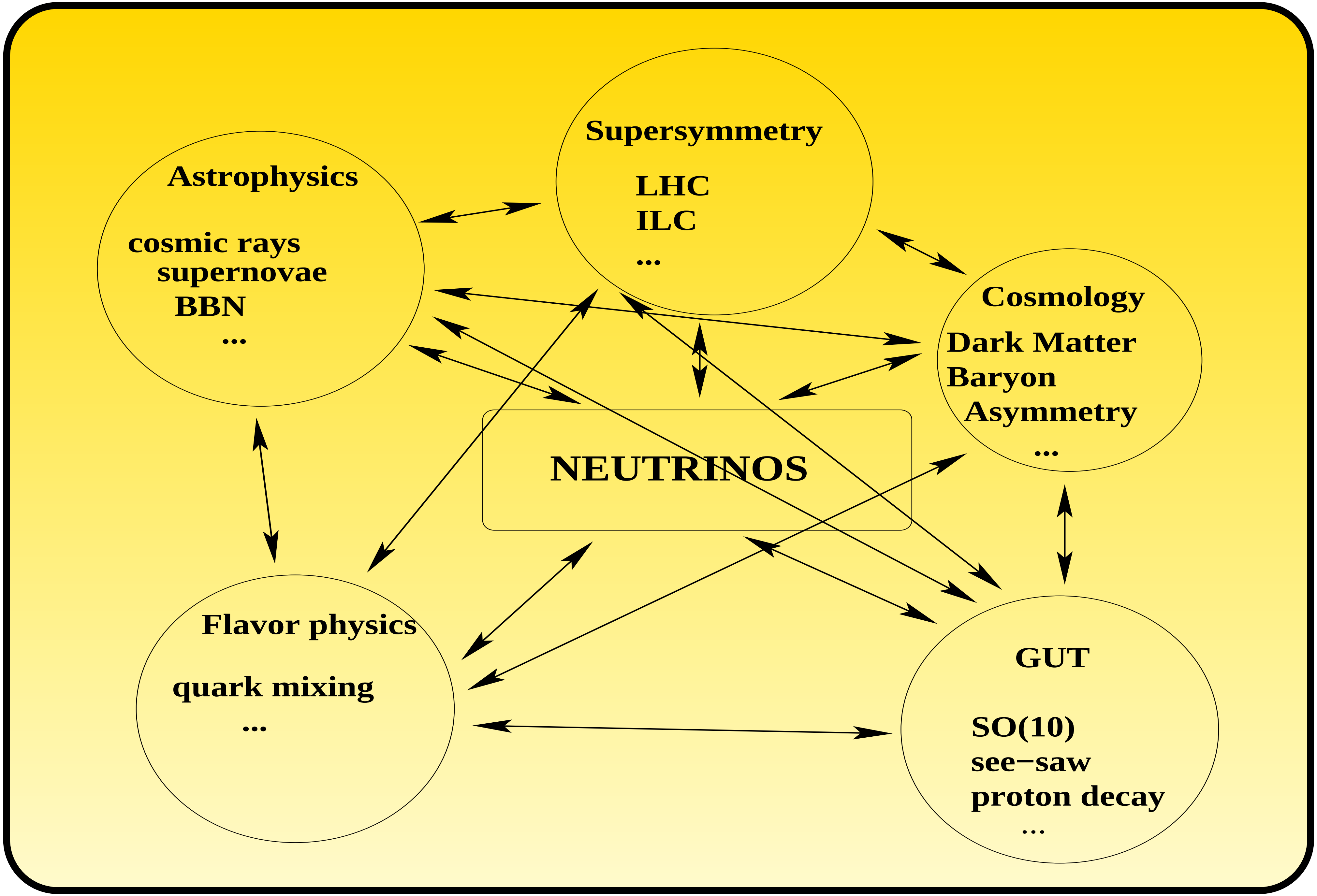
charge	3	25 (23)
Higgs	2	27 (25)
strong CP	1	28 (26)

- **plus new energy scale**
- **plus new representation**
- **plus new concepts**
- **plus**. . .


Supersymmetry
LHC
ILC
...
Astrophysics
cosmic rays
supernovae
BBN
...
Cosmology
Dark Matter
Baryon
Asymmetry
...
NEUTRINOS
Flavor physics
quark mixing
...
GUT
SO(10)
see−saw
proton decay
...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Neutrino parameters at low energy

$$\mathcal{L} = \frac{1}{2}\,\nu^T\,m_\nu\,\nu \quad \text{with} \quad m_\nu = U\,\mathrm{diag}(m_1, m_2, m_3)\,U^T$$

and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

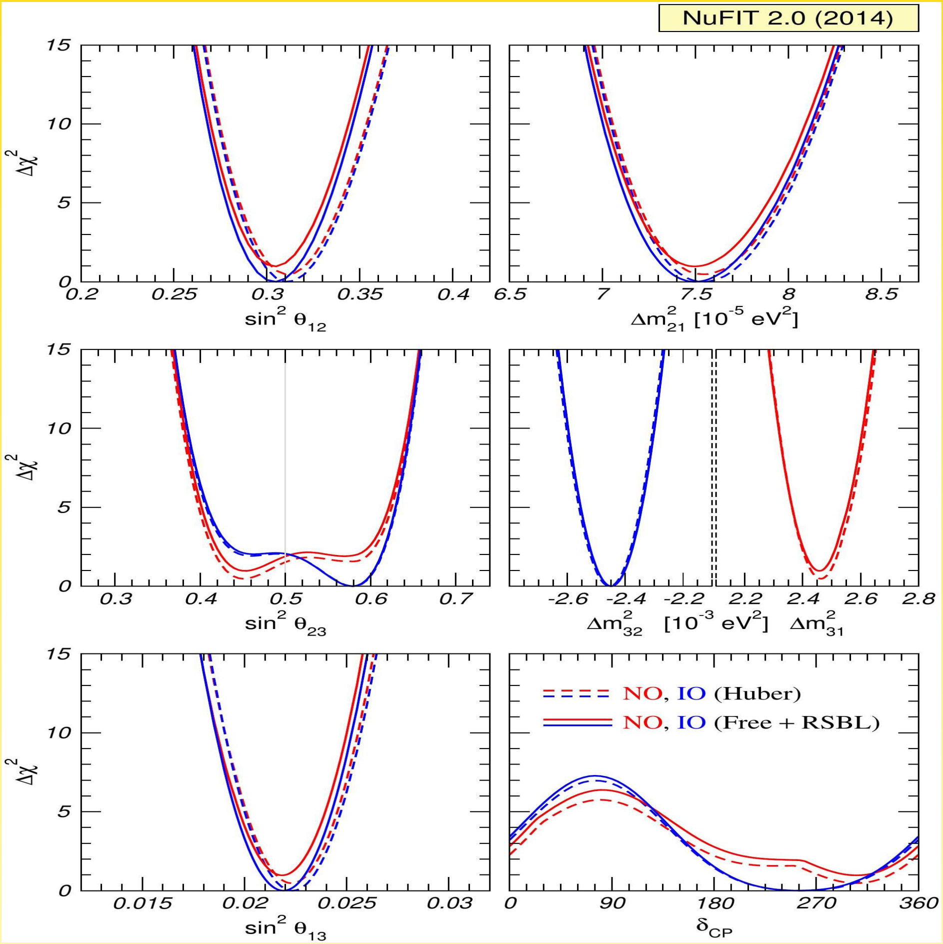
with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$ ($\leftrightarrow$ Majorana, violation of lepton number)

$\Rightarrow$ 3 angles, 3 phases, 3 masses

3 Tasks

- determine parameters
- check if minimal description ("3 Majorana neutrino paradigm") is correct
- explain parameters

Status of global fits



nu-fit.org, September 2014

PMNS matrix is given by

$$|U| = \begin{pmatrix} 0.801\ldots0.845 & 0.514\ldots0.580 & 0.137\ldots0.158 \\ 0.225\ldots0.517 & 0.441\ldots0.699 & 0.614\ldots0.793 \\ 0.246\ldots0.529 & 0.464\ldots0.713 & 0.590\ldots0.776 \end{pmatrix}$$

compare with CKM:

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.225220 \pm 0.000615 & 0.973434 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

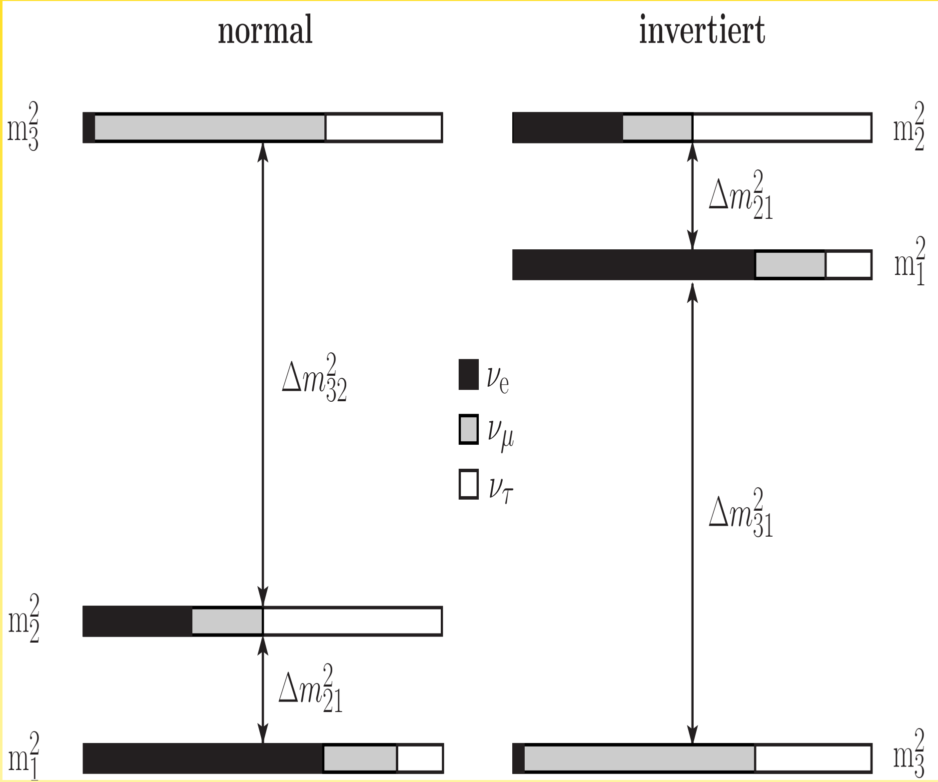
Why so different?? $\leftrightarrow$ **Flavour symmetries?**

Mass scale

$$\begin{pmatrix} u \\ d \end{pmatrix} \text{ with } m_u \simeq m_d \qquad \text{versus} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \text{ with } m_\nu \simeq 10^{-6}\, m_e$$

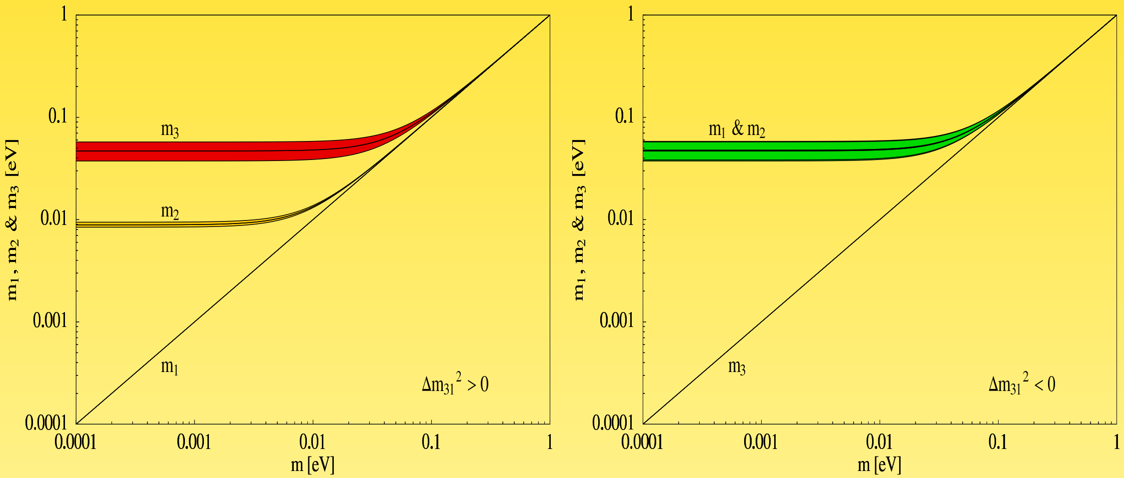
why m_ν tiny? $\Rightarrow$ Seesaw Mechanism

Mass scale and ordering



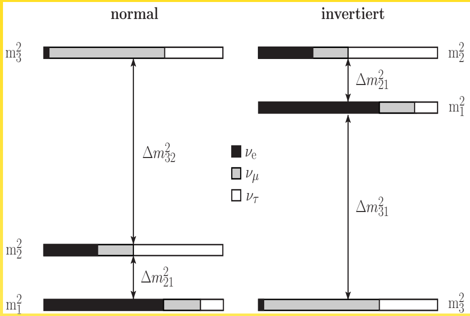
scale and normal/inverted fundamental input for model builders!

Mass scale and ordering



scale and normal/inverted fundamental input for model builders!

Mass scale and ordering



$$(m_\nu)_{\rm NH} \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad \text{versus} \quad (m_\nu)_{\rm IH} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

typical: $m_\nu \propto 1/\Lambda_{\rm NP}$

scale and normal/inverted fundamental input for model builders!

Status 24/06/2015

9 physical parameters in m_ν

- θ_{12} and $m_2^2 - m_1^2$
- θ_{23} and $|m_3^2 - m_2^2|$
- θ_{13}
- m_1, m_2, m_3
- $\mathrm{sgn}(m_3^2 - m_2^2)$
- Dirac phase δ
- Majorana phases α and β

Lessons

- consistent picture emerging
- there are 3 generation effects!
- about $1, 2\sigma$ hint for $\theta_{23} \neq \pi/4$, $\delta \neq 0$
- no hint for mass ordering
- future program of experiments to pin down, make more precise
- all is well...?

Sterile Neutrinos??

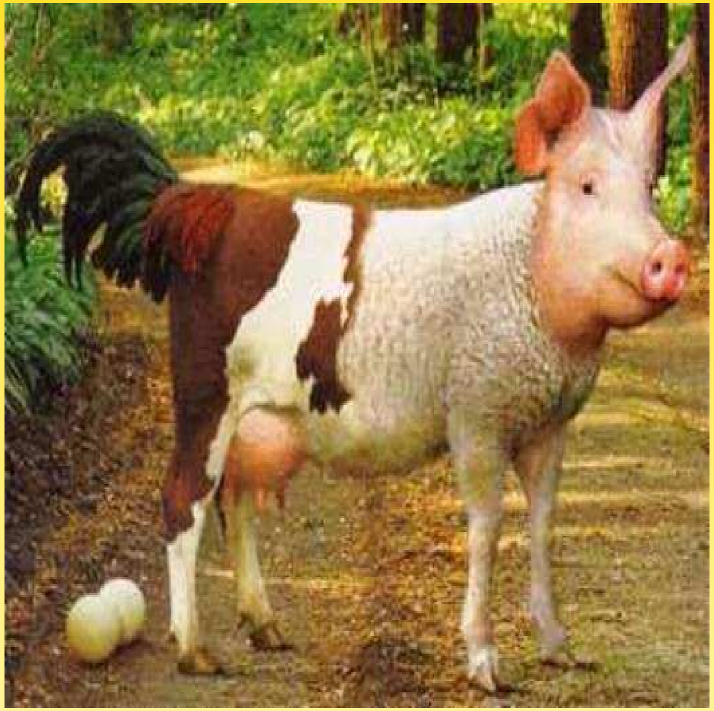
- LSND/MiniBooNE/gallium
- cosmology
- BBN
- r-process nucleosynthesis in Supernovae
- reactor anomaly

	$\Delta m^2_{41}[\mathrm{eV}^2]$	$\lvert U_{e4}\rvert$	$\lvert U_{\mu 4}\rvert$	$\Delta m^2_{51}[\mathrm{eV}^2]$	$\lvert U_{e5}\rvert$	$\lvert U_{\mu 5}\rvert$
3+2/2+3	0.47	0.128	0.165	0.87	0.138	0.148
1+3+1	0.47	0.129	0.154	0.87	0.142	0.163

or $\Delta m^2_{41} = 1.78$ eV^2 and $|U_{e4}|^2 = 0.151$

`Kopp, Maltoni, Schwetz, 1103.4570`

Sterile Neutrinos



- $\ll$ eV: missing upturn of $P_{ee}^{\odot}$
- eV: SBL Anomalies
- eV: $N_{\rm eff}$ (Cosmology, BBN), r-process
- **(eV: BICEP-2 and Planck)**
- keV: Warm Dark Matter
- TeV: Z-width, NuTeV
- 10^{10} GeV: Leptogenesis
- 10^{15} GeV: Seesaw Mechanism

New New Physics

- Sterile Neutrinos...
  - ...can mix with active neutrinos
  - ...can couple to Higgs
  - ...can couple to BSM physics
- good reasons to look for other exotic things...
  - ...long-range forces
  - ...non-standard interactions
  - ...CPT violation
  - ...Lorentz invariance violation
  - ... ...

Mass scale

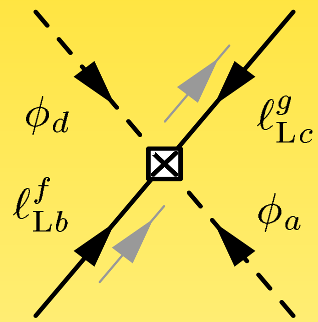
$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \text{with} \quad m_u \simeq m_d \qquad \text{versus} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \text{with} \quad m_\nu \simeq 10^{-6}\, m_e$$

why m_ν tiny? $\Rightarrow$ Seesaw Mechanism

Effective Suppression

There is only 1 dimension-5 operator (Weinberg 1979):

$$\frac{1}{\Lambda}\,\mathcal{O}_5 = \frac{c}{\Lambda}\,\overline{L^c}\,\tilde{\Phi}^*\,\tilde{\Phi}^\dagger\,L \xrightarrow{\text{EWSB}} \frac{c\,v^2}{2\,\Lambda}\,\overline{(\nu_L)^c}\,\nu_L \equiv \frac{1}{2}\,m_\nu\,\overline{(\nu_L)^c}\,\nu_L$$



can be UV-completed by introducing $N_R \sim (1,0)$ with couplings

$$1):\quad Y_\nu\,\bar{L}\,\Phi\,N_R \to m_D\,\bar{\nu}_L\,N_R$$

$$2):\quad M_R\,\bar{N}^c_R\,N_R$$

(one of only three tree-level realizations of Weinberg-operator)

Origin of small neutrino masses

Simplest concept[a] introduces 3 new aspects:

- fermionic singlets $N_R \sim (1,0)$
- new energy scale M_R ($\propto 1/m_\nu$)
- lepton number violation

$m_\nu = m_D^2/M_R = m_{\rm SM}^2/M_R = m_{\rm SM}\,\epsilon$ with $\epsilon = m_{\rm SM}/M_R$

[a] naturally realized in $SO(10)$ models!

Type I Seesaw Formalism

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

(arbitrary...) 6×6 (?) matrix, for $M_R \gg m_D$ diagonalized with

$$\mathcal{U}_\nu \simeq \begin{pmatrix} 1 - \frac{1}{2}BB^\dagger & B \\ -B^\dagger & 1 - \frac{1}{2}B^\dagger B \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V_R \end{pmatrix} \quad \text{with } B = m_D M_R^{-1}$$

light neutrinos:

$$m_\nu = -m_D\, M_R^{-1}\, m_D^T = U\, \mathrm{diag}(m_1, m_2, m_3)\, U^T$$

light-heavy mixing (unitarity violation):

$$\theta \sim \frac{m_D}{M_R} = \sqrt{\frac{m_\nu}{M_R}} \simeq 10^{-7} \left(\frac{\mathrm{TeV}}{M_R}\right)^{-1/2}$$

BUT: these are matrices, can be increased up to limits of $\mathcal{O}(10^{-2})$

Formalism: N^{n}LO-Terms

$$\tilde{m}_\nu = -m_D^T\, M_R^{-1}\, m_D + \frac{1}{2}\, m_D^T\, M_R^{-1}\, X\, M_R^{-1}\, m_D$$

$$\tilde{M}_R = M_R + \frac{1}{2}\, X$$

with $X \equiv A + A^T$, where $A \equiv m_D\, m_D^\dagger\, (M_R^*)^{-1}$ and NNLO terms

$$\tilde{m}_\nu^{\rm NNLO} = \frac{1}{2} m_D^T M_R^{-1} \left[\frac{1}{4} A M_R^{-1} A + \frac{1}{4} A^T M_R^{-1} A^T + \frac{1}{2} A^T M_R^{-1} A + \frac{1}{2} (M_R^*)^{-1} A^* A^T \right.$$
$$\left. + \frac{1}{2} A A^\dagger (M_R^*)^{-1} + A A^* (M_R^*)^{-1} + (M_R^*)^{-1} A^\dagger A^T \right] M_R^{-1} m_D$$

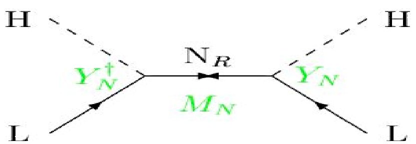
$$\tilde{M}_R^{\rm NNLO} = -\frac{1}{2} \left[A A^* (M_R^*)^{-1} + (M_R^*)^{-1} A^\dagger A^T + \frac{1}{4} A M_R^{-1} A + \frac{1}{4} A^T M_R^{-1} A^T \right]$$

`Hettmansperger, Lindner, W.R., JHEP` **`1104`**

The 3 basic seesaw models

i.e. tree level ways to generate the dim 5 operator

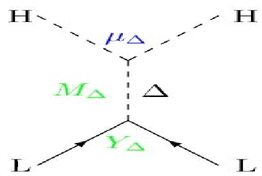
Right-handed singlet: (type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

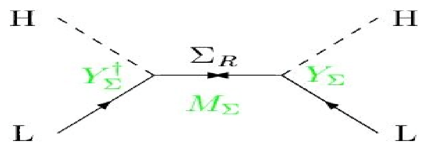
Scalar triplet: (type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet: (type-III seesaw)



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

slide by T. Hambye

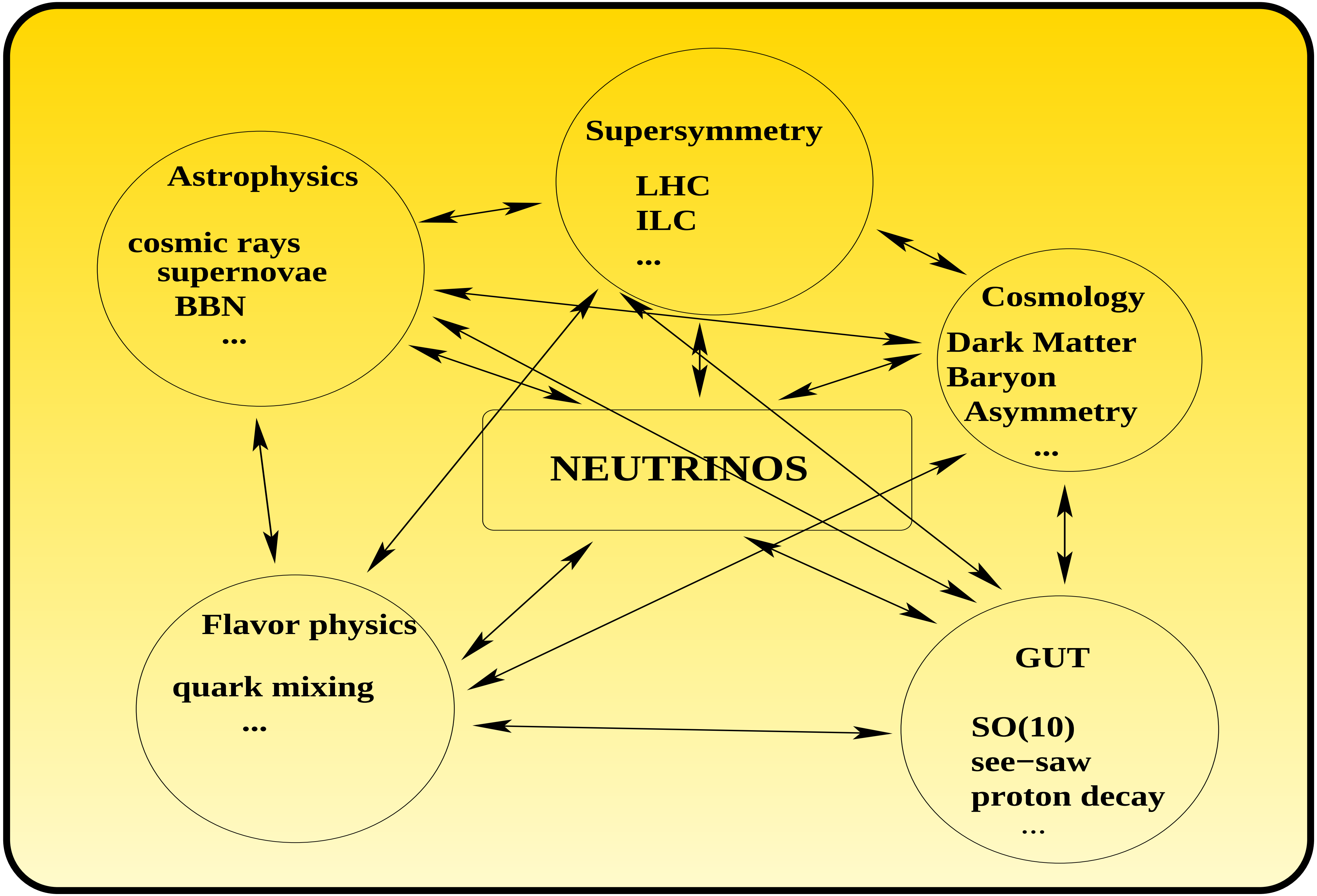
Tests of Seesaw?

naively, $M_R = m_D^2/m_\nu = v^2/\sqrt{\Delta m_{\rm A}^2} \sim 10^{15}$ GeV...

Seesaw portal: $\bar{L}\,\Phi\,N_R \longrightarrow m_D$:

Vertex of L, Φ, N_R!

- $N_R \to L\,\Phi$: leptogenesis
- $L_\alpha \to N_R\,\Phi \to L_\beta$: lepton flavor violation
- $\Phi \to N_R\,L \to \Phi$: vacuum stability, naturalness



Astrophysics
cosmic rays
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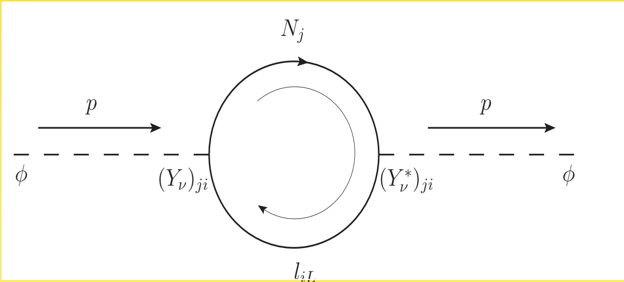
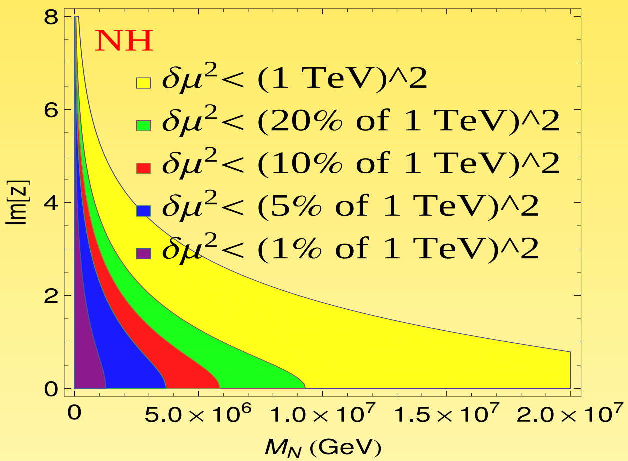
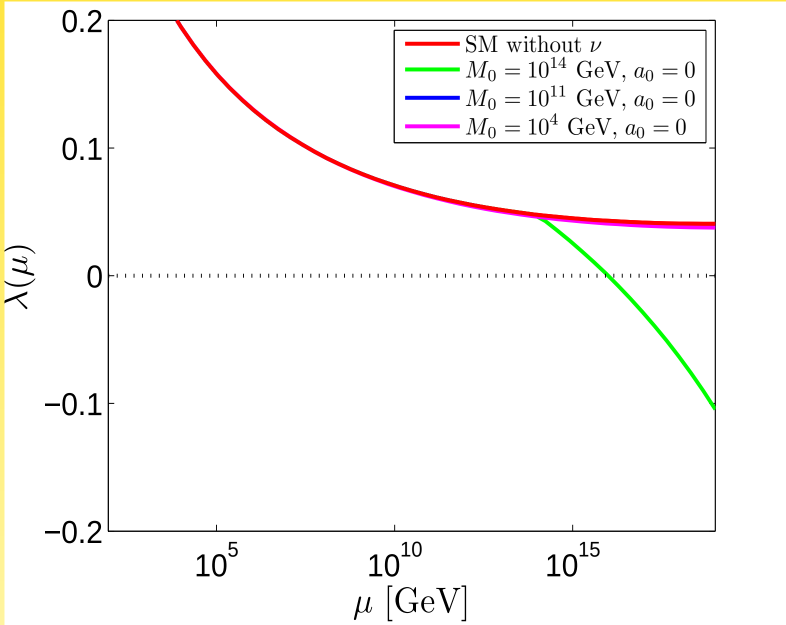
Higgs and Seesaw Mechanism

Naturallness:





Vacuum-stability:



Paths to Neutrino Mass

approach	ingredient	quantum number of messenger	$\mathcal{L}$	m_ν	scale
"SM" (Dirac mass)	RH ν	$N_R \sim (1,0)$	$h\overline{N_R}\Phi L$	hv	$h = \mathcal{O}(10^{-12})$
"effective" (dim 5 operator)	new scale + LNV	–	$h\,\overline{L^c}\,\Phi\,\Phi\,L$	$\frac{h\,v^2}{\Lambda}$	$\Lambda = 10^{14}$ GeV
"direct" (type II seesaw)	Higgs triplet + LNV	$\Delta \sim (3,-2)$	$h\overline{L^c}\Delta L + \mu\Phi\Phi\Delta$	hv_T	$\Lambda = \frac{1}{h\mu}M_\Delta^2$
"indirect 1" (type I seesaw)	RH ν + LNV	$N_R \sim (1,0)$	$h\overline{N_R}\Phi L + \overline{N_R}M_R N_R^c$	$\frac{(hv)^2}{M_R}$	$\Lambda = \frac{1}{h}M_R$
"indirect 2" (type III seesaw)	fermion triplets + LNV	$\Sigma \sim (3,0)$	$h\overline{\Sigma}\,L\Phi + \mathrm{Tr}\overline{\Sigma}M_\Sigma\Sigma$	$\frac{(hv)^2}{M_\Sigma}$	$\Lambda = \frac{1}{h}M_\Sigma$

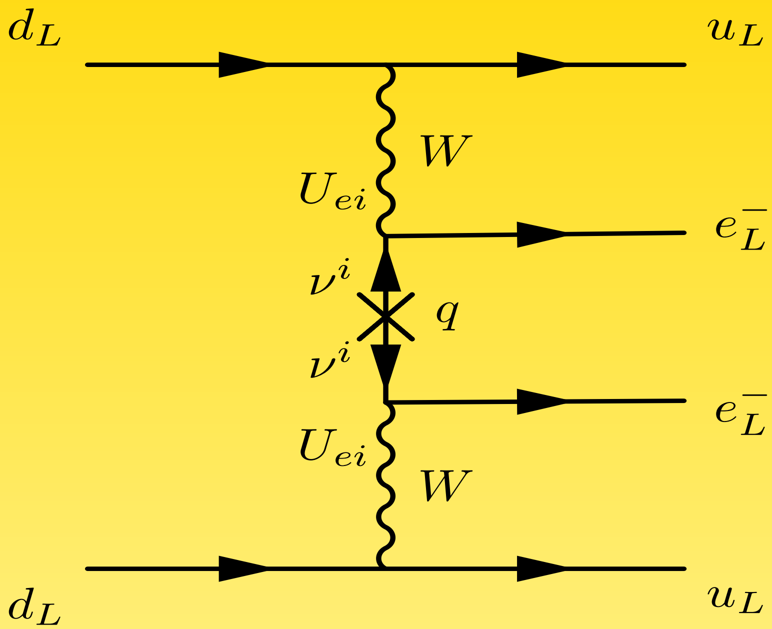
plus seesaw variants (linear, double, inverse,. . .)

plus radiative mechanisms

plus extra dimensions

plusplusplus

Consequence of almost all mechanisms: lepton number violation



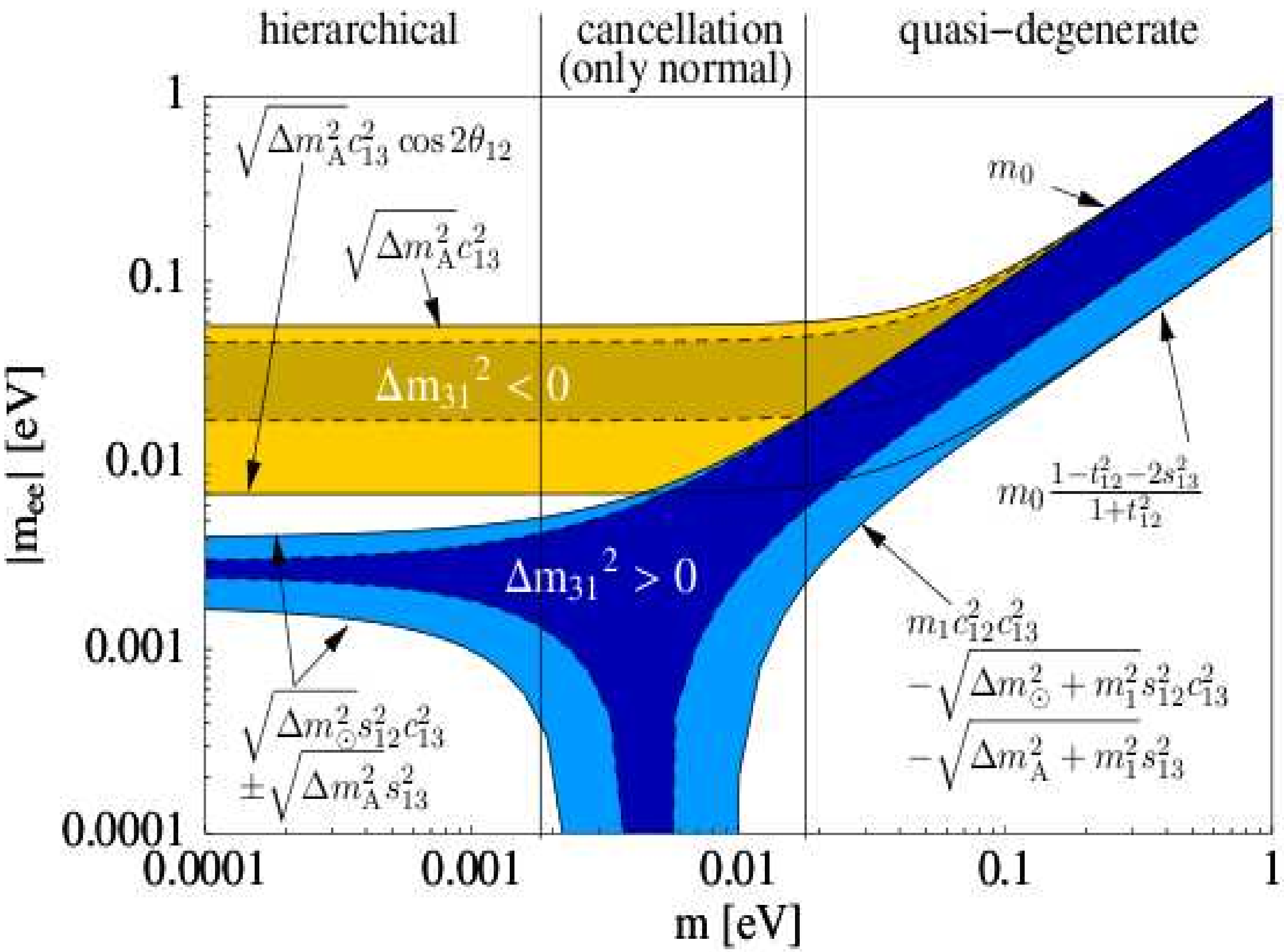
Neutrinoless double beta decay $(A, Z) \to (A, Z+2) + 2\,e^-$

$$|m_{ee}| \equiv \left|\sum U_{ei}^2\, m_i\right| = \left||U_{e1}|^2\, m_1 + |U_{e2}|^2\, m_2\, e^{2i\alpha} + |U_{e3}|^2\, m_3\, e^{2i\beta}\right|$$

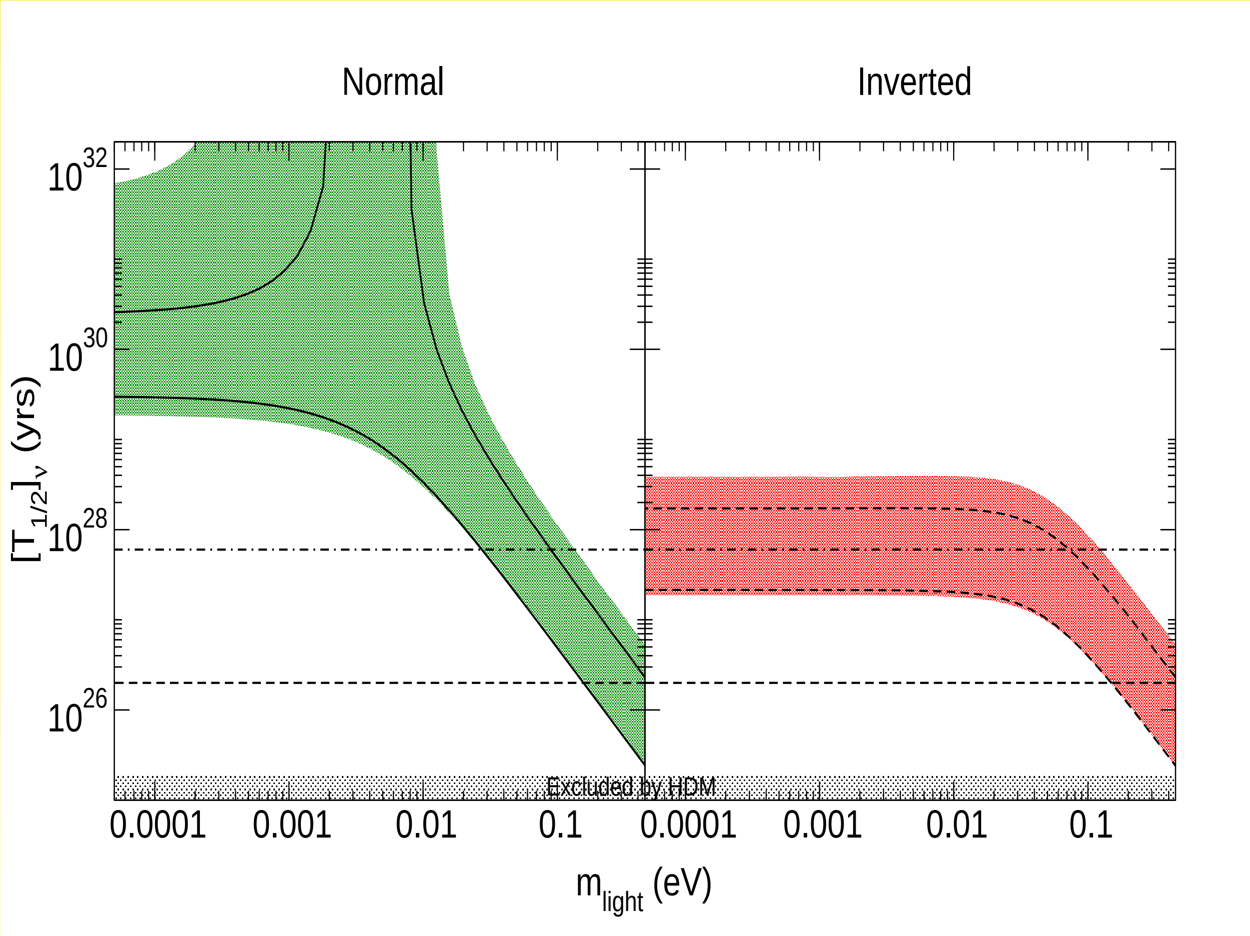
$$= f\left(\theta_{12}, |U_{e3}|, m_i, \mathrm{sgn}(\Delta m^2_{\mathrm{A}}), \alpha, \beta\right)$$

7 of 9 parameters!

Usual plot. . .


hierarchical
cancellation (only normal)
quasi−degenerate
$\sqrt{\Delta m_A^2 c_{13}^2}\cos 2\theta_{12}$
$\sqrt{\Delta m_A^2 c_{13}^2}$
m_0
$\Delta m_{31}{}^2 < 0$
$\Delta m_{31}{}^2 > 0$
$m_0 \frac{1-t_{12}^2-2s_{13}^2}{1+t_{12}^2}$
$m_1 c_{12}^2 c_{13}^2 - \sqrt{\Delta m_\odot^2 + m_1^2 s_{12}^2 c_{13}^2} - \sqrt{\Delta m_A^2 + m_1^2 s_{13}^2}$
$\sqrt{\Delta m_\odot^2 s_{12}^2 c_{13}^2} \pm \sqrt{\Delta m_A^2 s_{13}^2}$
|m_ee| [eV]
m [eV]
1
0.1
0.01
0.001
0.0001
0.0001
0.001
0.01
0.1
1

...Lifetime vs. smallest mass



Exciting times

HDM-limit from 2001 improved in 2012/13...

Name	Isotope	source = detektor; kalorimetric with ΔE high	ΔE low	topology	source $\neq$ detector topology
AMoRE	^{100}Mo	✓	–	–	–
CANDLES	^{48}Ca	–	✓	–	–
COBRA	^{116}Cd (und ^{130}Te)	–	–	✓	–
CUORE	^{130}Te	✓	–	–	–
CUPID	^{82}Se / ^{100}Mo / ^{116}Cd / ^{130}Te	✓	–	–	–
DCBA/MTD	^{82}Se / ^{150}Nd	–	–	–	✓
EXO	^{136}Xe	–	–	✓	–
GERDA	^{76}Ge	✓	–	–	–
KamLAND-Zen	^{136}Xe	–	✓	–	–
LUCIFER	^{82}Se / ^{100}Mo / ^{130}Te	✓	–	–	–
LUMINEU	^{100}Mo	✓	–	–	–
MAJORANA	^{76}Ge	✓	–	–	–
MOON	^{82}Se / ^{100}Mo / ^{150}Nd	–	–	–	✓
NEXT	^{136}Xe	–	–	✓	–
SNO+	^{130}Te	–	✓	–	–
SuperNEMO	^{82}Se / ^{150}Nd	–	–	–	✓
XMASS	^{136}Xe	–	✓	–	–

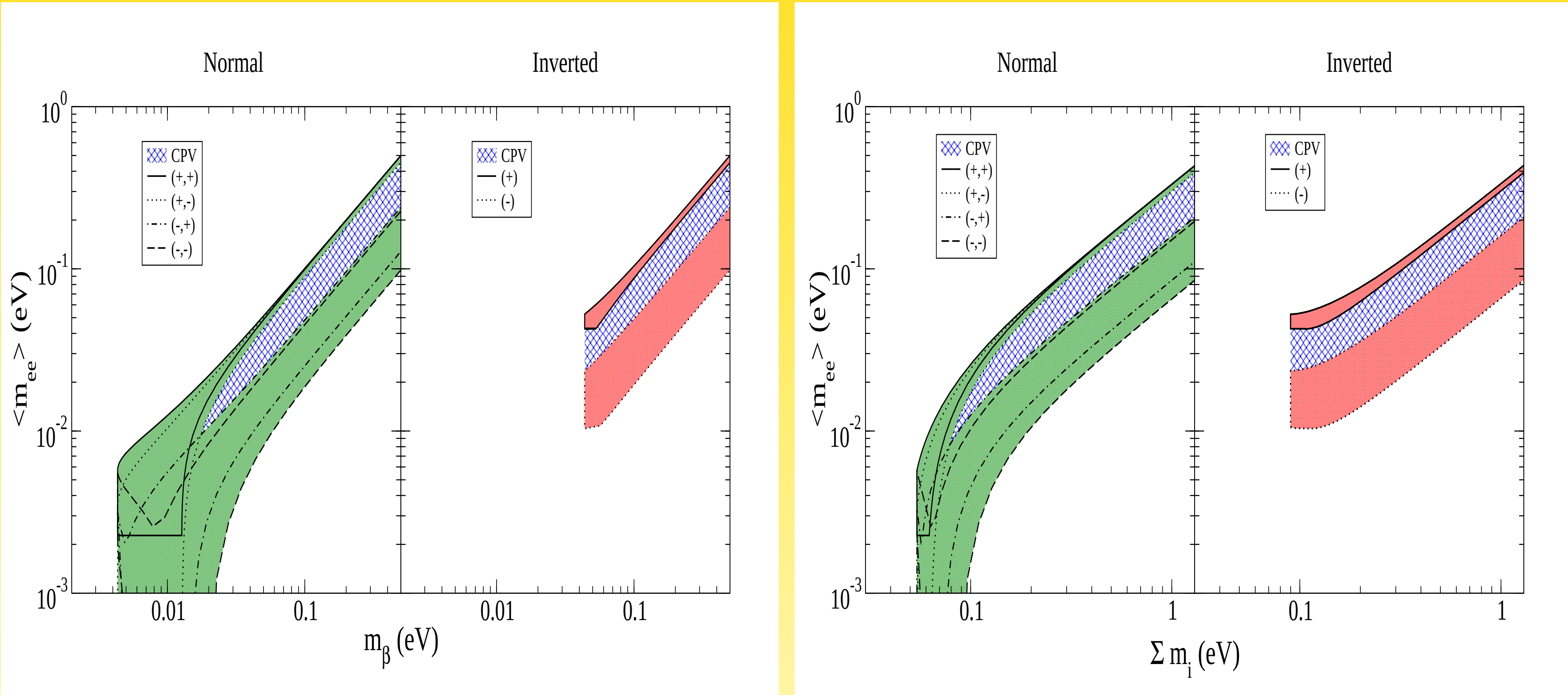
Neutrino mass

$$m(\text{heaviest}) \gtrsim \sqrt{|m_3^2 - m_1^2|} \simeq 0.05 \text{ eV}$$

3 **complementary** methods to measure:

Method	Observable	cur. [eV]	near/far [eV]	pro	con
Kurie	$\sqrt{\sum \|U_{ei}\|^2 m_i^2}$	2.3	$0.2/0.1$	model-indep.; theo. clean	final?; weakest
Cosmo.	$\sum m_i$	0.7	$0.3/0.05$	best; NH/IH	systemat.; model-dep.
$0\nu\beta\beta$	$\|\sum U_{ei}^2 m_i\|$	0.3	$0.1/0.05$	fundament.; NH/IH	model-dep.; theo. dirty

Complementarity!



$$|m_{ee}| = |U_{ei}^2 \, m_i| \;,\; m_\beta = \sqrt{|U_{ei}|^2 \, m_i^2} \text{ and } \Sigma = \sum m_i$$

Sterile Neutrinos and $0\nu\beta\beta$

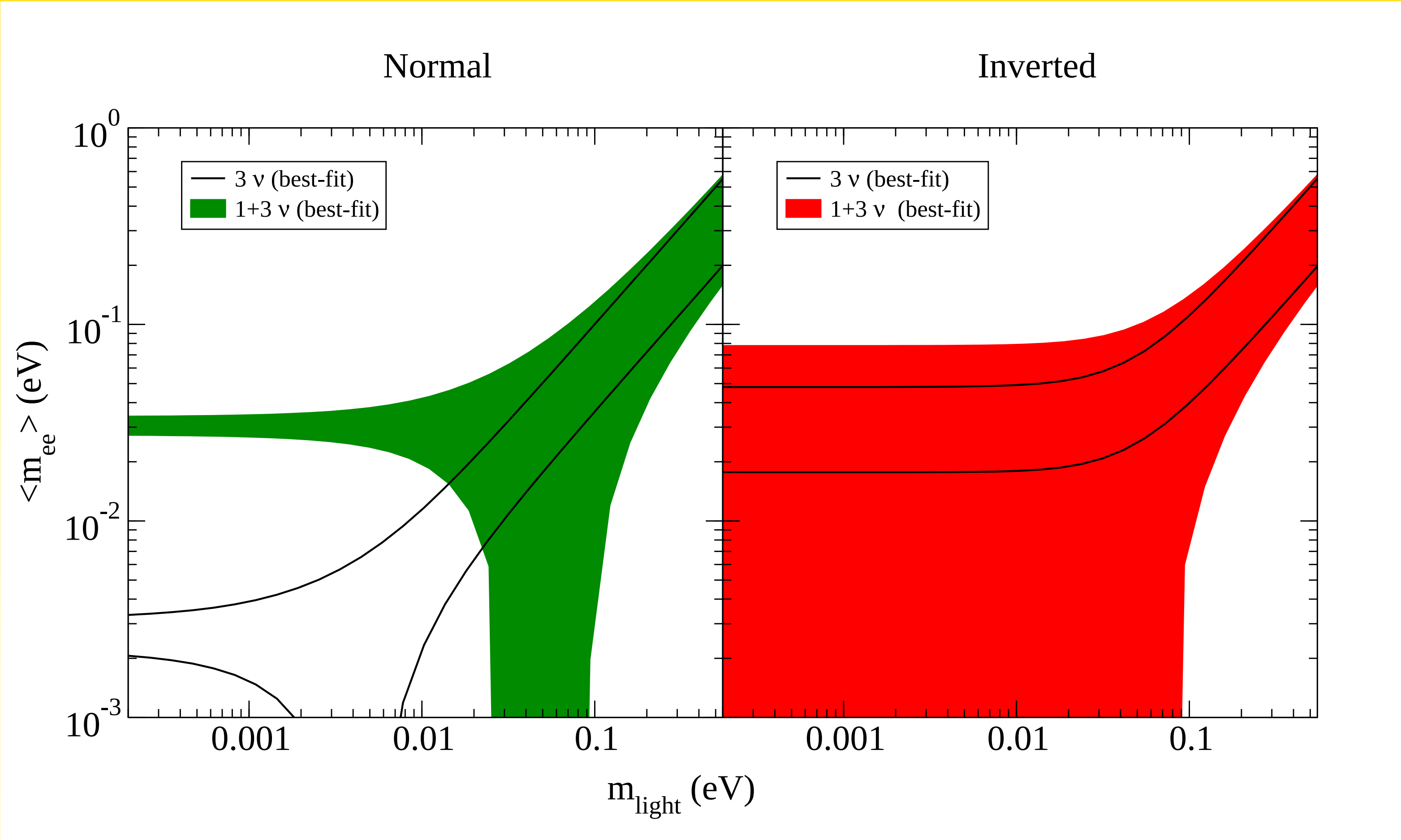
- recall: $|m_{ee}|^{\rm act}_{\rm NH}$ can vanish and $|m_{ee}|^{\rm act}_{\rm IH} \sim 0.03$ eV cannot vanish
- $|m_{ee}| = |\underbrace{|U_{e1}|^2\, m_1 + |U_{e2}|^2\, m_2\, e^{2i\alpha} + |U_{e3}^2|\, m_3\, e^{2i\beta}}_{m_{ee}^{\rm act}} + \underbrace{|U_{e4}|^2\, m_4\, e^{2i\Phi_1}}_{m_{ee}^{\rm st}}|$
- sterile contribution to $0\nu\beta\beta$ (assuming 1+3):

$$|m_{ee}|^{\rm st} \simeq \sqrt{\Delta m^2_{\rm st}}\, |U_{e4}|^2 \simeq 0.03 \text{ eV} \left\{ \begin{array}{c} \gg |m_{ee}|^{\rm act}_{\rm NH} \\ \simeq |m_{ee}|^{\rm act}_{\rm IH} \end{array} \right.$$

- $\Rightarrow$ $|m_{ee}|_{\rm NH}$ cannot vanish and $|m_{ee}|_{\rm IH}$ can vanish!

usual phenomenology gets completely turned around!

Light Steriles: Usual plot gets completely turned around!



Interpretation of Neutrinoless Double Beta Decay

- **Standard Interpretation:**

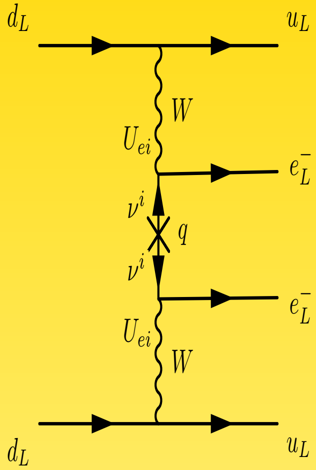
  Neutrinoless Double Beta Decay is mediated by light and massive Majorana neutrinos (the ones which oscillate) and all other mechanisms potentially leading to $0\nu\beta\beta$ give negligible or no contribution

- **Non-Standard Interpretations:**

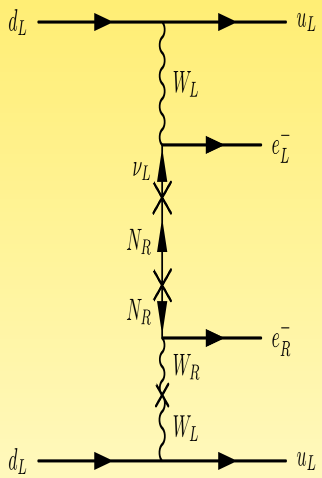
  There is at least one other mechanism leading to Neutrinoless Double Beta Decay and its contribution is at least of the same order as the light neutrino exchange mechanism

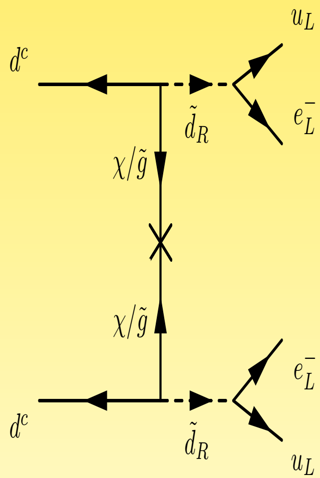
`W.R., Int. J. Mod. Phys. E20, 1833-1930 (2011)`

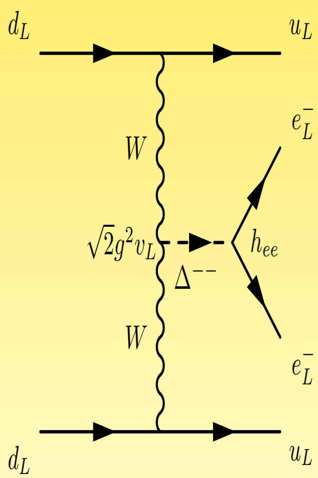
- **Standard Interpretation:**

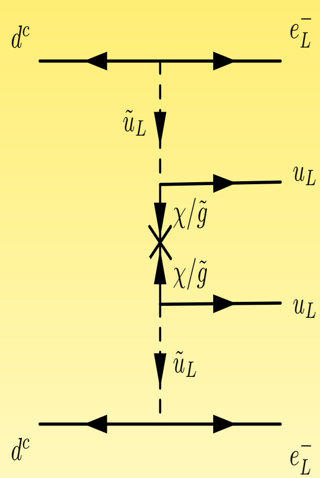


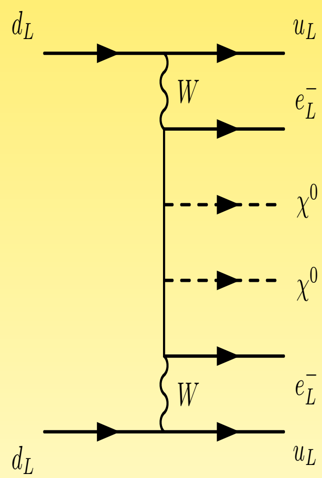
- **Non-Standard Interpretations:**











Why should we expect Lepton Number Violation?

- L and B *accidentally* conserved in SM
  - L can be made global symmetry...
  - L can be made local symmetry...
- $(100-\epsilon)$ % of all models BSM violate L by 1 or 2 units
- effective theory: $\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\,\mathcal{L}_{\rm LNV} + \frac{1}{\Lambda^2}\,\mathcal{L}_{\rm LFV,\,BNV,\,LNV} + \ldots$
- baryogenesis: B is violated
- B, L often connected in GUTs
- GUTs have seesaw and Majorana neutrinos

$\Rightarrow$ Lepton Number Violation as important as Baryon Number Violation

$0\nu\beta\beta$ is **NOT** a neutrino mass experiment!!

Energy Scale:

Note: *standard amplitude* for light Majorana neutrino exchange:

$$\mathcal{A}_{\rm l} \simeq G_F^2 \, \frac{|m_{ee}|}{q^2} \simeq 7 \times 10^{-18} \left(\frac{|m_{ee}|}{0.5 \text{ eV}} \right) \text{GeV}^{-5} \simeq 2.7 \text{ TeV}^{-5}$$

if new heavy particles are exchanged:

$$\mathcal{A}_{\rm h} \simeq \frac{c}{M^5}$$

$\Rightarrow$ for 0$\nu\beta\beta$ holds:

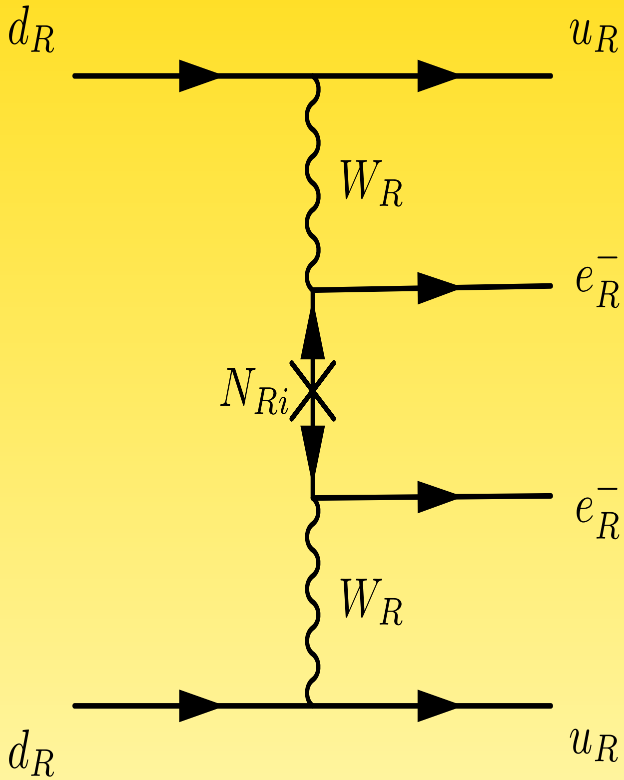
$$\boxed{1 \text{ eV} = 1 \text{ TeV}}$$

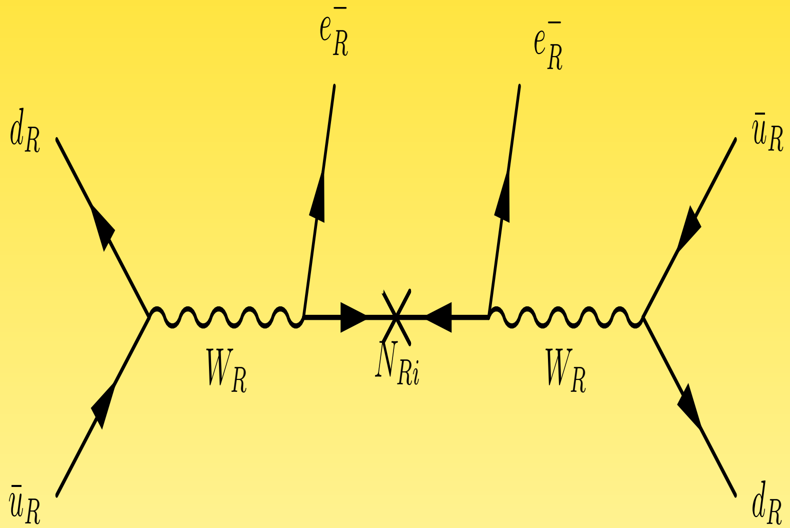
$\Rightarrow$ Phenomenology in colliders, LFV

$$T^{0\nu}_{1/2}(m_\nu = 1\,\mathrm{eV}) = T^{0\nu}_{1/2}(M = 1\,\mathrm{TeV})$$

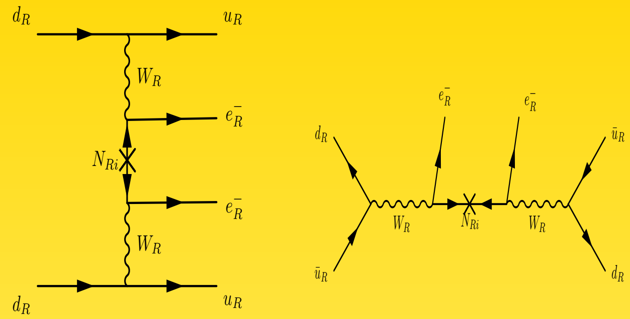
- RPV SUSY
- left-right symmetric theories
- heavy neutrinos
- Color-octets
- Leptoquarks
- effective operators
- extra dimensions
- ...

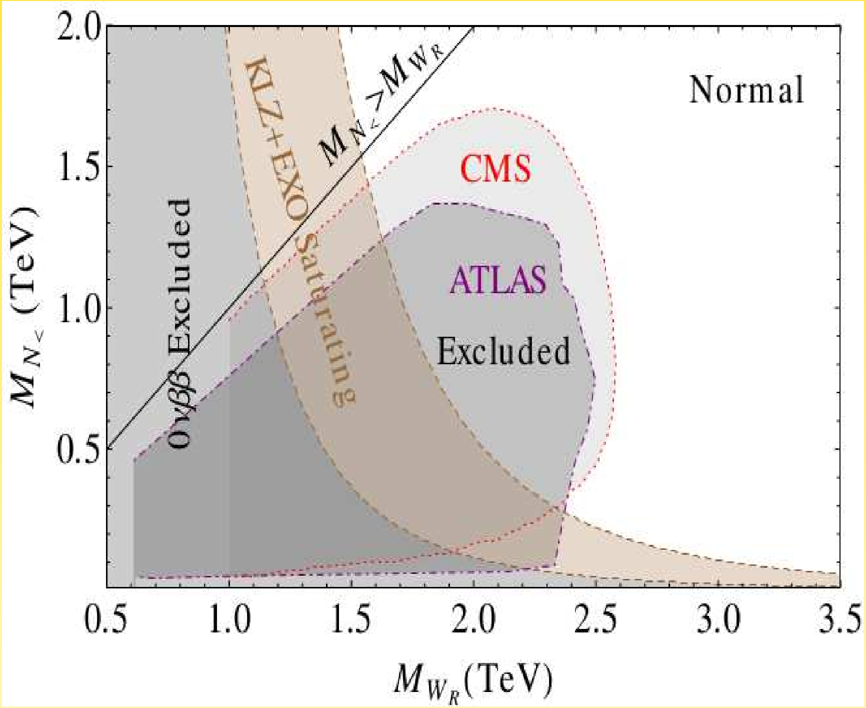
Left-right symmetry





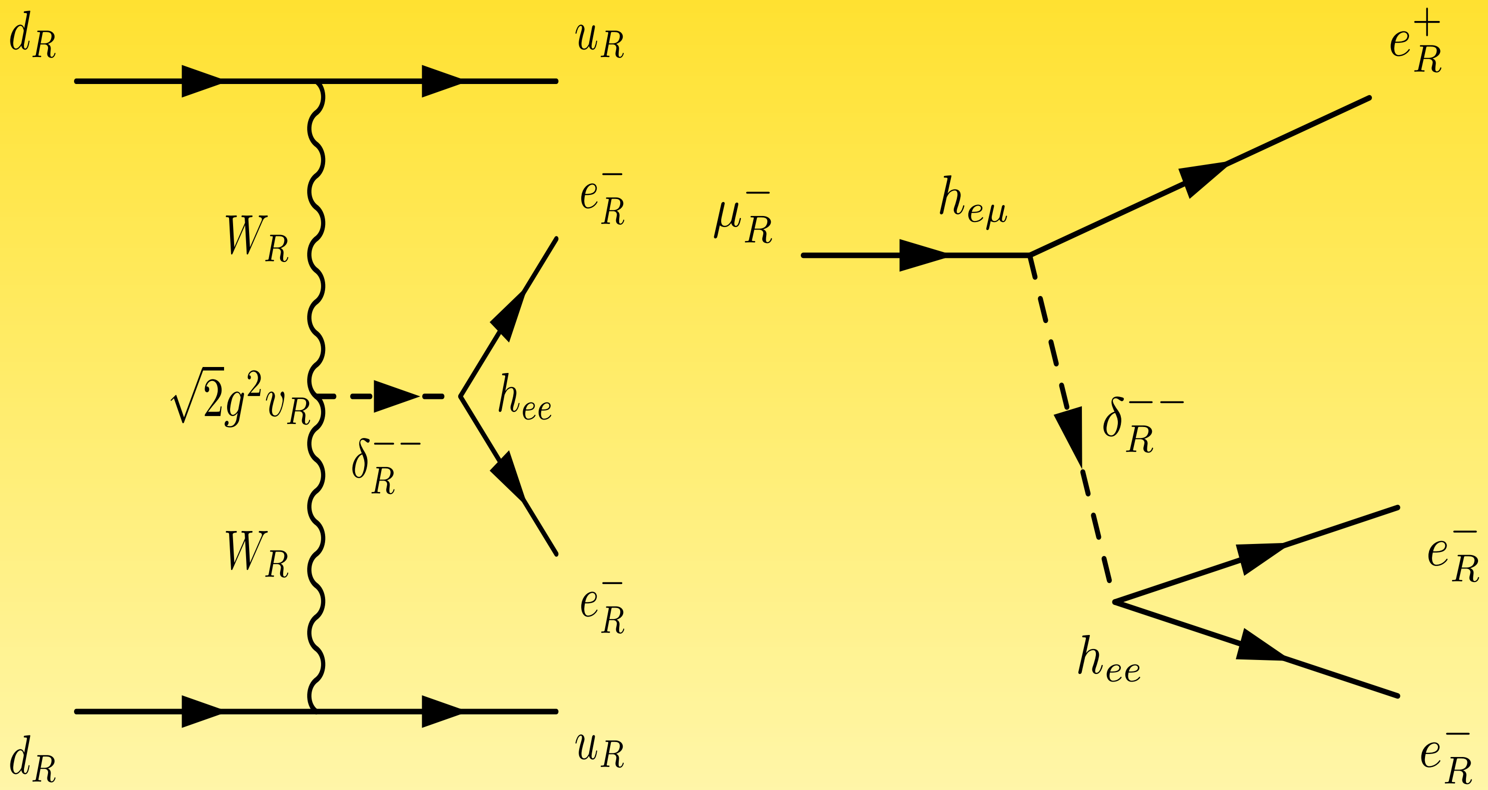
`Senjanovic, Keung, 1983`



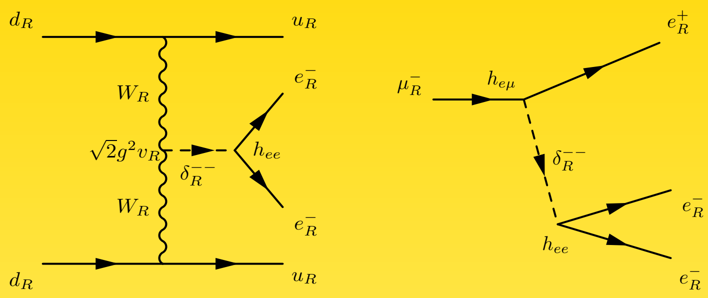


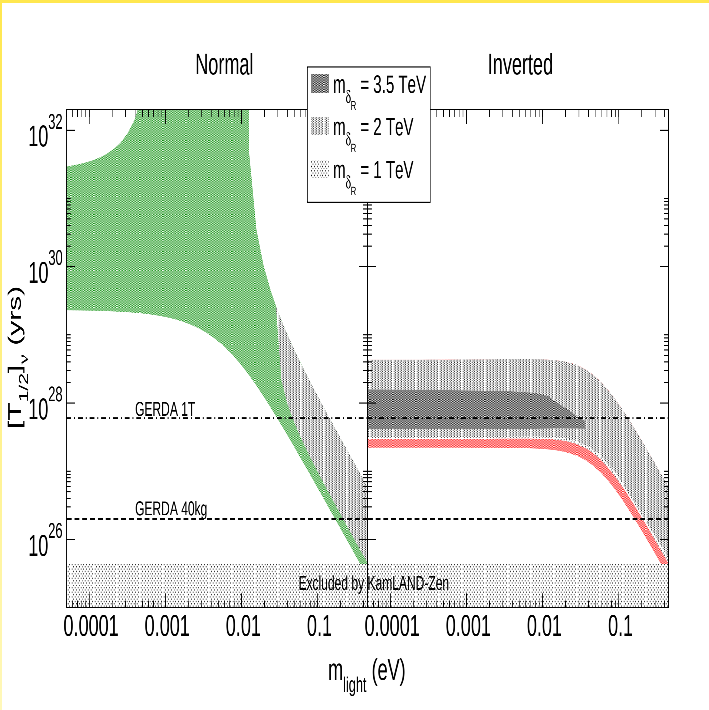
Bhupal Dev, Goswami, Mitra, W.R.; Hirsch *et al.*, Senjanovic *et al.*

Constraints from Lepton Flavor Violation



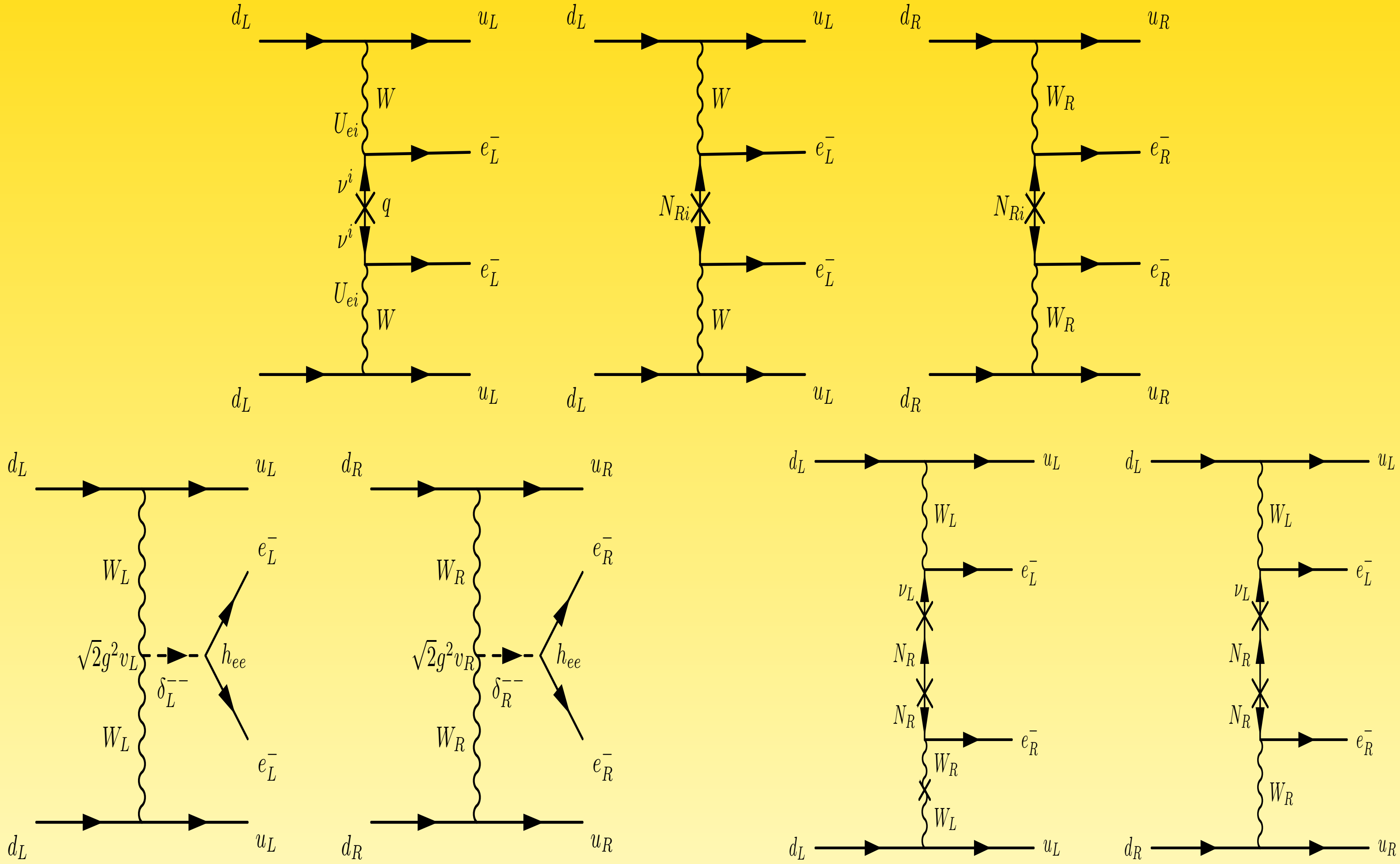
Constraints from Lepton Flavor Violation



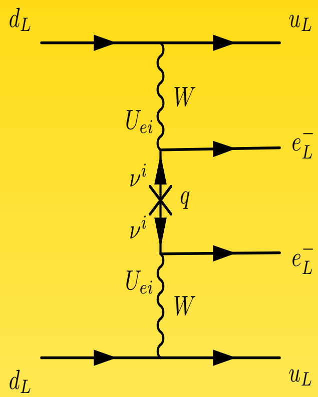


Barry, W.R., JHEP **1309**

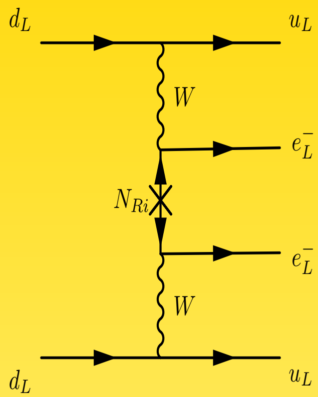
Left-right symmetry



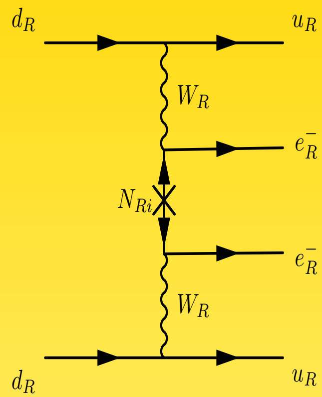
Left-right symmetry



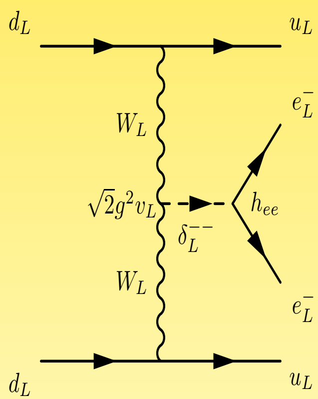
$$U_{ei}^2\, m_i$$



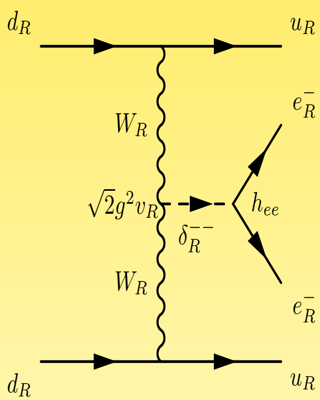
$$\frac{S_{ei}^2}{M_i}$$



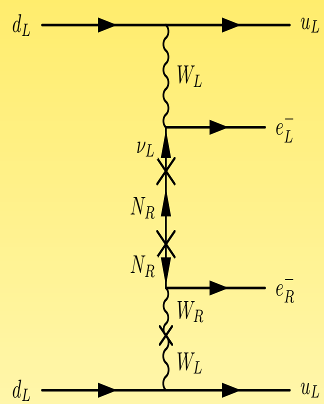
$$\frac{V_{ei}^2}{M_{W_R}^4\, M_i}$$



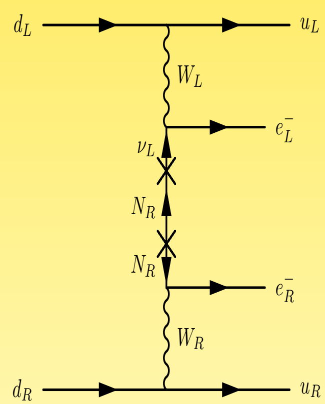
$$\frac{U_{ei}^2\, m_i}{M_{\Delta_L}^2}$$



$$\frac{V_{ei}^2\, M_i}{M_{W_R}^4\, M_{\Delta_R}^2}$$

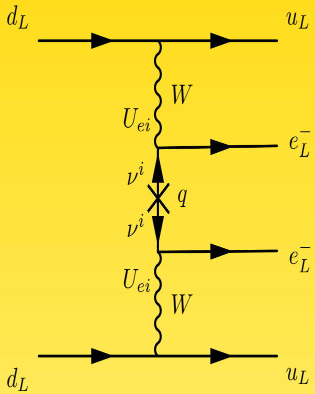


$$U_{ei}\, T_{ei} \tan\zeta$$

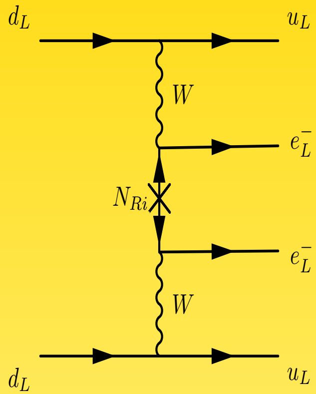


$$\frac{U_{ei}\, T_{ei}}{M_{W_R}^2}$$

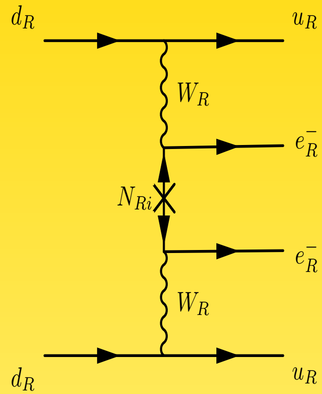
Left-right symmetry



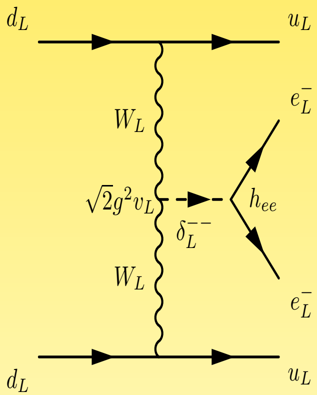
0.4 eV



2×10^{-8} GeV^{-1}



4×10^{-16} GeV^{-5}



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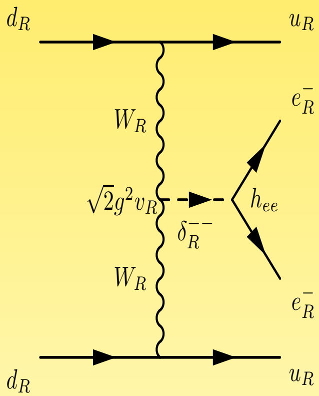
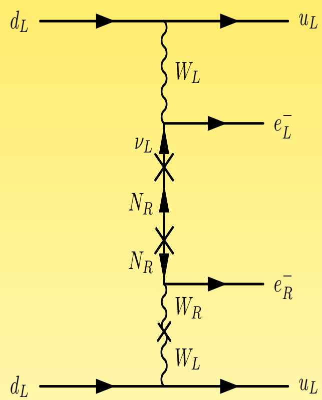
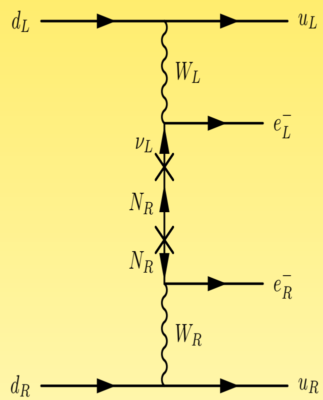


10^{-15} GeV^{-5}

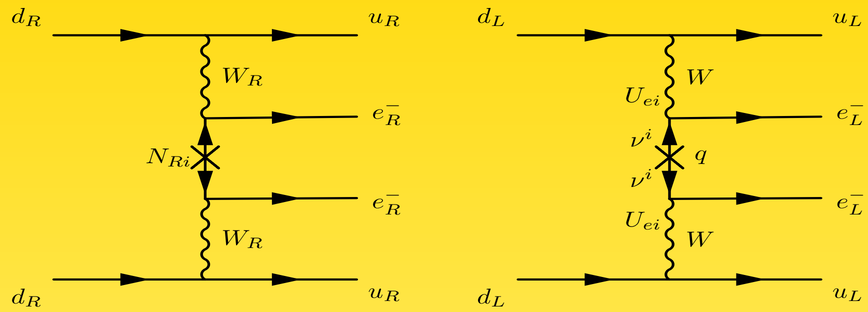


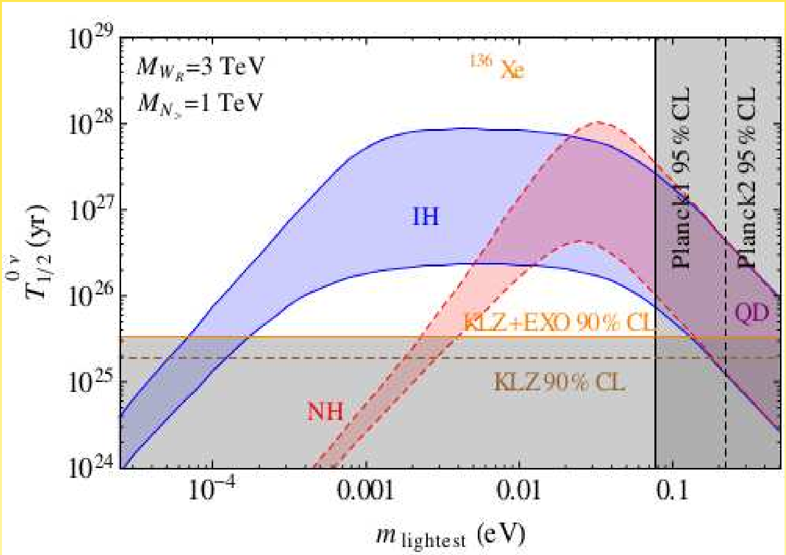
6×10^{-9}



1.4×10^{-10} GeV^{-2}

Adding diagrams





$\Rightarrow$ lower bound on $m(\text{lightest}) \gtrsim$ meV

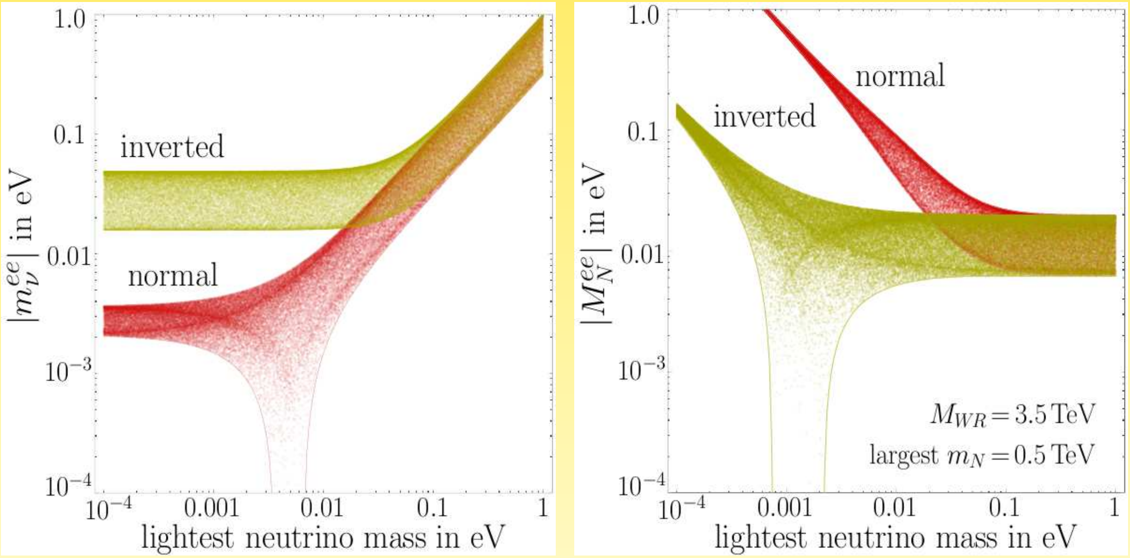
Bhupal Dev, Goswami, Mitra, W.R., PR**D88**

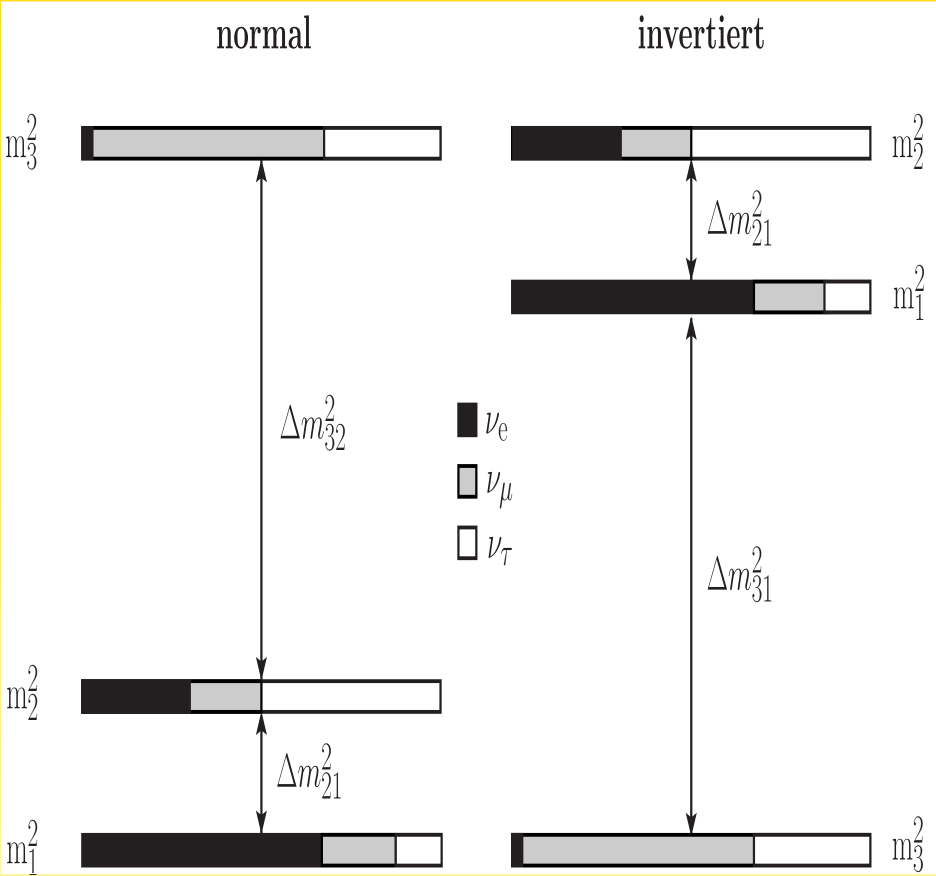
Type II dominance (`Tello` *et al.*, `1011.3522`)

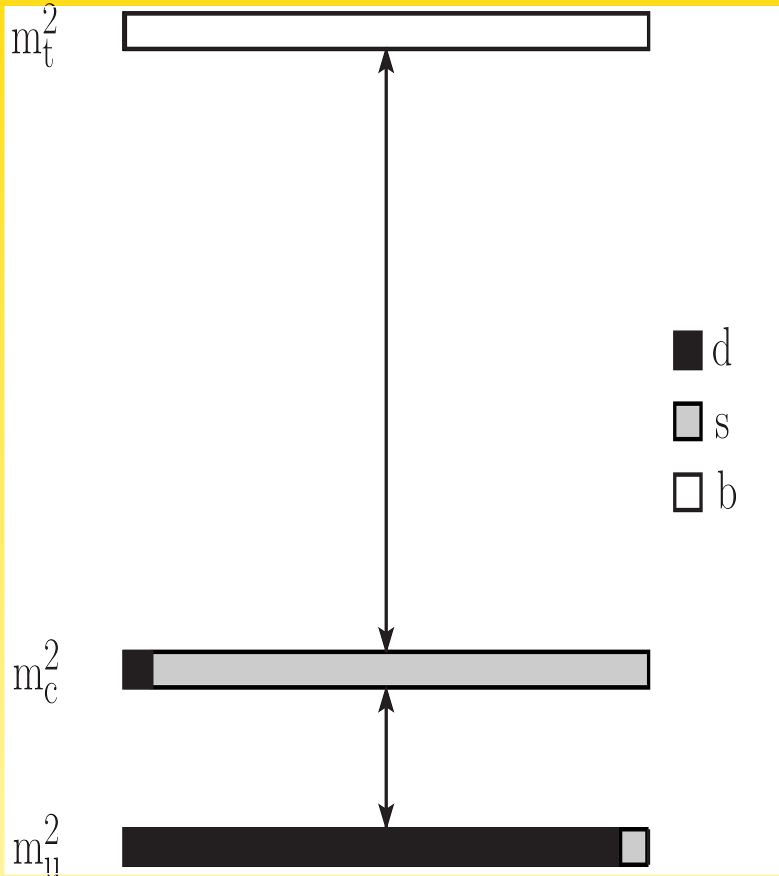
$$m_\nu = m_L - m_D\, M_R^{-1}\, m_D^T = v_L\, f - \frac{v^2}{v_R}\, Y_D\, f^{-1}\, Y_D^T \longrightarrow v_L\, f$$

$\Rightarrow m_\nu$ fixes M_R and exchange of N_R with W_R fixed in terms of PMNS:

$$\Rightarrow \mathcal{A}_{N_R} \simeq G_F^2 \left(\frac{m_W}{M_{W_R}}\right)^4 \sum \frac{V_{ei}^2}{M_i} \propto \sum \frac{U_{ei}^2}{m_i}$$







Why so different?? $\leftrightarrow$ Flavour symmetries?

Tri-bimaximal Mixing

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

`Harrison, Perkins, Scott (2002)`

with mass matrix

$$(m_\nu)_{\rm TBM} = U_{\rm TBM}\, m_\nu^{\rm diag}\, U_{\rm TBM}^T = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

$$A = \frac{1}{3}\left(2\, m_1 + m_2\, e^{-2i\alpha}\right) , \quad B = \frac{1}{3}\left(m_2\, e^{-2i\alpha} - m_1\right) , \quad D = m_3\, e^{-2i\beta}$$

$\Rightarrow$ Flavor symmetries...

Flavor symmetry issues

- discrete or continuous?
- Abelian or non-Abelian?
- broken at high scale?
- broken at electroweak scale?

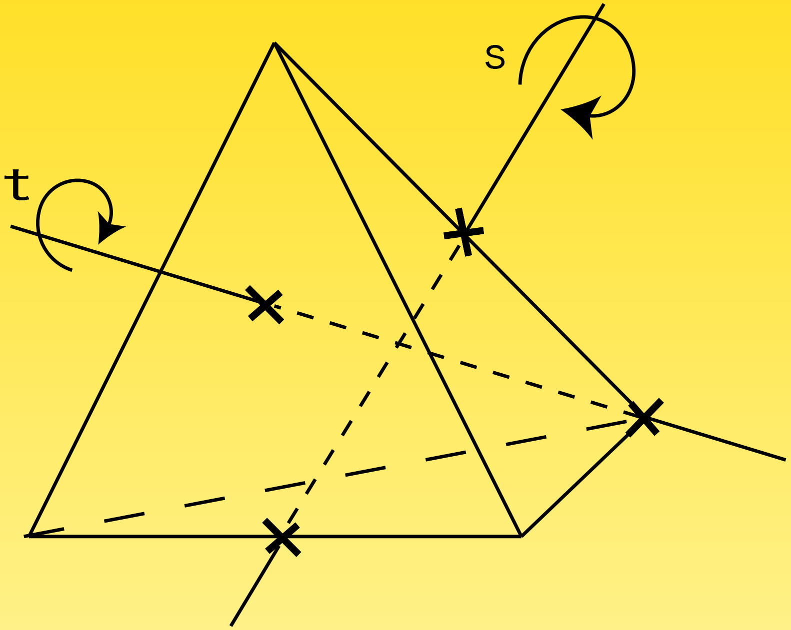
How to choose the discrete Non-Abelian group

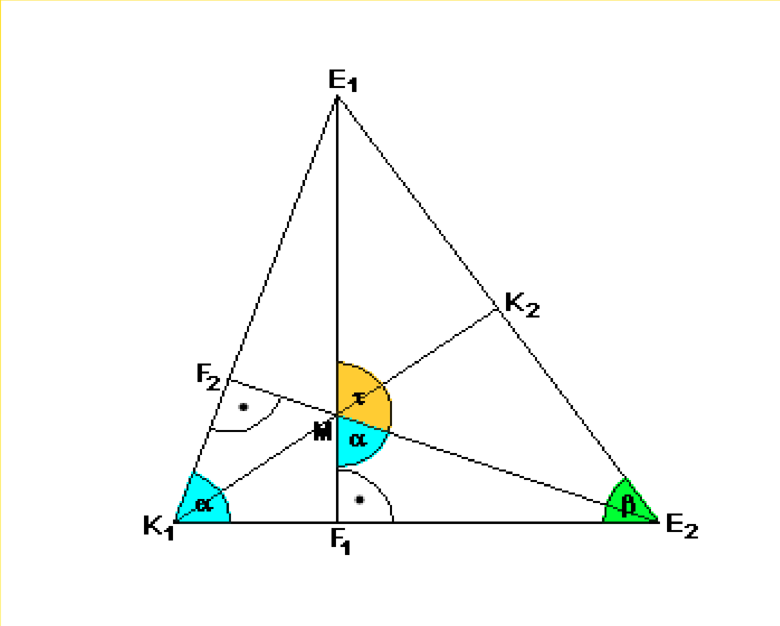
Group	d	Irr. Repr.'s	Presentation
$D_3 \sim S_3$	6	1, $1'$, 2	$A^3 = B^2 = (AB)^2 = 1$
D_4	8	$1_1, \ldots 1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$
D_5	10	1, $1'$, 2, 2'	$A^5 = B^2 = (AB)^2 = 1$
D_6	12	$1_1, \ldots, 1_4$, 2, 2'	$A^6 = B^2 = (AB)^2 = 1$
D_7	14	1, $1'$, 2, $2'$, $2''$	$A^7 = B^2 = (AB)^2 = 1$
A_4	12	1, $1'$, $1''$, 3	$A^3 = B^2 = (AB)^3 = 1$
$A_5 \sim PSL_2(5)$	60	1, 3, $3'$, 4, 5	$A^3 = B^2 = (BA)^5 = 1$
T'	24	1, $1'$, $1''$, 2, $2'$, $2''$, 3	$A^3 = (AB)^3 = R^2 = 1,\ B^2 = R$
S_4	24	1, $1'$, 2, 3, $3'$	$BM : A^4 = B^2 = (AB)^3 = 1$ $TB : A^3 = B^4 = (BA^2)^2 = 1$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, \ldots, 1_9, 3, \overline{3}$	
$PSL_2(7)$	168	$1, 3, \overline{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \overline{1'}, 3, \overline{3}$	$A^7 = B^3 = 1,\ AB = BA^4$

`Altarelli, Feruglio, 1002.0211`

Minimal Choice

A_4 is isomorphic to symmetry group of tetrahedron:





- smallest group with 3-dim irrep.
- has 3 one-dimensional irreps. 1, $1'$, $1''$
- angle between two faces: $\alpha = 2\,\theta_{\mathrm{TBM}}$, where $\sin^2\theta_{\mathrm{TBM}} = \frac{1}{3}$

Tests of Flavor Symmetry Models

- sometimes: $U = U_\ell^\dagger \, U_\nu$ with $U_\ell \simeq R_{12}$ and neutrino sector:

$$U_\nu = \begin{pmatrix} c_{12}^\nu & s_{12}^\nu & 0 \\ \cdot & \cdot & \sqrt{\frac{1}{2}} \\ \cdot & \cdot & \sqrt{\frac{1}{2}} \end{pmatrix} \text{ and } \sin^2\theta_{12}^\nu = \left\{ \begin{array}{ccc} \frac{1}{2} & \text{bimaximal} & S_4 \\ \frac{1}{3} & \text{tri-bimaximal} & A_4, T' \\ \frac{1}{1+\varphi^2} & \text{Golden Ratio A} & A_5 \\ \frac{3-\varphi}{4} & \text{Golden Ratio B} & D_{10} \\ \frac{1}{4} & \text{hexagonal} & D_{12} \end{array} \right.$$

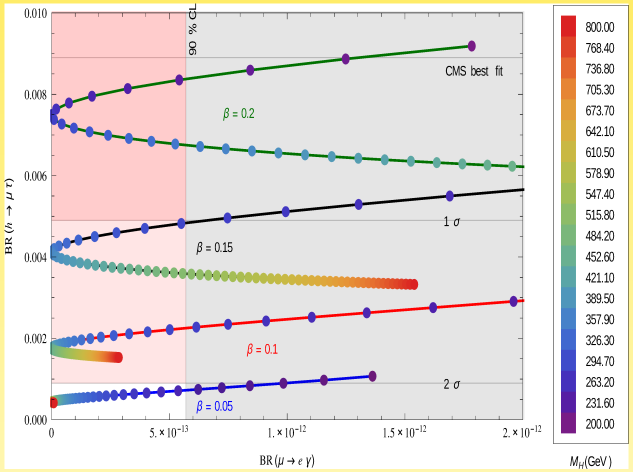
gives sum-rule $\sin^2\theta_{12} = \sin^2\theta_{12}^\nu + |U_{e3}| \, \sin 2\theta_{12}^\nu \, \cos\delta$

- sometimes: neutrino mass sum-rules, e.g. $\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$

- sometimes: models broken at electroweak scale. . .

Chasing the Higgs Ambulance I

	ℓ	e_R	μ_R	τ_R	χ	Φ	ξ
A_4	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$
Z_4	i	i	i	i	1	-1	-1
$\mathrm{SU}(2)_L$	2	1	1	1	2	1	1
$\mathrm{U}(1)_Y$	$-1/2$	-1	-1	-1	$1/2$	0	0



Holthausen, Heeck, W.R., Shimizu, NPB896

Chasing the Higgs Ambulance II

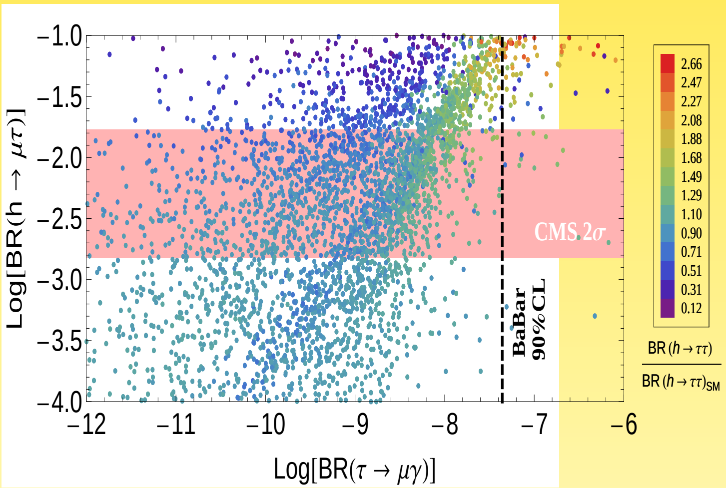
	L_e	L_μ	L_τ	e_R	μ_R	τ_R	N_e	N_μ	N_τ	Φ_1	Φ_2	S
$\mathrm{U}(1)_{L_\mu - L_\tau}$	0	1	-1	0	1	-1	0	1	-1	-2	0	1
$\mathrm{SU}(2)_L$	2	2	2	1	1	1	1	1	1	2	2	1
$\mathrm{U}(1)_Y$	$-1/2$	$-1/2$	$-1/2$	-1	-1	-1	0	0	0	$1/2$	$1/2$	0

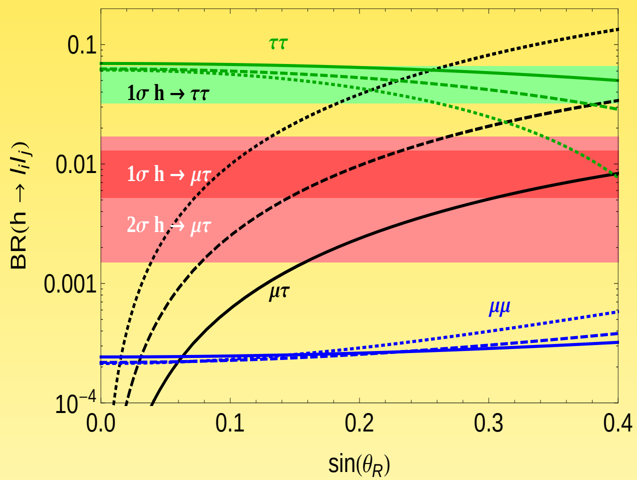
$$\mathcal{L} = y_\alpha \, \overline{L^\alpha} \, \delta_{\alpha\beta} \, \ell_R^\beta \, \Phi_2 + \xi_{\tau\mu} \, \overline{\tau_L} \, \mu_R \, \Phi_1$$

Holthausen, Heeck, W.R., Shimizu, NPB896

Chasing the Higgs Ambulance II

	L_e	L_μ	L_τ	e_R	μ_R	τ_R	N_e	N_μ	N_τ	Φ_1	Φ_2	S
$\mathrm{U}(1)_{L_\mu - L_\tau}$	0	1	-1	0	1	-1	0	1	-1	-2	0	1
$\mathrm{SU}(2)_L$	2	2	2	1	1	1	1	1	1	2	2	1
$\mathrm{U}(1)_Y$	$-1/2$	$-1/2$	$-1/2$	-1	-1	-1	0	0	0	$1/2$	$1/2$	0





Holthausen, Heeck, W.R., Shimizu, NPB896

Open Neutrino Questions

- Testable with current experiments:
  - Dirac or Majorana?
  - CP violated?
  - mass ordering normal or inverted? Mass scale?
  - θ_{23} maximal?
  - light sterile neutrinos?
- Very hard to address experimentally:
  - scale of neutrino mass generation?
  - explanation of mixing pattern?
  - unification with quarks possible?
  - neutrino CP related to baryon asymmetry?

Summary

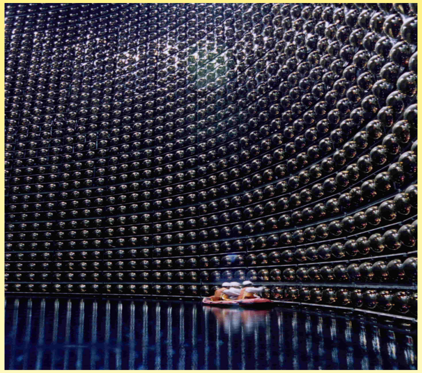
- precision era!
- testing the standard paradigm!
- testing models!
- reaching beyond pure neutrino sector!
- influence on new fields!
- exciting new results in sight!

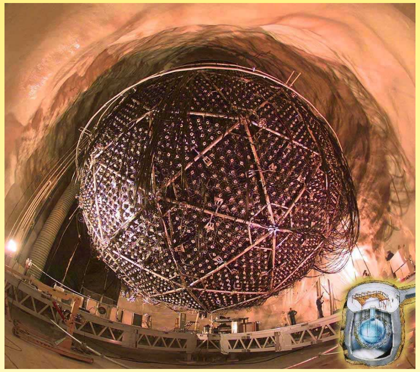
Thank you!

Mixing Angles

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$





$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{atmospheric and} \\ \text{LBL accelerator}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{solar and} \\ \text{LBL reactor}}}$$

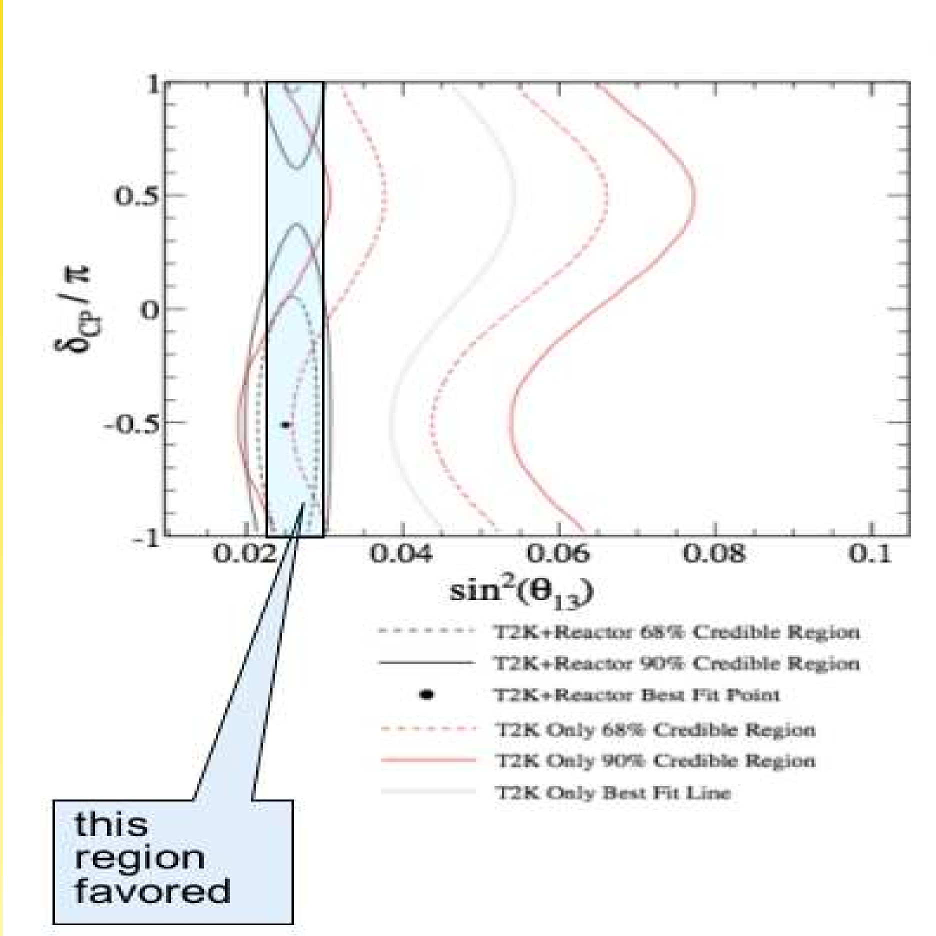
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}}+\epsilon \\ 0 & \sqrt{\frac{1}{2}}+\epsilon & \sqrt{\frac{1}{2}} \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & \epsilon \\ 0 & 1 & 0 \\ \epsilon & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}}+\epsilon & 0 \\ -\sqrt{\frac{1}{3}}+\epsilon & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2\theta_{23} = \tfrac{1}{2} - \epsilon) \qquad (\sin^2\theta_{13} = \epsilon^2) \qquad (\sin^2\theta_{12} = \tfrac{1}{3} - \epsilon^2)$$

$$\Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\odot}$$

Status of global fits: T2K

- ν_μ disappearence: θ_{23}
- ν_e appearence: θ_{13} and δ
- reactor $\bar{\nu}_e$ disappearence as input: δ



rather mild preference for $\delta = 3\pi/2$

Recent Results

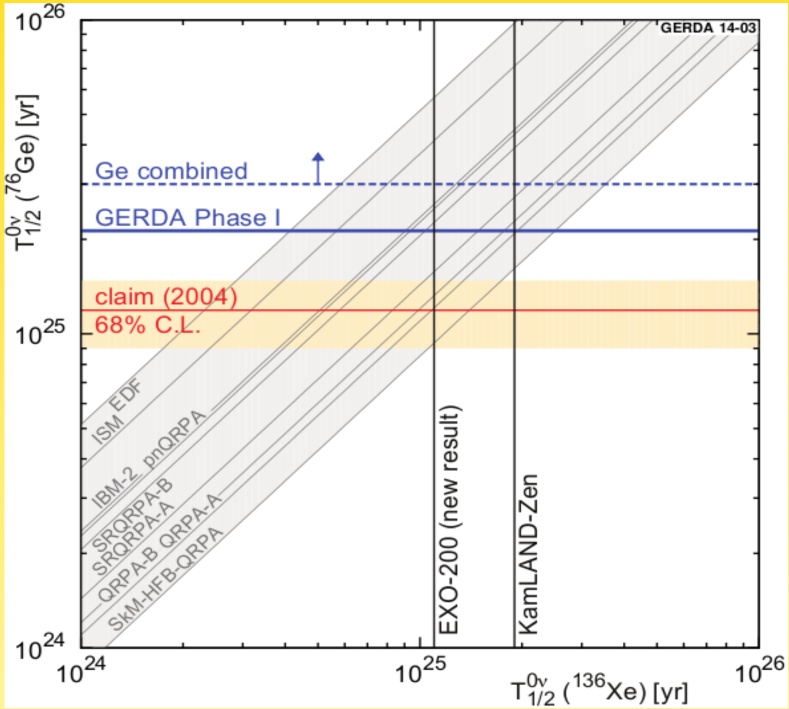
$$\Gamma^{0\nu} = G(Q,Z)\,|\mathcal{M}(A,Z)\,\eta|^2$$

- ^{76}Ge:
  - GERDA: $T_{1/2} > 2.1 \times 10^{25}$ yrs
  - GERDA + IGEX + HDM: $T_{1/2} > 3.0 \times 10^{25}$ yrs
- ^{136}Xe:
  - EXO-200: $T_{1/2} > 1.1 \times 10^{25}$ yrs **(first run with less exposure:** $T_{1/2} > 1.6 \times 10^{25}$ **yrs. . .)**
  - KamLAND-Zen: $T_{1/2} > 2.6 \times 10^{25}$ yrs

Xe-limit is stronger than Ge-limit when:

$$T_{\rm Xe} > T_{\rm Ge}\,\frac{G_{\rm Ge}}{G_{\rm Xe}}\,\left|\frac{\mathcal{M}_{\rm Ge}}{\mathcal{M}_{\rm Xe}}\right|^2\ \text{yrs}$$

NME	^{76}Ge		^{136}Xe	
	GERDA	comb	KLZ	comb
EDF(U)	**0.32**	**0.27**	**0.13**	–
ISM(U)	**0.52**	**0.44**	**0.24**	–
IBM-2	**0.27**	**0.23**	**0.16**	–
pnQRPA(U)	**0.28**	**0.24**	**0.17**	–
SRQRPA-B	**0.25**	**0.21**	**0.15**	–
SRQRPA-A	**0.31**	**0.26**	**0.23**	–
QRPA-A	*0.28*	*0.24*	*0.25*	–
SkM-HFB-QRPA	*0.29*	*0.24*	*0.28*	–



GERDA

Bhupal Dev, Goswami, Mitra, W.R., Phys. Rev. **D88**

Predictions of $SO(10)$ theories

Yukawa structure of $SO(10)$ models depends on Higgs representations

10_H ($\leftrightarrow H$), $\overline{126}_H$ ($\leftrightarrow F$), 120_H ($\leftrightarrow G$)

Gives relation for mass matrices:

$$
\begin{aligned}
m_{\text{up}} &\propto r(H + sF + it_u\, G) \\
m_{\text{down}} &\propto H + F + iG \\
m_D &\propto r(H - 3sF + it_D\, G) \\
m_\ell &\propto H - 3F + it_l\, G \\
M_R &\propto r_R^{-1} F
\end{aligned}
$$

Numerical fit including RG, Higgs, θ_{13}

$10_H + \overline{126}_H$: 19 free parameters

$10_H + \overline{126}_H + 120_H$: 18 free parameters

20 (19) observables to be fitted

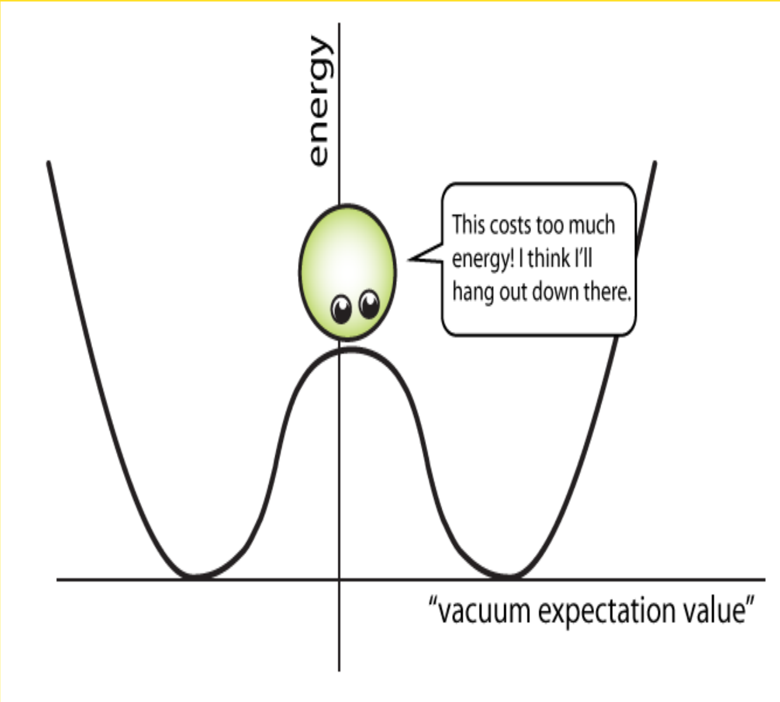
Predictions of $SO(10)$ theories

model	Fit	$\lvert m_{ee} \rvert$ [meV]	m_0 [meV]	M_3 [GeV]	χ^2
$10_H + \overline{126}_H$	NH	0.49	2.40	3.6×10^{12}	23.0
$10_H + \overline{126}_H$ + SS	NH	0.44	6.83	1.1×10^{12}	3.29
$10_H + \overline{126}_H + 120_H$	NH	2.87	1.54	9.9×10^{14}	11.2
$10_H + \overline{126}_H + 120_H$ + SS	NH	0.78	3.17	4.2×10^{13}	6.9×10^{-6}
$10_H + \overline{126}_H + 120_H$	IH	35.52	30.2	1.1×10^{13}	13.3
$10_H + \overline{126}_H + 120_H$ + SS	IH	24.22	12.0	1.2×10^{13}	0.6

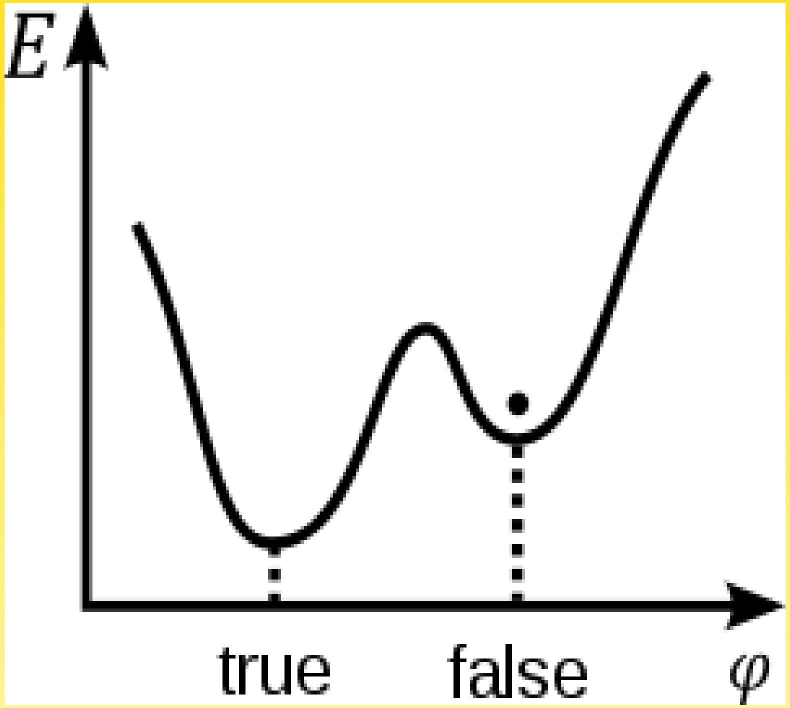
Dueck, W.R., JHEP **1309**

Phenomenology of heavy singlets: Higgs

Higgs potential is rather flat $\Rightarrow$ corrections are dangerous



$\longrightarrow$

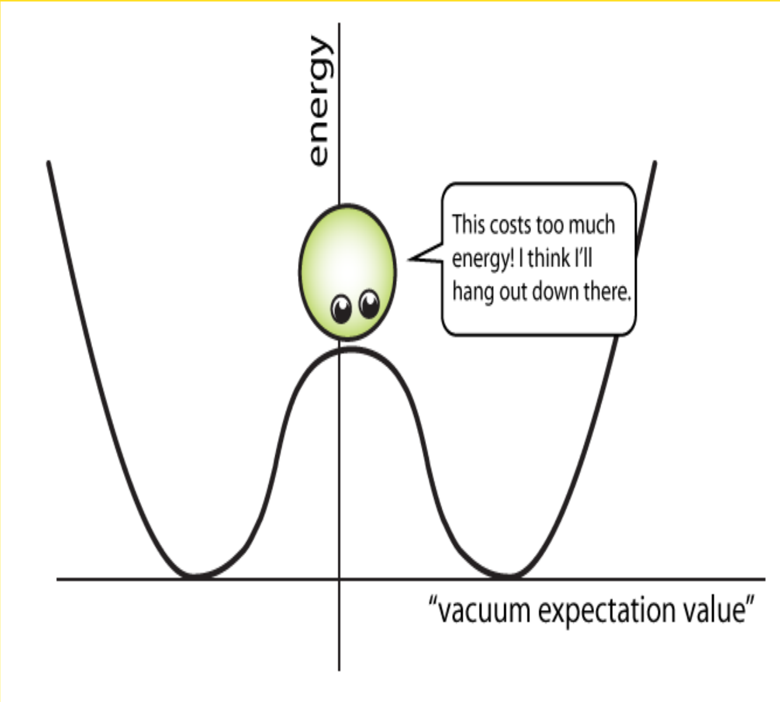


vacuum probably metastable: could tunnel to true vacuum

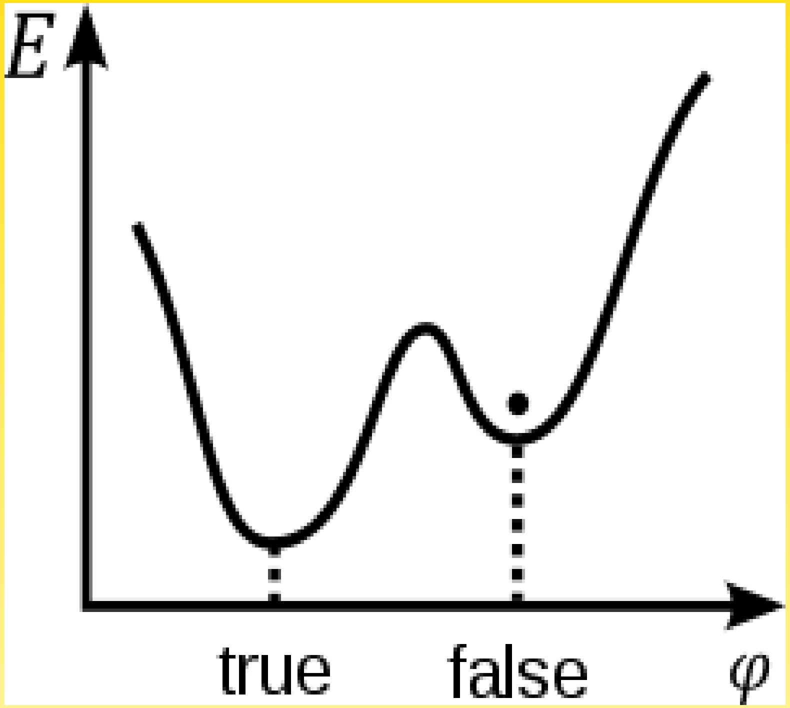
replacing all fields and forces with new fields and forces

Phenomenology of heavy singlets: Higgs

Higgs potential is rather flat $\Rightarrow$ corrections are dangerous



$\longrightarrow$



vacuum probably metastable: could tunnel to true vacuum

replacing all fields and forces with new fields and forces

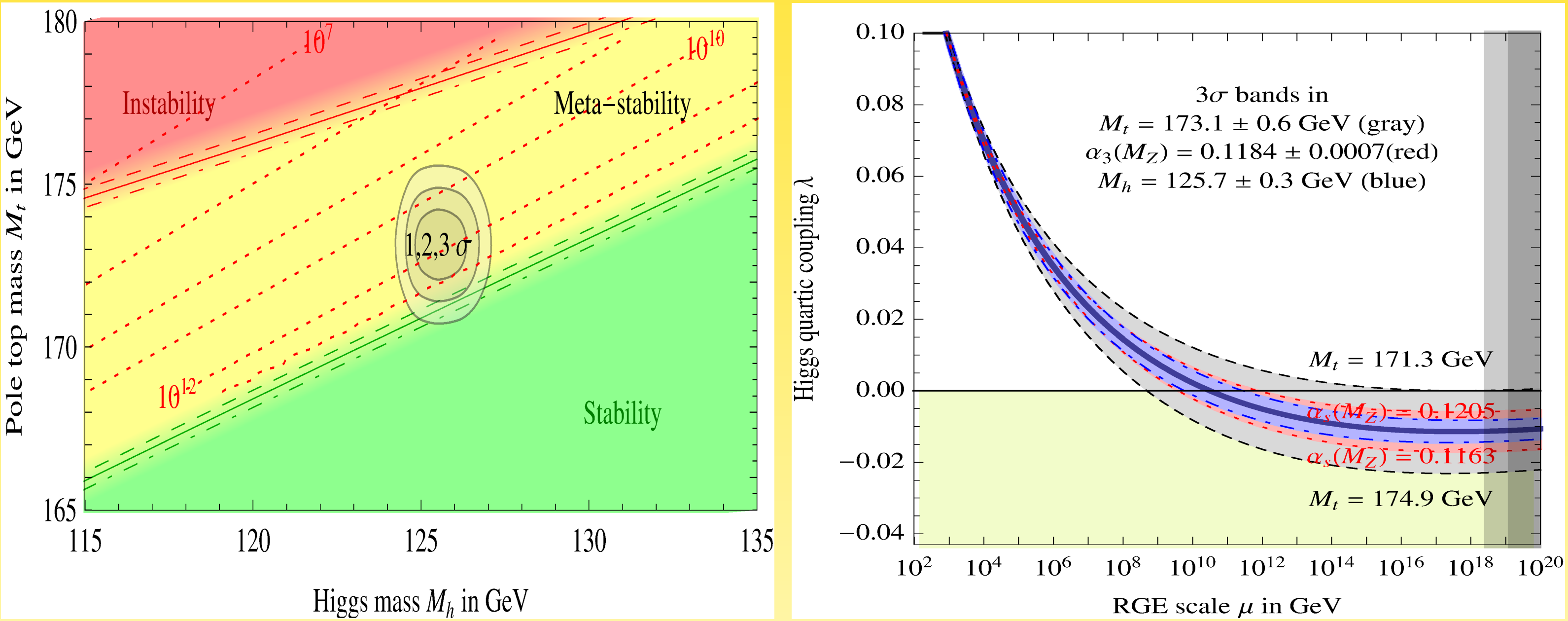
(not good)

Phenomenology of heavy singlets: Higgs

Higgs coupling λ is driven to negative values by top Yukawa:

$$\beta_\lambda \propto -24\,\mathrm{Tr}\left(Y_u^\dagger Y_u\right)^2 \propto m_t^4$$

vacuum stability



`Degrassi` *et al.*; `1205.6497`

dependence on top mass, α_s

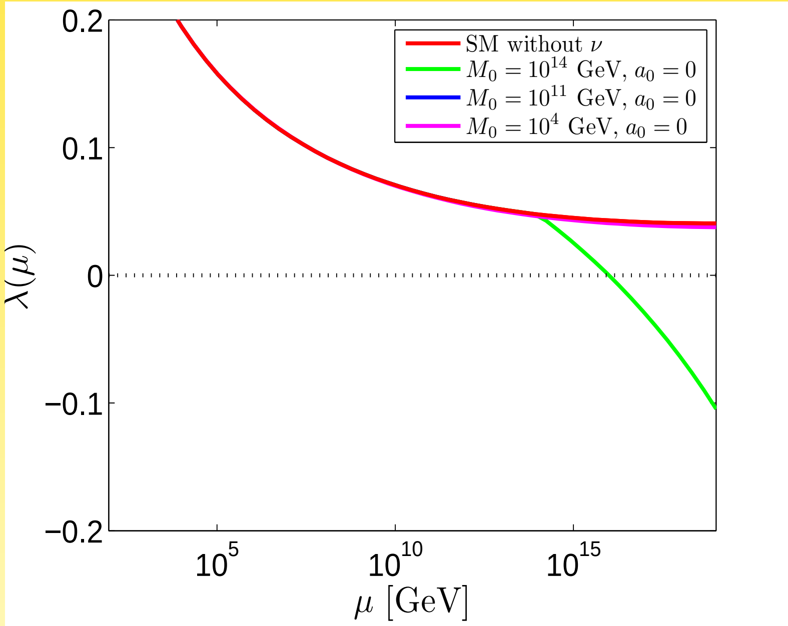
Phenomenology of heavy singlets: Higgs

Lepton-Higgs-N_R vertex: Dirac Yukawa $\bar{L}\, Y_\nu\, N_R$ contribution

$$\Delta\beta_\lambda = -8\,\mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right)^2 \propto m_D^4$$

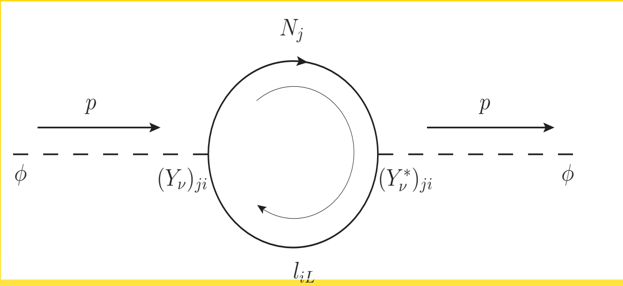
Casas *et al.*; Strumia *et al.*; W.R., Zhang

makes vacuum stability condition worse!



(also effect if tricks are played to produce TeV-scale N_R at colliders)

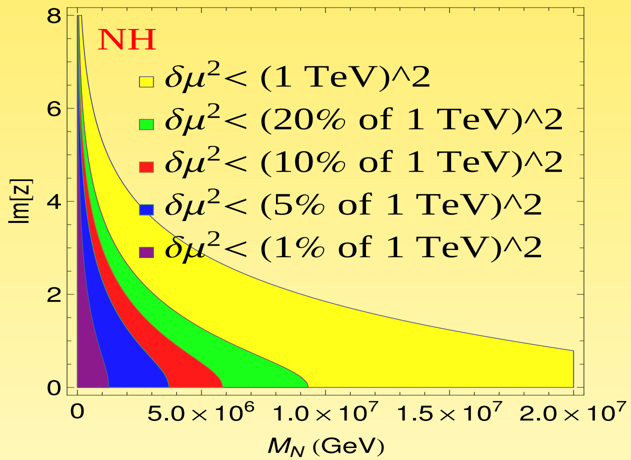
Naturalness issues (Vissani, PRD57; Volkas *et al.* PRD91)



$$\delta\mu^2 \approx \frac{1}{4\pi^2 v^2}\,\mathrm{Tr}\left[m_D^\dagger\,\mathrm{diag}(M_1^2, M_2^2, M_3^2)\, m_D\right] \overset{!}{\lesssim} \mathrm{TeV}^2 \Rightarrow M_R \lesssim 10^7\,\mathrm{GeV}$$

most minimal case $M_1 = M_2 = M$, parametrize m_D as

$$m_D = \sqrt{M}\,R\,\mathrm{diag}(0, \sqrt{m_2}, \sqrt{m_3})\,U^\dagger \quad \text{with } R = \begin{pmatrix} 0 & \cos z & \sin z \\ 0 & -\sin z & \cos z \end{pmatrix}$$



Goswami, W.R., to appear

Alternative to Standard Seesaw I: inverse seesaw

basis (ν_L, N_R^c, S)

$$M = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}$$

$$\Rightarrow m_\nu = m_D\,(M^T)^{-1}\,\mu\,M^{-1}\,m_D^T = 0.1\left(\frac{m_D}{10^2\,\mathrm{GeV}}\right)^2\left(\frac{10^4\,\mathrm{GeV}}{M}\right)^2\left(\frac{\mu}{\mathrm{keV}}\right)\,\mathrm{eV}$$

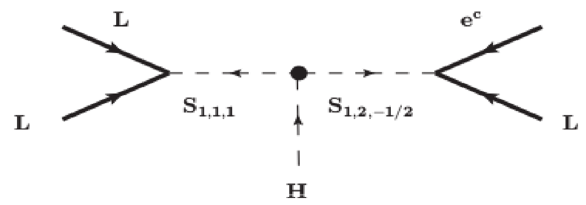
- 't Hooft...
- unitarity violation $U^\dagger U - 1 \simeq \frac{m_D^2}{M^2}$
- keV-scale $\leftrightarrow$ warm dark matter

Alternative to Standard Seesaw III: $d = 7$ operator

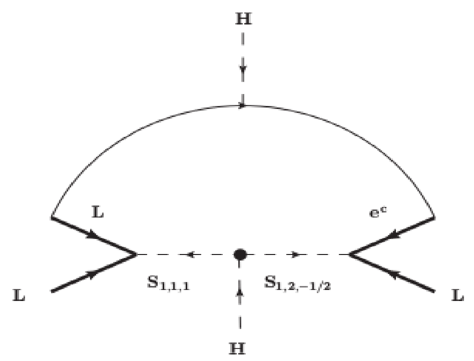
$$\mathcal{O} = L\, L\, L\, e^c\, H$$

open up to generate 1-loop neutrino mass model:

One more example, open $d = 7$:



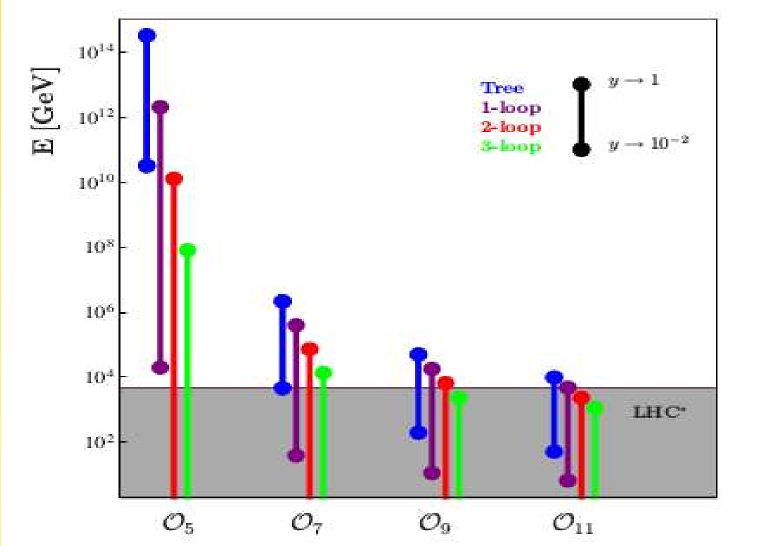
Close using SM Yukawa interaction:



Theoretical Expectation of Neutrino Mass Generation Scale

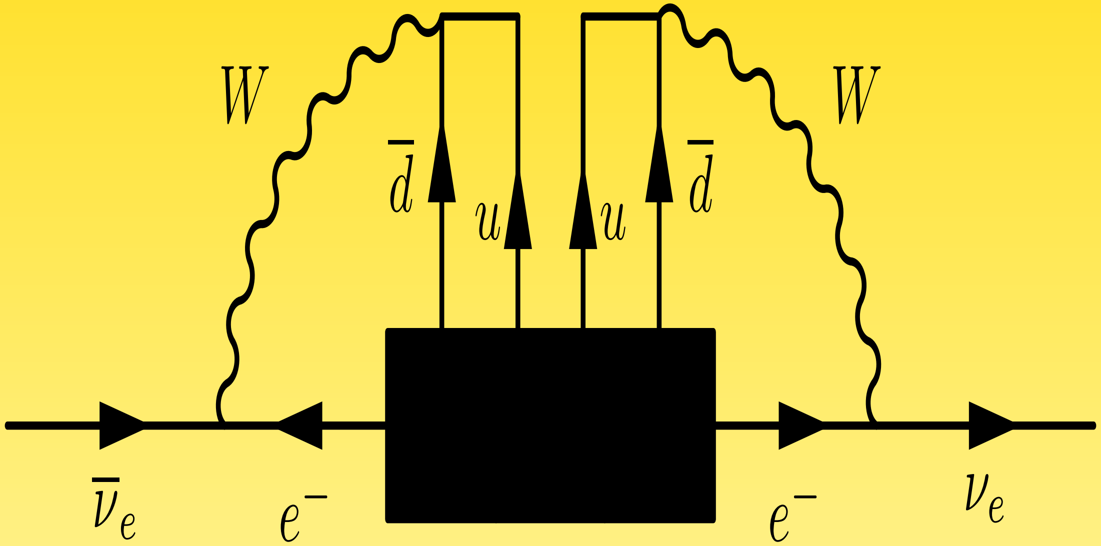
$$m_\nu = \frac{(y\,v)^2}{\Lambda}\left(\frac{y^2}{16\pi^2}\right)^n \left(\frac{y\,v}{\Lambda}\right)^{d-5} \epsilon$$

- classical seesaw
- n-loop
- dimension d
- nearly conserved L



M. Hirsch, talk at WIN2015

Schechter-Valle theorem: observation of $0\nu\beta\beta$ implies Majorana neutrinos



is 4 loop diagram: $m_\nu^{\mathrm{BB}} \sim \dfrac{G_F^2}{(16\pi^2)^4}\mathrm{MeV}^5 \sim 10^{-25}\,\mathrm{eV}$

Distinguishing Mechanisms

The inverse problem of $0\nu\beta\beta$

1.) Other observables (LHC, LFV, KATRIN, cosmology,...)

2.) Decay products (individual e^- energies, angular correlations, spectrum,...)

3.) Nuclear physics (multi-isotope, 0νECEC, $0\nu\beta^+\beta^+$,...)

Left-right Symmetry

6 neutrinos with flavor states n'_L and mass states $n_L = (\nu_L, N_R^c)^T$

$$n'_L = \begin{pmatrix} \nu'_L \\ {\nu_R}^c \end{pmatrix} = \begin{pmatrix} K_L \\ K_R \end{pmatrix} n_L = \begin{pmatrix} U & S \\ T & V \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

Right-handed currents:

$$\mathcal{L}_{CC}^{\rm lep} = \frac{g}{\sqrt{2}} \left[\overline{\ell_L} \gamma^\mu K_L n_L (W_{1\mu}^- + \xi e^{i\alpha} W_{2\mu}^-) + \overline{\ell_R} \gamma^\mu K_R n_L^c (-\xi e^{-i\alpha} W_{1\mu}^- + W_{2\mu}^-) \right]$$

(K_L and K_R are 3×6 mixing matrices)

plus: gauge boson mixing

$$\begin{pmatrix} W_L^\pm \\ W_R^\pm \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi\, e^{i\alpha} \\ -\sin\xi\, e^{-i\alpha} & \cos\xi \end{pmatrix} \begin{pmatrix} W_1^\pm \\ W_2^\pm \end{pmatrix}$$

Connection to Neutrinos

Majorana mass matrices $M_L = f_L\, v_L$ from $\langle \Delta_L \rangle$ and $M_R = f_R\, v_R$ from $\langle \Delta_R \rangle$
(with $f_L = f_R = f$)

$$\mathcal{L}^{\nu}_{\rm mass} = -\frac{1}{2} \left(\overline{\nu'_L} \ \overline{\nu'_R}^{\,c} \right) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu'^{\,c}_L \\ \nu'_R \end{pmatrix} \Rightarrow m_\nu = M_L - M_D\, M_R^{-1}\, M_D^T$$

useful special cases

(i) type I dominance: $m_\nu = M_D\, M_R^{-1}\, M_D^T = M_D\, f_R^{-1}/v_R\, M_D^T$

(ii) type II dominance: $m_\nu = f_L\, v_L$

for case (i): mixing of light neutrinos with heavy neutrinos of order

$$|S_{\alpha i}| \simeq |T^T_{\alpha i}| \simeq \sqrt{\frac{m_\nu}{M_i}} \lesssim 10^{-7} \left(\frac{\rm TeV}{M_i} \right)^{1/2}$$

small (or enhanced up to 10^{-2} by cancellations)

Right-handed Currents in Double Beta Decay

$$(A,Z) \to (A,Z+2) + 2e^-$$

$$\mathcal{L}_{CC}^{\rm lep} = \frac{g}{\sqrt{2}} \sum_{i=1}^{3} \Big[\overline{e_L} \gamma^\mu (U_{ei} \nu_{Li} + S_{ei} N_{Ri}^c)(W_{1\mu}^- + \xi e^{i\alpha} W_{2\mu}^-)$$

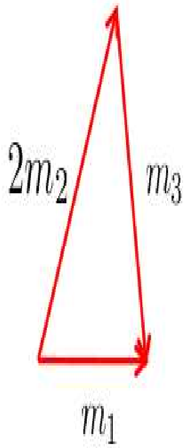
$$+ \overline{e_R} \gamma^\mu (T_{ei}^* \nu_{Li}^c + V_{ei}^* N_{Ri})(-\xi e^{-i\alpha} W_{1\mu}^- + W_{2\mu}^-) \Big]$$

$$\mathcal{L}_Y^\ell = -\overline{L}_L'^c i\sigma_2 \Delta_L f_L L_L' - \overline{L}_R'^c i\sigma_2 \Delta_R f_R L_R'$$

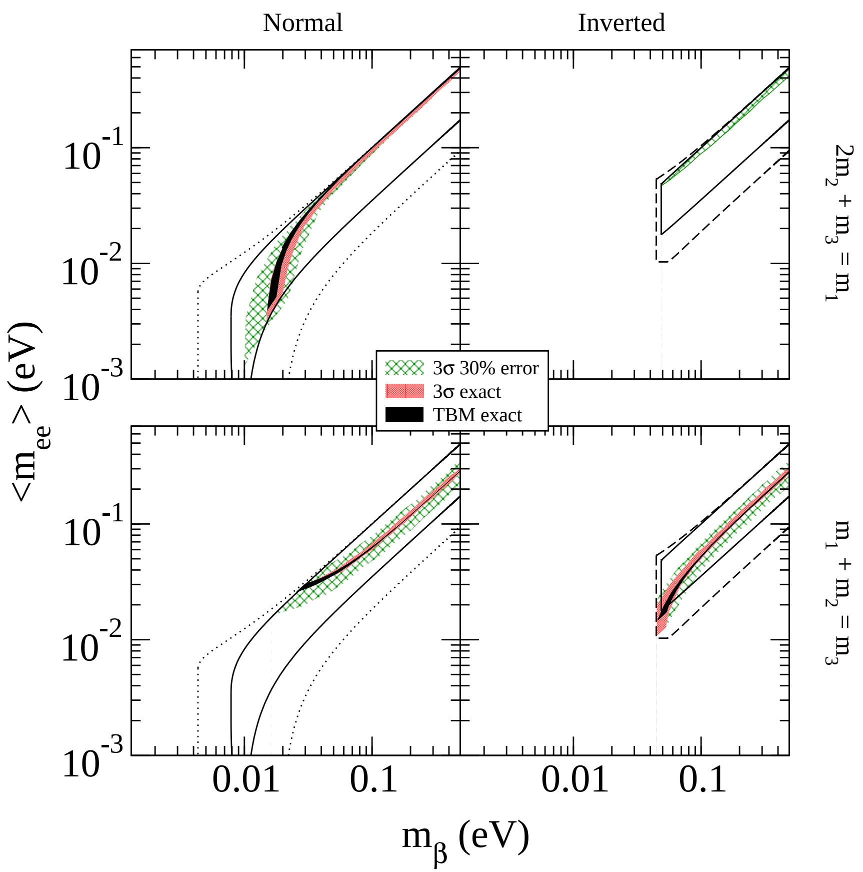
classify diagrams:

- mass dependent diagrams (same helicity of electrons)
- triplet exchange diagrams (same helicity of electrons)
- momentum dependent diagrams (different helicity of electrons)

Flavor Symmetry Models: sum-rules



Sum-rule	Flavour symmetry
$2m_2 + m_3 = m_1$	$A_4, T', (S_4)$
$m_1 + m_2 = m_3$	$S_4, (A_4)$
$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}$	A_4, T'
$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$	S_4



constraints on masses and Majorana phases

Barry, W.R., Nucl. Phys. **B842**

Master Formula: $2 \times 2 = 3 + 1$

$SU(2)_L \times U(1)_Y$ with $2 \otimes 2 = 3 \oplus 1$:

$$\overline{L}\,\tilde{\Phi} \sim (2,+1) \otimes (2,-1) = (3,0) \oplus (1,0)$$

To make a singlet, couple $(1,0)$ or $(3,0)$, because $3 \otimes 3 = 5 \oplus 3 \oplus 1$

Alternatively:

$$\overline{L}\,L^c \sim (2,+1) \otimes (2,+1) = (3,+2) \oplus (1,+2)$$

To make singlet, couple to $(1,-2)$ or $(3,-2)$. However, singlet combination $(1,+2)$ is $\overline{\nu}\,\ell^c - \overline{\ell}\,\nu^c$, which cannot generate neutrino mass term

$\Longrightarrow$	$(1,0)$	or	$(3,-2)$	or	$(3,0)$
	type I		type II		type III