Freiburg Lectures 2016

Graduiertenkolleg "Mass and Symmetries after the Discovery of the Higgs Particle at the LHC"

Tracking
and Tracking Detectors

Norbert Wermes
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Outline

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Lecture 1

Tracking

- momentum measurement
- vertex measurement
- influence of multiple scattering
- errors and what to do ...

Lectures 2 & 3 & 4

Tracking Detectors

- the signal (and the noise)
- spatial resolution with structured electrodes
- gaseous detectors
- semiconductor detectors
- pixel detectors ... status and future

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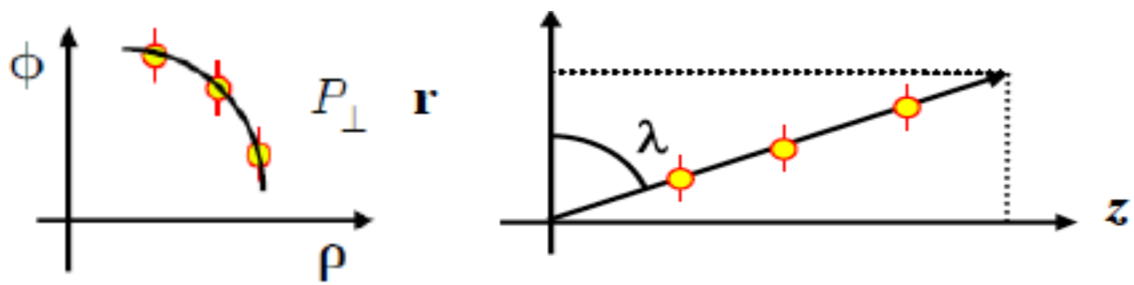
Lecture 1

Tracking



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Content Lecture 1

- Tasks of trackers
- Motion in magnetic field
- (Tracking) Spectrometers
  - **Dipole**
  - **Solenoid**
  - **Toroid**
- some examples

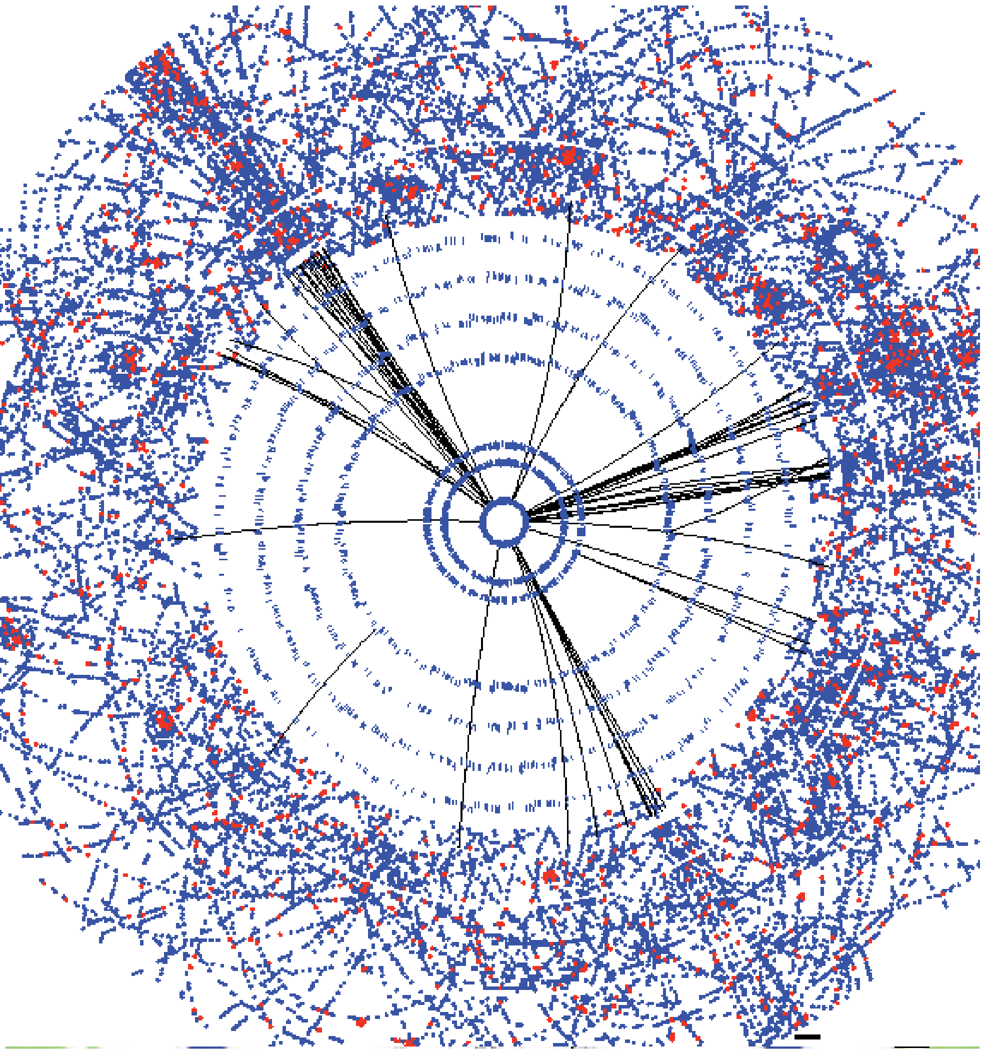
- Trajectories
  - straight lines
  - helix
- Track models
  - straight line fits
  - circular fits
  - matrix formalism

- Use cases
  - Dipole tracking
  - Solenoid tracking
  - **Momentum resolution**
  - Determination of the sign of charge

- **Multiple scattering**
  - Momentum resolutions at low momenta

- Vertex detection
  - **Extrapolation error**
  - Impact parameter resolution at low momenta

"Tracking"



$H \to b\bar{b}$

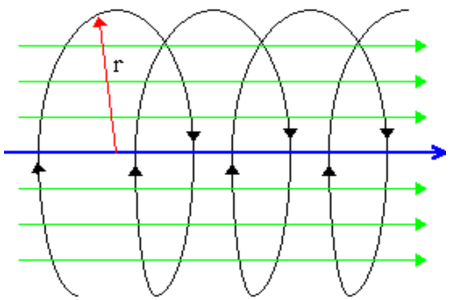
- find track patterns in an ensemble of measured hits
  - algorithmic pattern recognition (including noise hits -> Kalman filter)
- find best estimate for the trajectory of a particle's path
- measure the curvature -> momentum
- measure the (polar) angle
- extrapolate to origin -> vertex (1^{st}, 2^{nd})
- estimate the precision of the momentum or vertex measurement
- determine the effect of scattering in matter in the precision
- include systematic effects (like mis-alignment)

see also **Lecture by Markus Elsing** next week

Aims .. cont'd

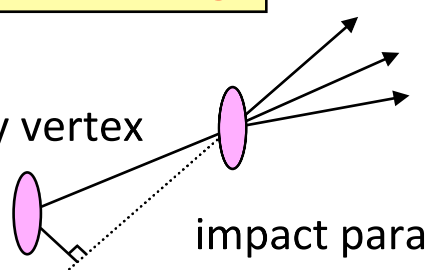
- Recognition and reconstruction of charged particles trajectories (tracks)
- Measure (not a full list, but addressed here)

momentum (in magnetic field)



p = 0.3·B·r

lifetime and lifetime "tag"



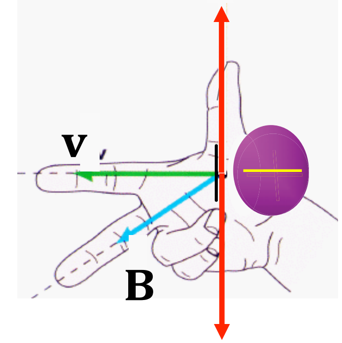
secondary vertex

primary vertex

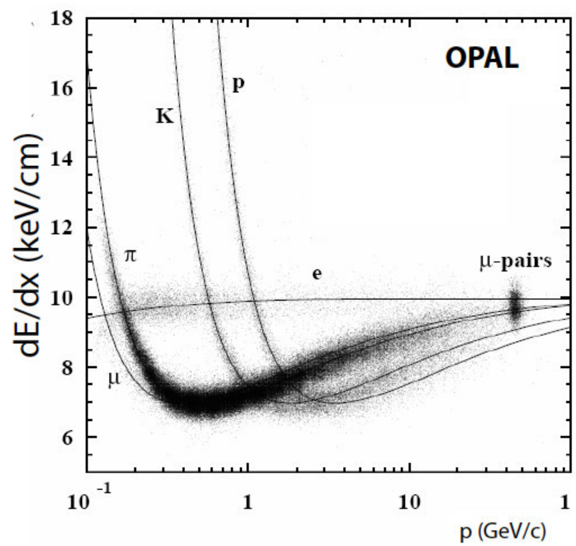
impact parameter

the sign of the charge

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$



particle ID (mass),
not necessarily with the same detector



specific ionisation loss

$$p = m_o\,\gamma\,\beta$$

$$dE/dx = \mathrm{fct}(\gamma\beta)$$

not topic of lecture

A spectrometer -> momentum measurement in B-field

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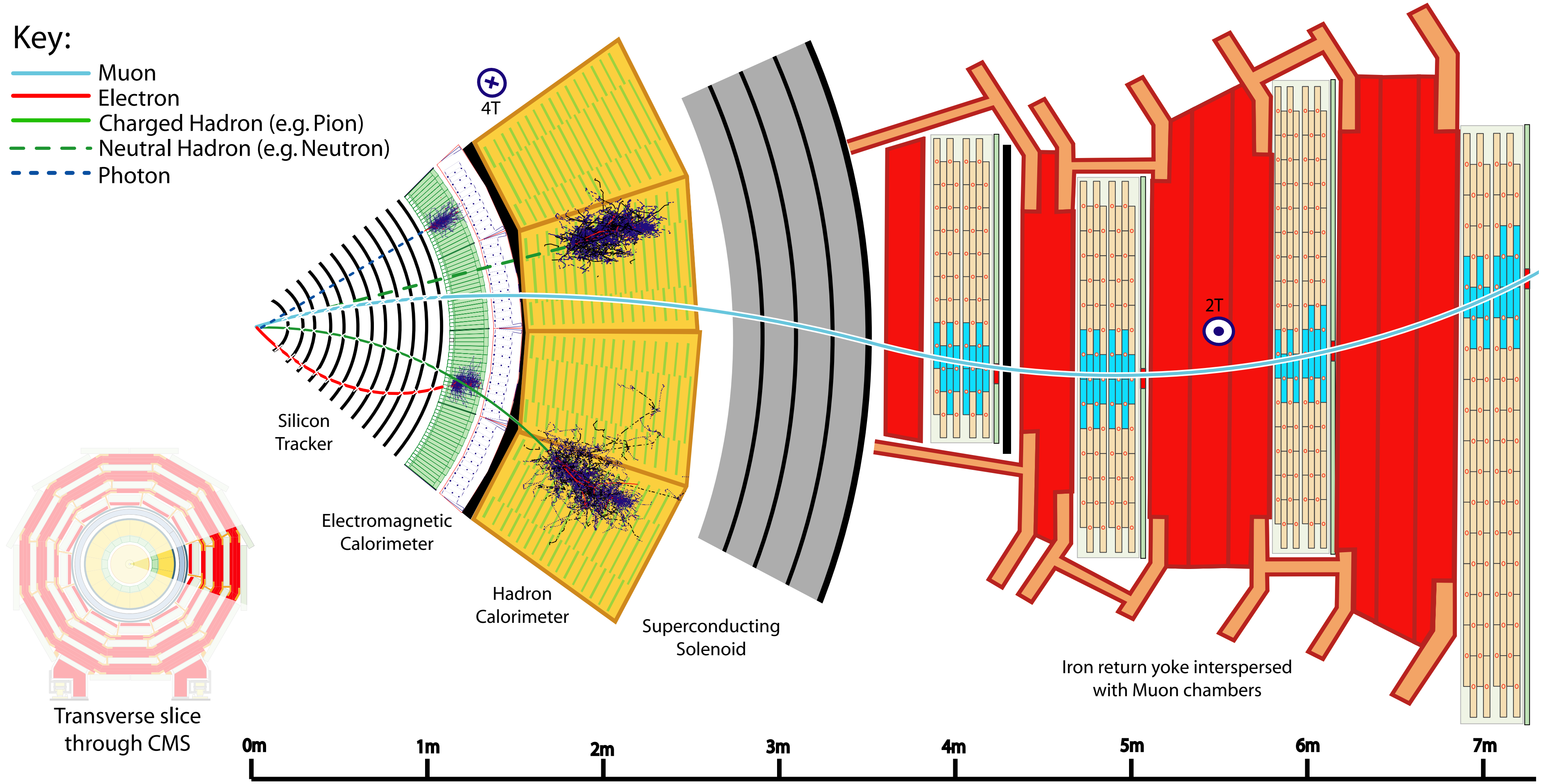
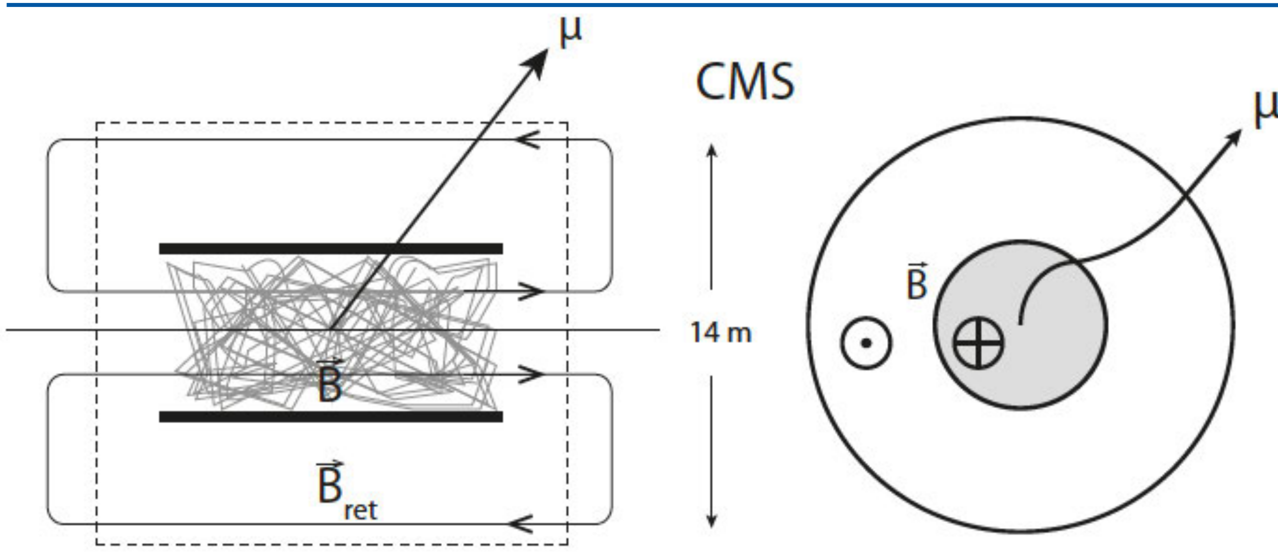


Example: Muon Tracking in CMS and ATLAS

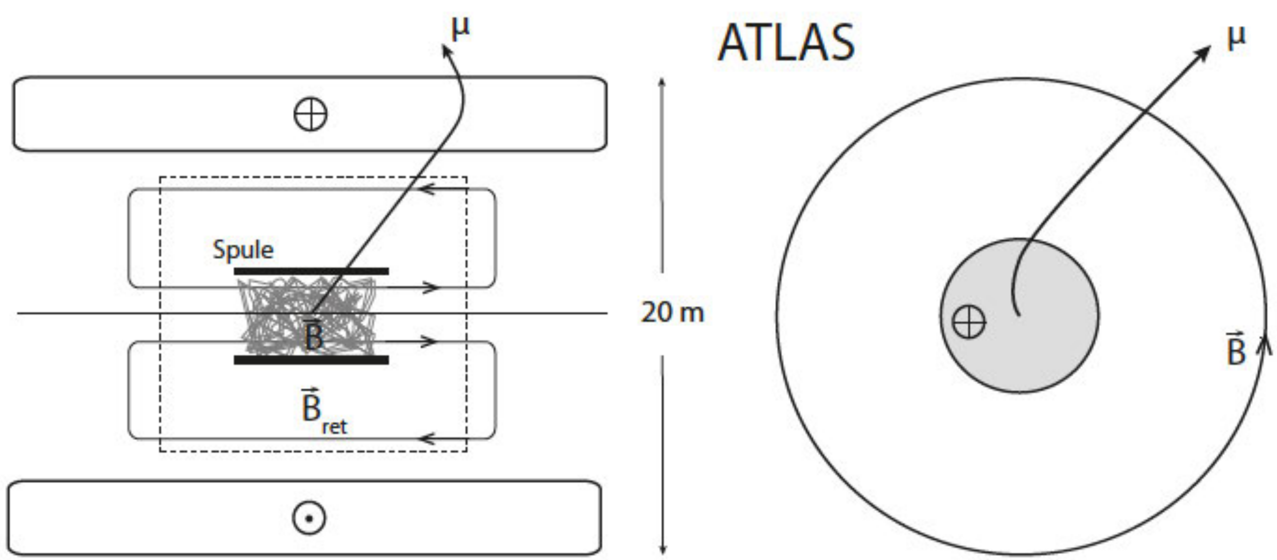


CMS

measurement of momentum in tracker <u>and</u> B return flux;

Solenoid with Fe flux return

Property: μ tracks point back to vertex in r-z plane

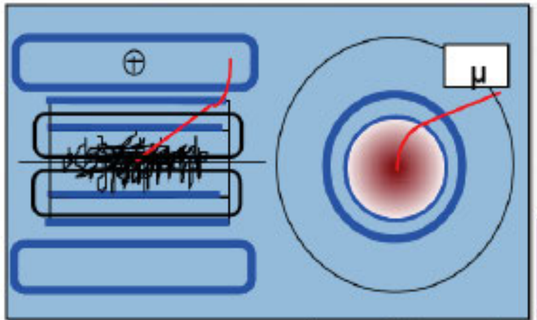


ATLAS

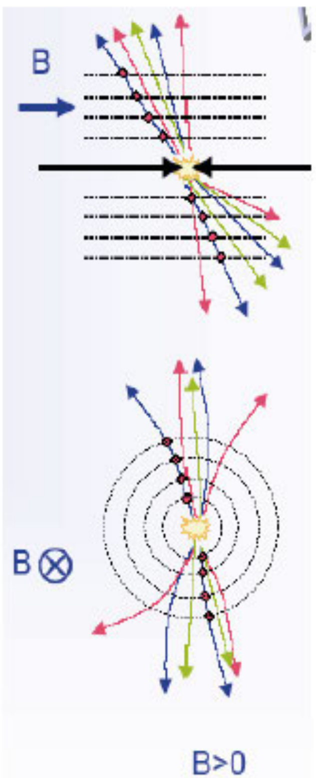
standalone μ momentum measurement; Air-core toroid safe for high multiplicities;

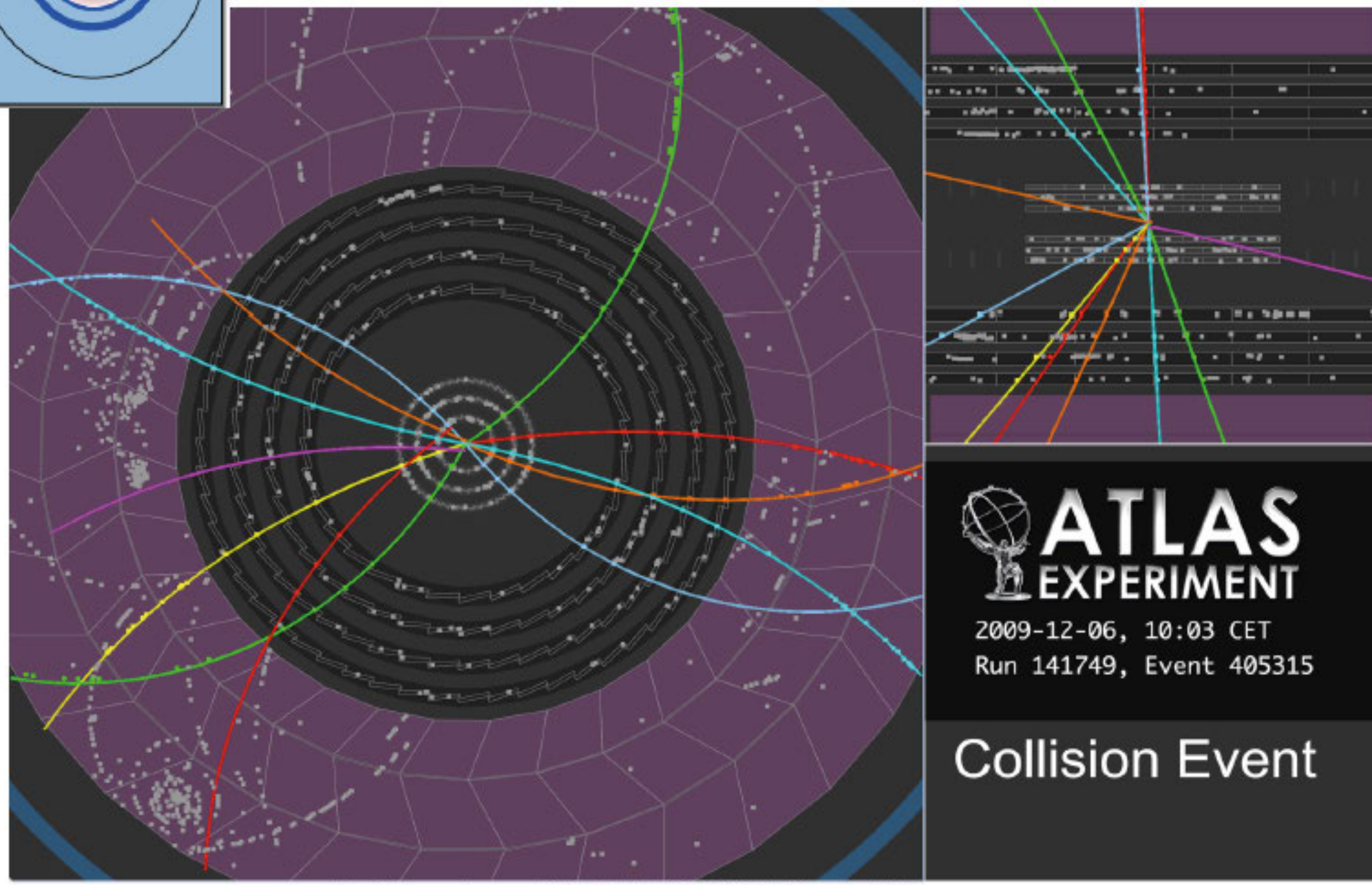
Property: σ_p flat with η

Tracking in ATLAS



ATLAS: (air-core) toroid magnet
+ inner solenoid





http://atlas.web.cern.ch/Atlas/public/EVTDISPLAY/events.html

Equation of Motion in a (homogeneous) magnetic field

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$$\vec{F} = \dot{\vec{p}} = q\left(\vec{v} \times \vec{B}\right) \quad \Rightarrow \quad \boxed{\dot{\vec{v}} = \frac{q}{\gamma m}\left(\vec{v} \times \vec{B}\right)}$$

differential equation in $\vec{v}$

- solution is a rotating vector $\vec{v}_T$ in plane perpendicular to $\vec{B}$ $\quad \vec{v}_{||}$ is unchanged

$$B_1 = B_2 = 0,\ B_3 = B > 0$$

$$\omega_B = \frac{|q|\,B}{\gamma m}$$

$$\Rightarrow$$

$$\begin{aligned} v_1 &= v_T \cos(\eta\,\omega_B\,t + \psi) \\ v_2 &= -v_T \sin(\eta\,\omega_B\,t + \psi) \\ v_3 &= v_3 \end{aligned} \qquad \eta = \frac{q}{|q|}$$

equations also hold relativistically $\quad \omega_B = \omega_B(\gamma); E = \gamma m$

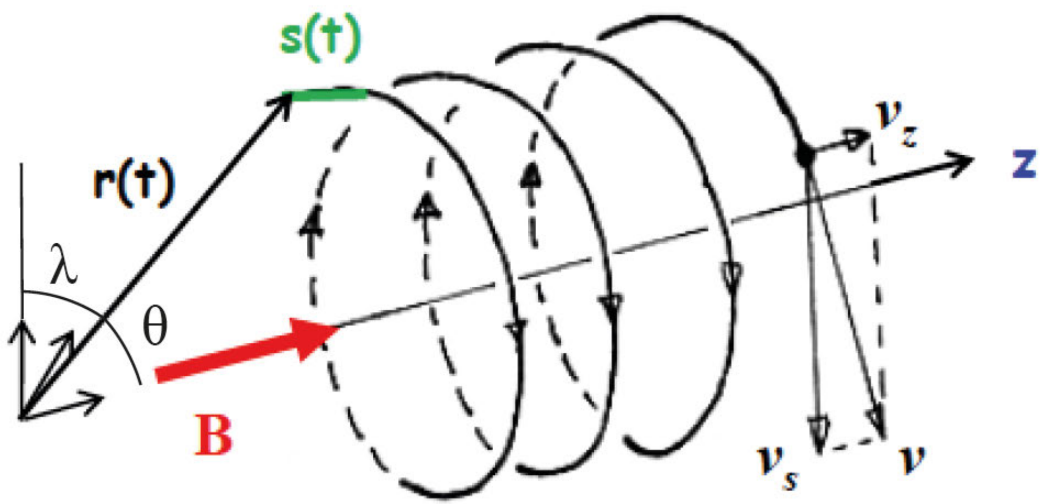
- integration yields spatial trajectory

$$\begin{aligned} x_1 &= \frac{v_T}{\eta\,\omega_B} \sin(\eta\,\omega_B\,t + \psi) + x_{10} \\ x_2 &= \frac{v_T}{\eta\,\omega_B} \cos(\eta\,\omega_B\,t + \psi) + x_{20} \\ x_3 &= v_3\,t + x_{30} \end{aligned}$$

curvature radius

$$\boxed{R} = \sqrt{(x_1 - x_{10})^2 + (x_2 - x_{20})^2} = \frac{v_T}{\omega_B} = \frac{\gamma\,m\,v_T}{|q|\,B} = \boxed{\frac{p_T}{|q|\,B}}$$

$p_T = 0.3 \times B \times R$



$$\sin\theta = \frac{p_T}{p}$$

$$\cos\lambda = \frac{p_T}{p} = \pi/2 - \theta$$

(dip angle)

using $q = 1.6 \times 10^{-19} C$ $\quad J = \frac{1}{1.6\ 10^{-19}} eV \quad \frac{s}{m} = \frac{1}{c} \cdot 3 \cdot 10^8$

$$p(GeV/c) = 0.3\ B(T)\ R(m); \qquad p = \frac{p_T}{sin\theta}$$

q -> 0.3 |z|

Magnetic Spectrometers (= mom. measurement)

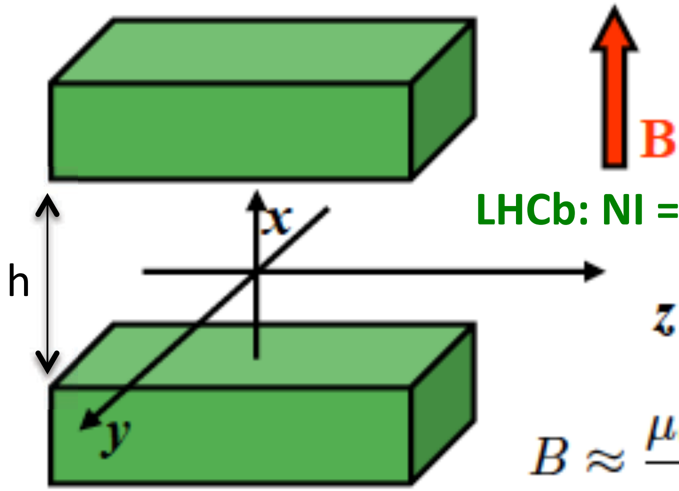
Almost all HEP experiments at accelerators have a magnetic spectrometer to measure the momentum of charged particles.

main configurations:

- **dipole**
- **solenoid**
- **toroid**

$$N\,I = \frac{1}{\mu_0\,\mu_{\mathrm{Luft}}}\int_{\mathrm{Luft}} \vec{B}\,d\vec{l} + \frac{1}{\mu_0\,\mu_{\mathrm{Fe}}}\int_{\mathrm{Fe}} \vec{B}\,d\vec{l}$$

Dipole

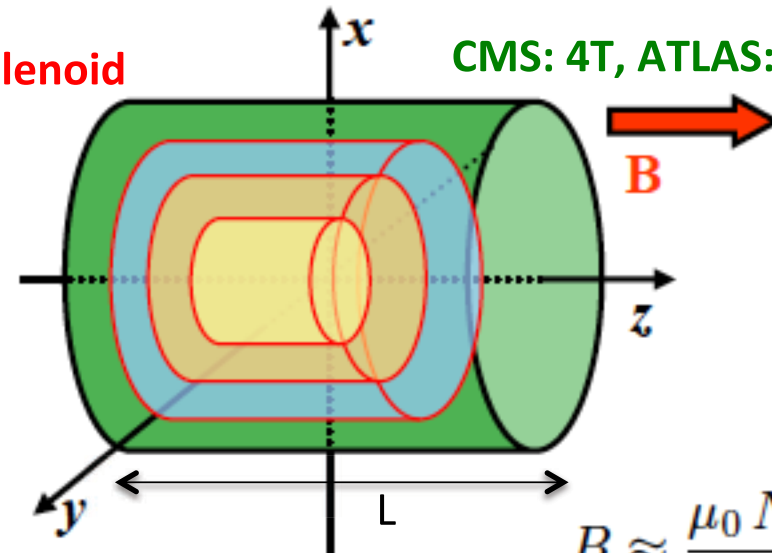


LHCb: NI = 4Tm

$$B \approx \frac{\mu_0\,N\,I}{h}$$

Solenoid

CMS: 4T, ATLAS: 2T



$$B \approx \frac{\mu_0\,N\,I}{L}$$

❑ **rectangular symmetry**

- deflection in y - z plane
- tracking detectors are arranged in parallel planes along z
- measurement of curved trajectories in y - z planes at fixed z

❑ **cylindrical symmetry**

- deflection in x –y (r - ϕ) plane
- tracking detectors are arranged in cylindrical shells along r
- measurement of curved trajectories in r-ϕ planes at fixed r

Magnetic Spectrometers (= mom. measurement)

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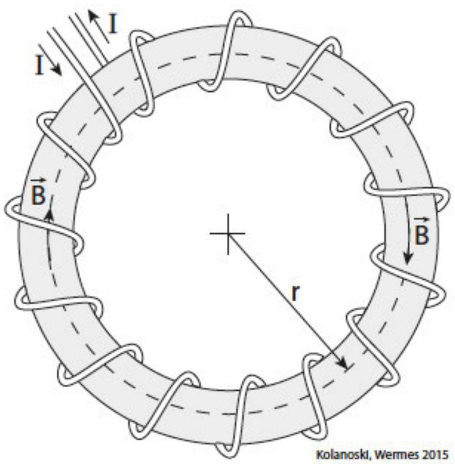
Almost all HEP experiments at accelerators have a magnetic spectrometer to measure the momentum of charged particles.

main configurations:

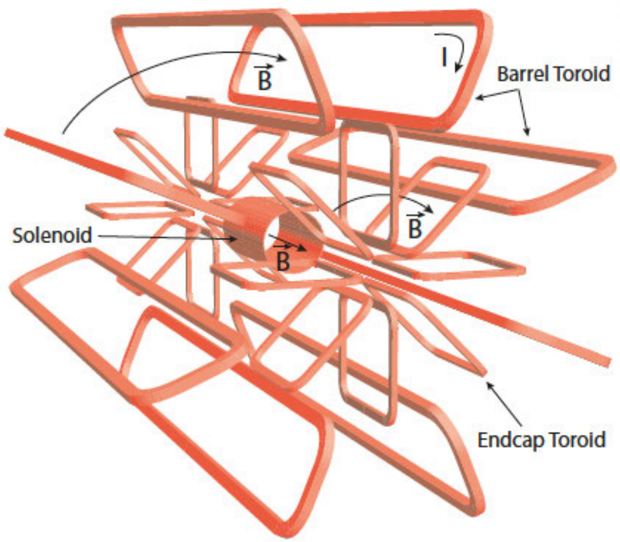
- **dipole**
- **solenoid**
- **toroid**

$$N\,I = \frac{1}{\mu_0\,\mu} \oint \vec{B}\,d\vec{l} = \frac{1}{\mu_0\mu} B(r)\,2\pi r$$

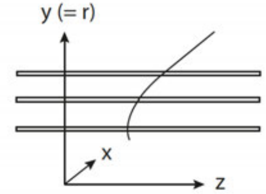
Toroid



(a)



(b)



ATLAS: 0.5 T

- **azimuthal symmetry**
  - deflection in (r - z) - plane
  - tracking detectors are (in ATLAS) also arranged in cylindrical shells
  - but measurement of curved trajectories in r-z planes at fixed r

Example: Tracking in LHCb (Dipole)

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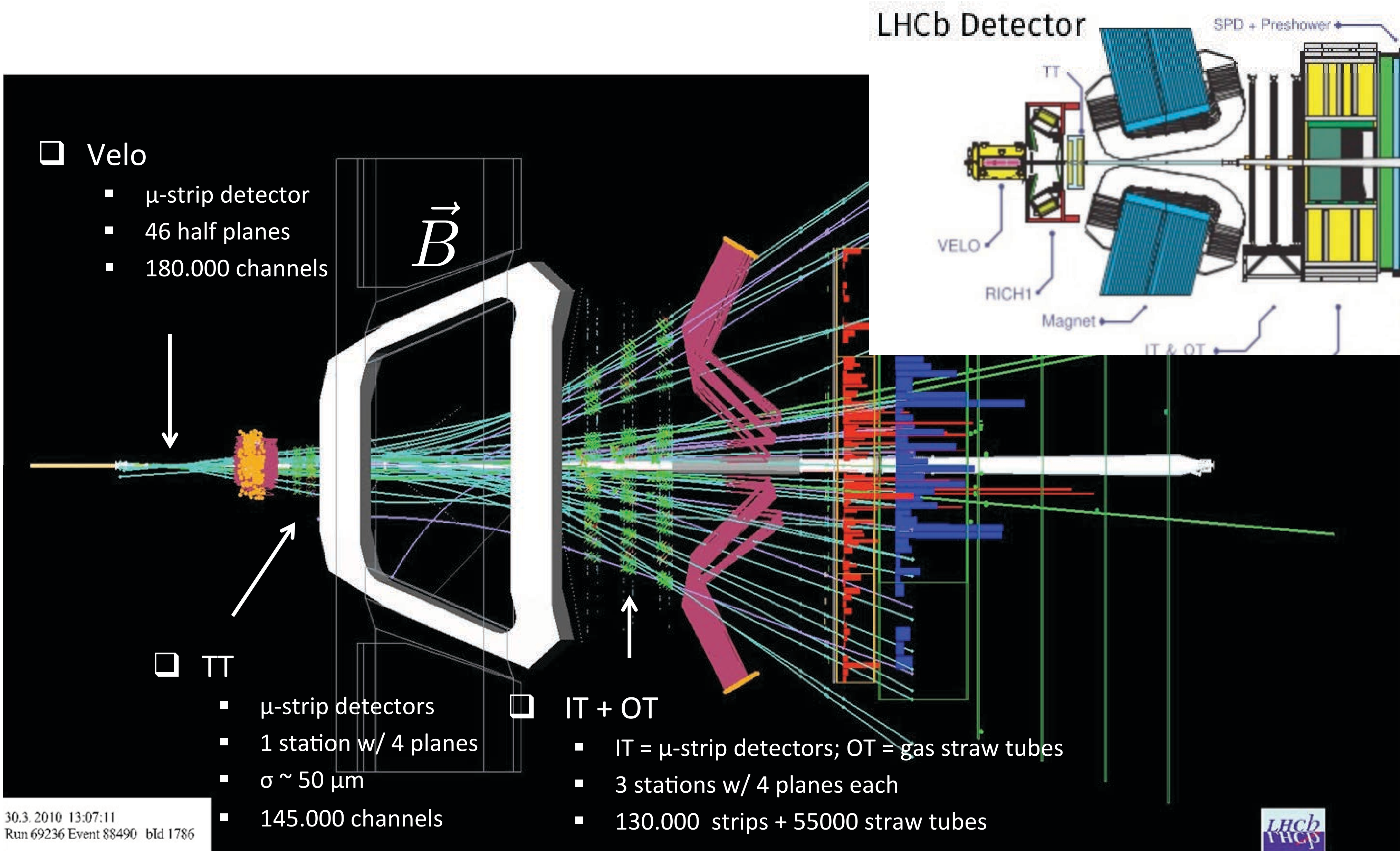
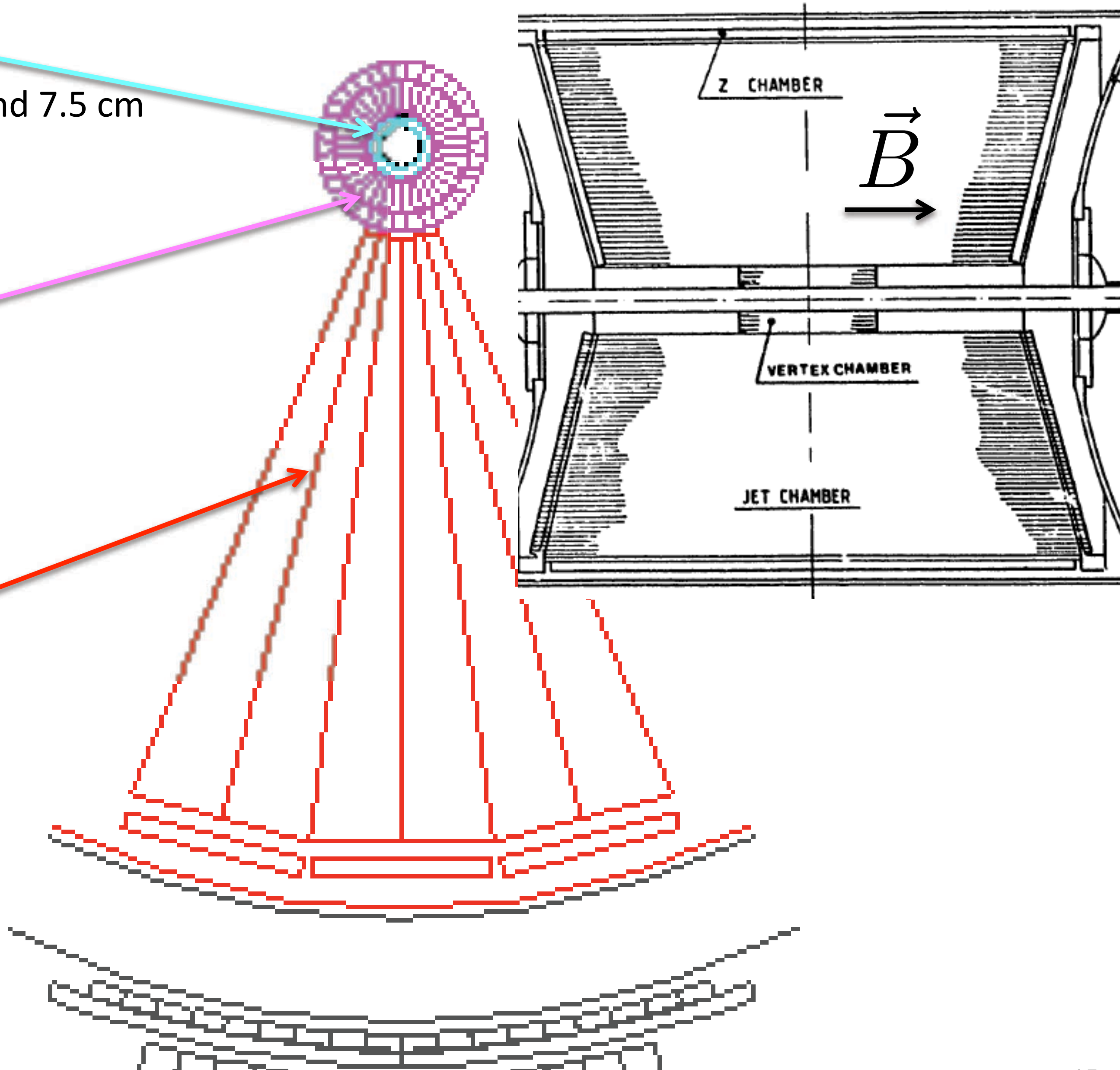


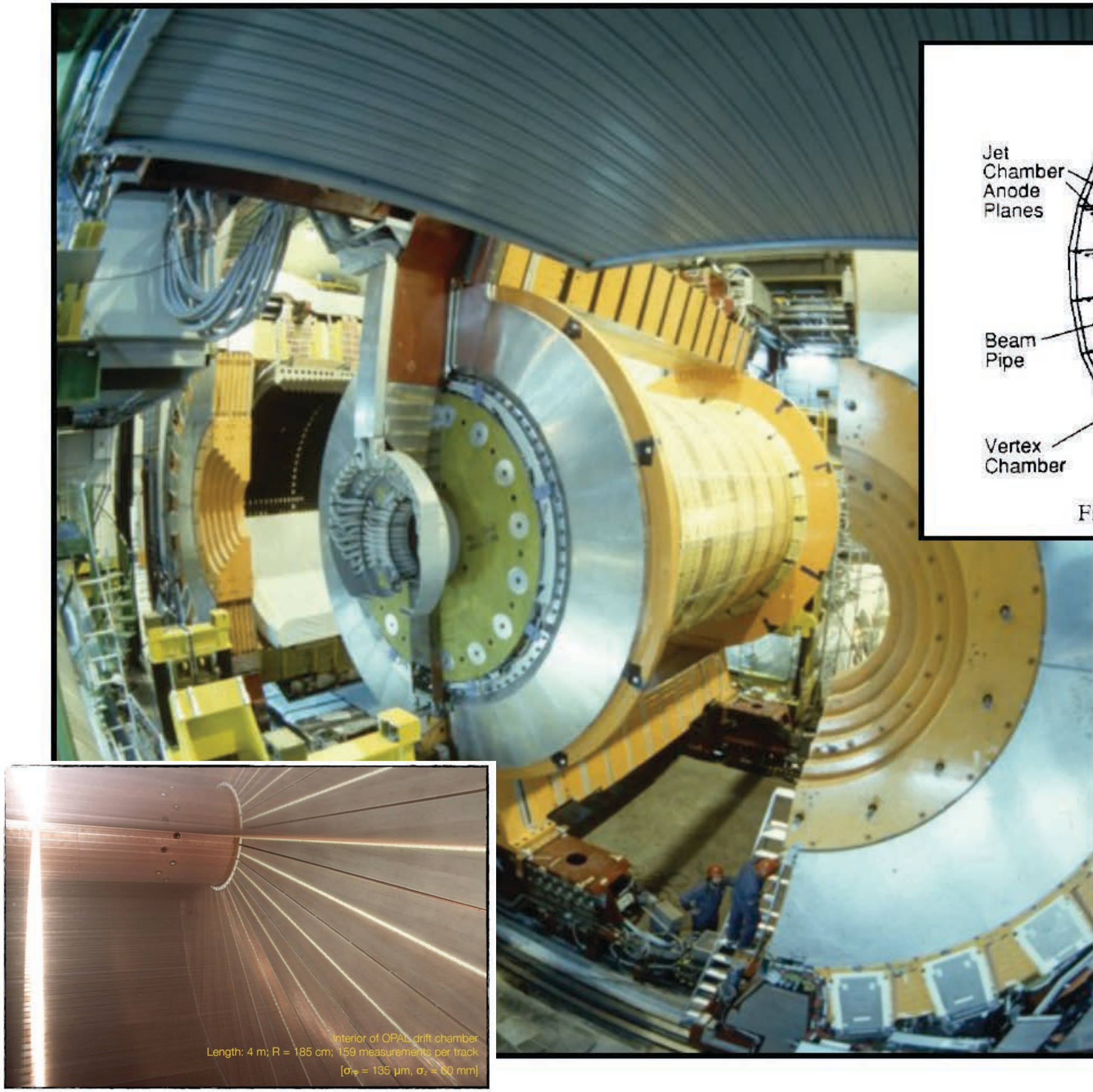
- Velo
  - µ-strip detector
  - 46 half planes
  - 180.000 channels
- TT
  - µ-strip detectors
  - 1 station w/ 4 planes
  - σ ~ 50 µm
  - 145.000 channels
- IT + OT
  - IT = µ-strip detectors; OT = gas straw tubes
  - 3 stations w/ 4 planes each
  - 130.000 strips + 55000 straw tubes

Example: Tracking in OPAL (LEP) (solenoid)

- **Micro Vertex Detector (Si)**
  - 2 double-barrels @ r = 6.1 and 7.5 cm
  - strip pitch 25 µm
  - $\sigma_{r\phi}$= 5-10 µm σ_z = 5-10 µm
- **Vertex Chamber (gas)**
  - 648 wires
  - 36 axial and 36 stereo cells
  - stereo angle 4°
  - $\sigma_{r\phi}$= 50 µm, σ_z= ~700 µm
- **Jet Chamber (gas)**
  - 7600 wires in 24 sectors
  - 159 points on track
  - $\sigma_{r\phi}$= 120 µm
    σ_z= 4 cm (per wire)



OPAL Jet Chamber



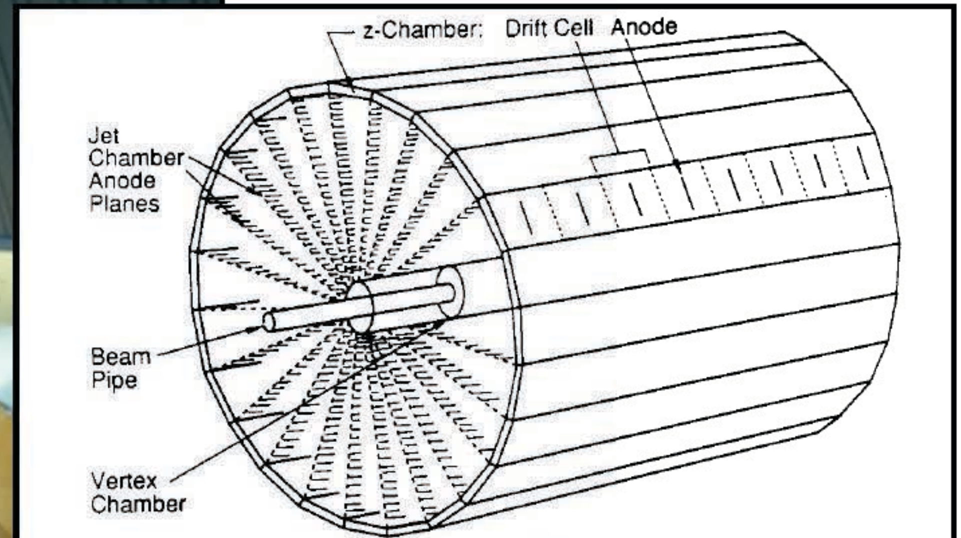
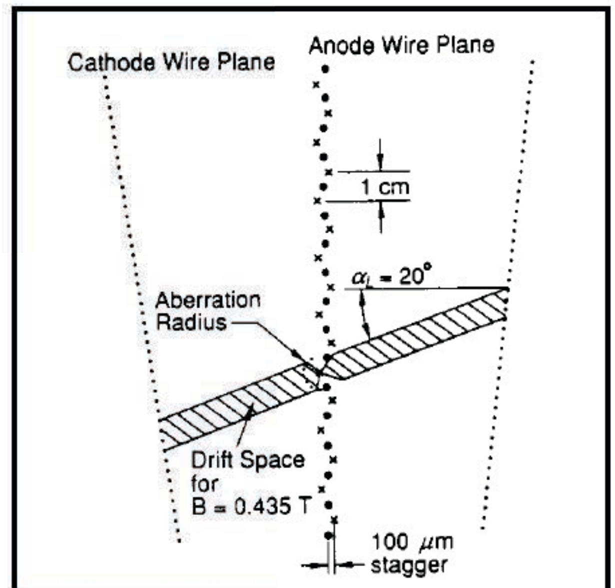


Fig. 1. Schematic view of the OPAL central detector.



Example: Tracking in CMS (solenoid)

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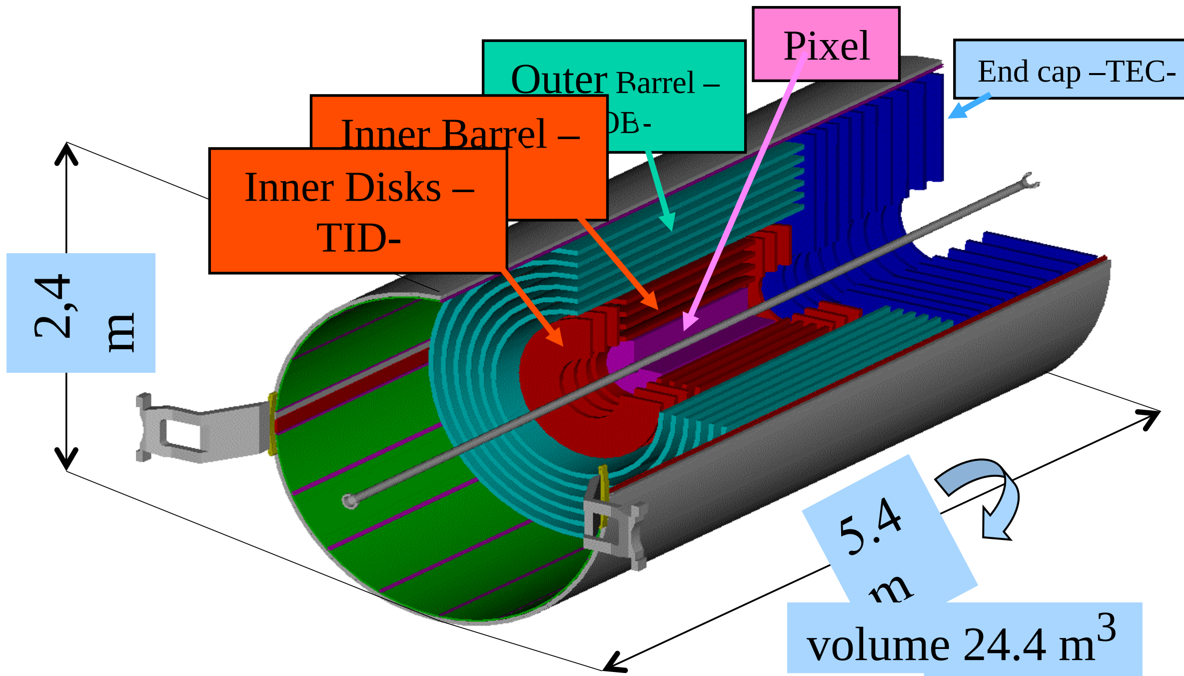
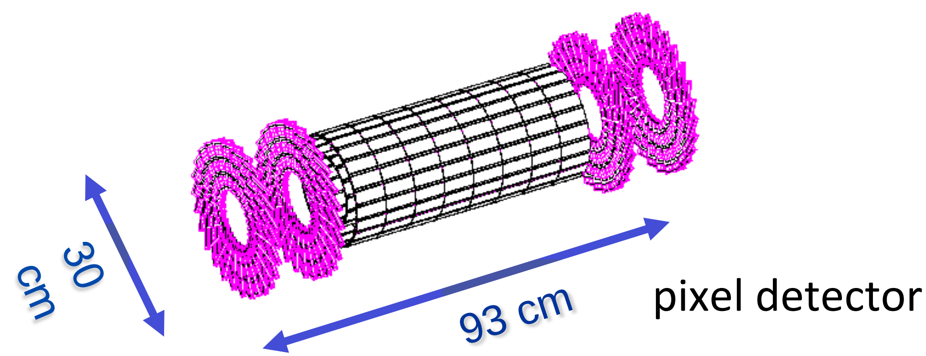
- **Pixel Detector**
  - 2 barrels, 2 disks: 40×10^6 pixels
  - barrel radii: 4.1, ~10 cm
  - pixel size 100×150 μm
  - $\sigma_{r\phi}$= 10 μm σ_z = 30 μm
- **Inner Silicon Strip Tracker**
  - 4 barrels, many disks: 2×10^6 strips
  - strip pitch 80,120 μm
  - $\sigma_{r\phi}$= 25 μm σ_z = 230 μm
- **Outer Silicon Strip Tracker**
  - 6 barrels, many disks: 7×10^6 strips
  - barrel radii: max 120 cm
  - strip pitch 80, 120 μm
  - $\sigma_{r\phi}$= 35 μm σ_z = 530 μm



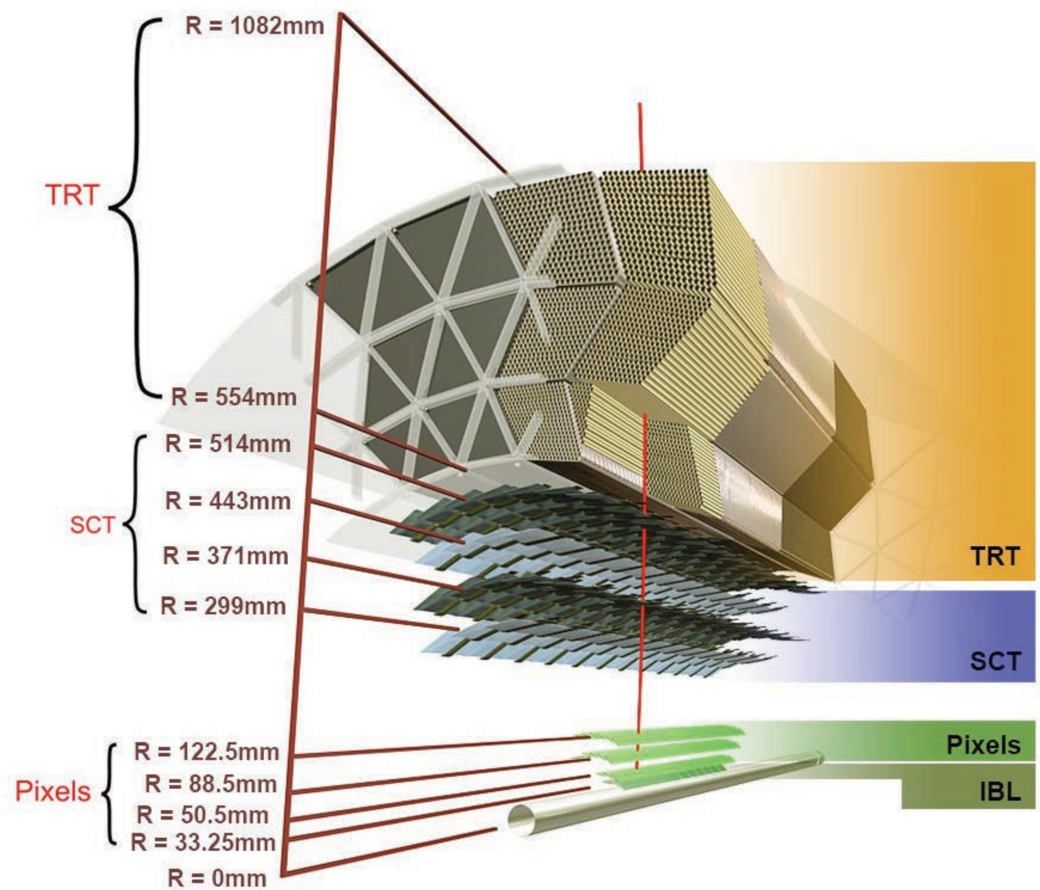


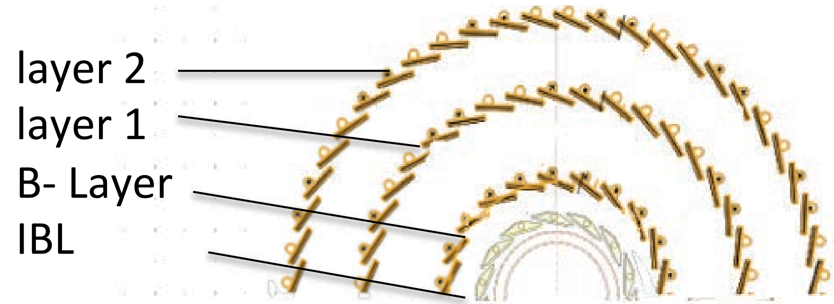
Example: Tracking in ATLAS (solenoid)

- **Pixel Detector**
  - 4 barrels, 3+3 disks: 80×10^6 pixels
  - barrel radii: 3.3, 5.1, 8.9, 12.3 cm
  - pixel size 50×400 µm (50x250 µm)
  - $\sigma_{r\phi}$= 8-10 µm σ_z = 100 µm (70 µm)

- **SCT**
  - 4 barrels, disks: 6.3×10^6 strips
  - barrel radii:30, 37, 44 ,51 cm
  - strip pitch 80 µm
  - stereo angle ~40 mr
  - $\sigma_{r\phi}$= 16 µm σ_z = 580 µm

- **TRT**
  - barrel: 55 cm < R < 105 cm
  - 36 layers of straw tubes
  - $\sigma_{r\phi}$= 170 µm
  - 400.000 channels





Example: Tracking in ATLAS μ-Spectrometer (toroid)

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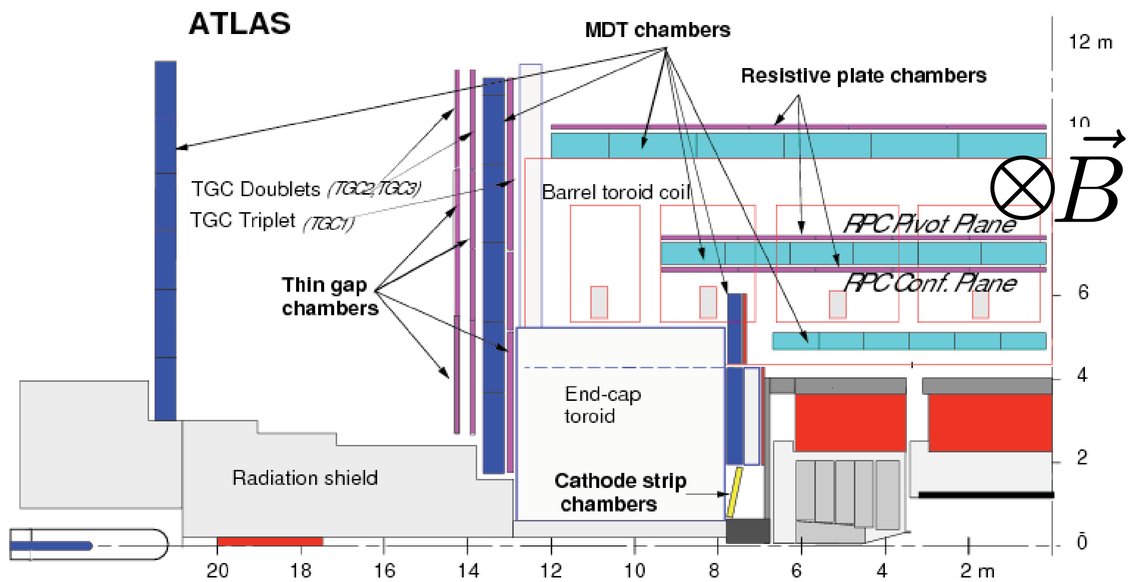
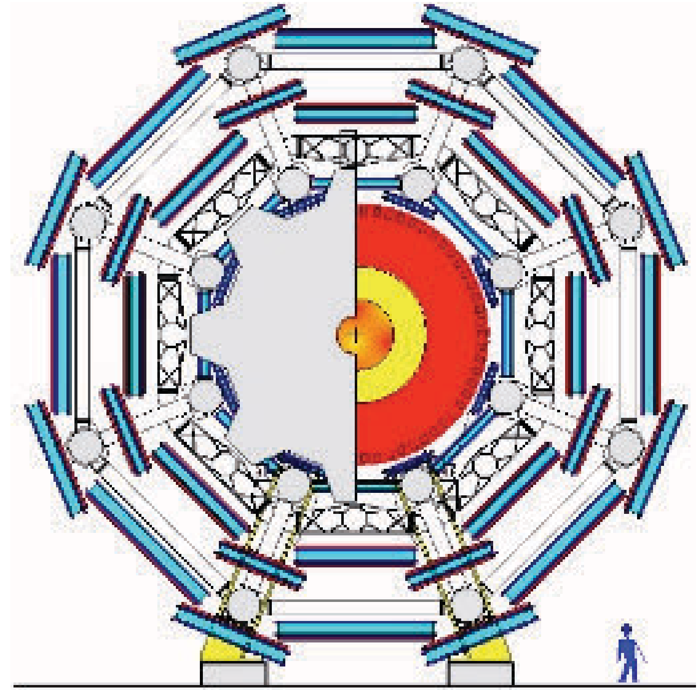
- **MDT = monitored drift tubes**
  - precision tracking
  - σ_z = 35 μm (intrinsic)
  - 1088 chambers, 350 000 channels
- **CSC = cathode strip chambers**
  - precision tracking
  - σ_r = 40 μm, σ_φ = 5 mm (intr.)
  - 32 chambers, 31 000 channels



- **RPC = resistive plate chambers**
  - triggering
  - 600 chambers, 370 000 channels
  - σ_z = 10 mm, σ_ϕ = 10 mm
- **TGC = Thin Gap Chambers**
  - triggering
  - 3600 chambers, 320 000 channels
  - σ_r = 2-6 mm, σ_z = 3-7 mm



(see lecture 2)

Trajectories

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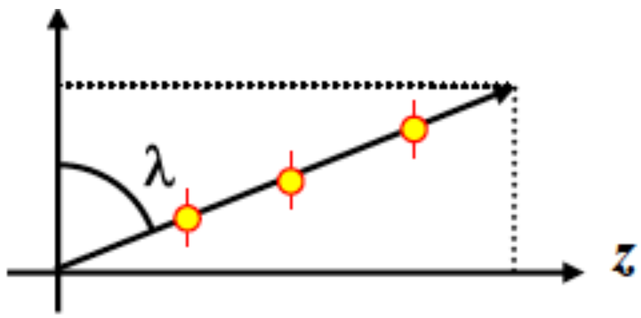
- straight in space

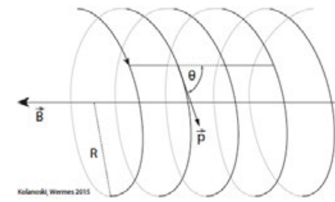
R

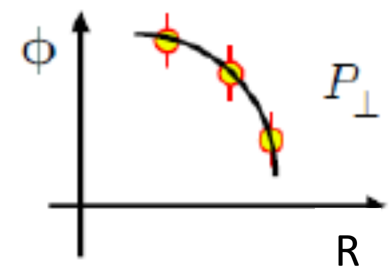


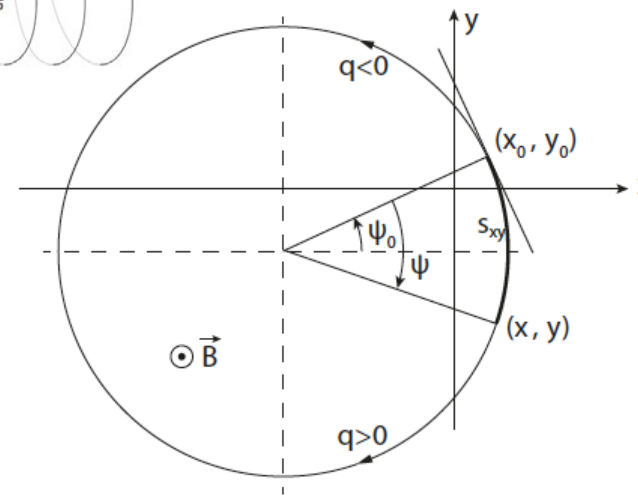
$$y = t_{yx}x + y_0$$
$$z = t_{zx}x + z_0$$

4 parameters

- helix







$$x = \boxed{x_0} + \boxed{R}\left(\cos(\boxed{\psi_0} - \eta\psi) - \cos\psi_0\right)$$
$$y = \boxed{y_0} + R\left(\sin(\psi_0 - \eta\psi) - \sin\psi_0\right)$$
$$z = \boxed{z_0} + \frac{\psi R}{\tan\boxed{\theta}}.$$

6->5 parameters
w/ $d_0^2 = x_0^2 + y_0^2$

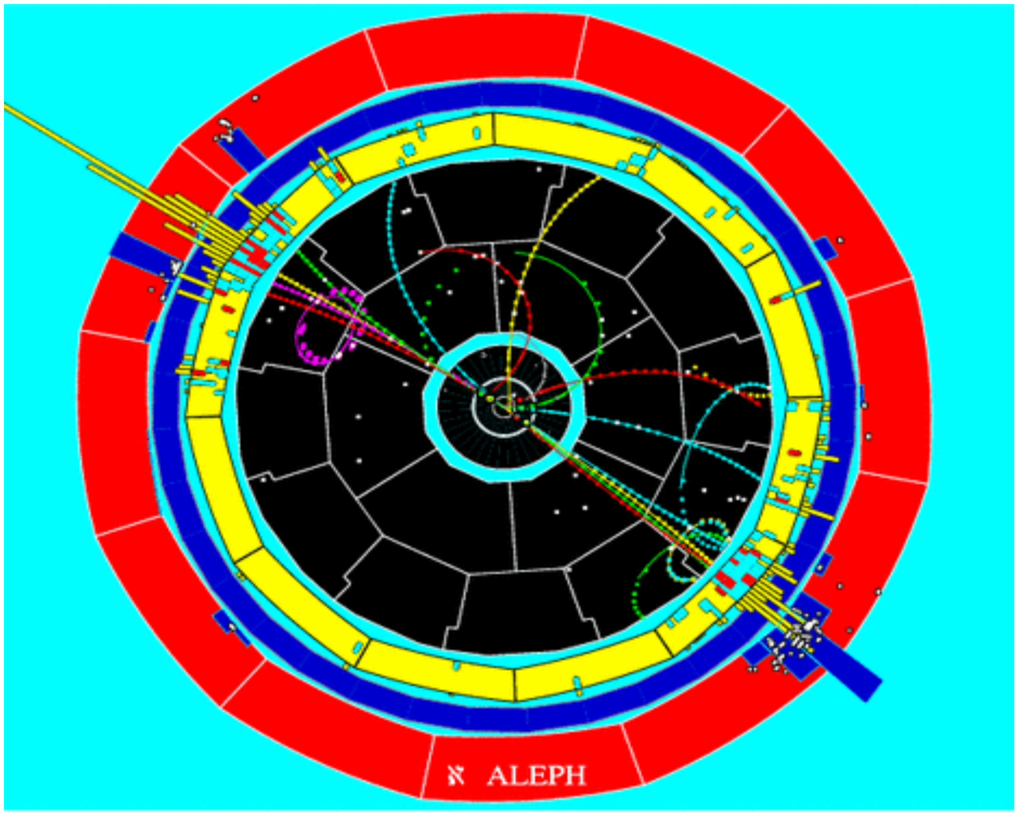
The representation of the circular projection can be series expanded (large R) => parabolic eq.

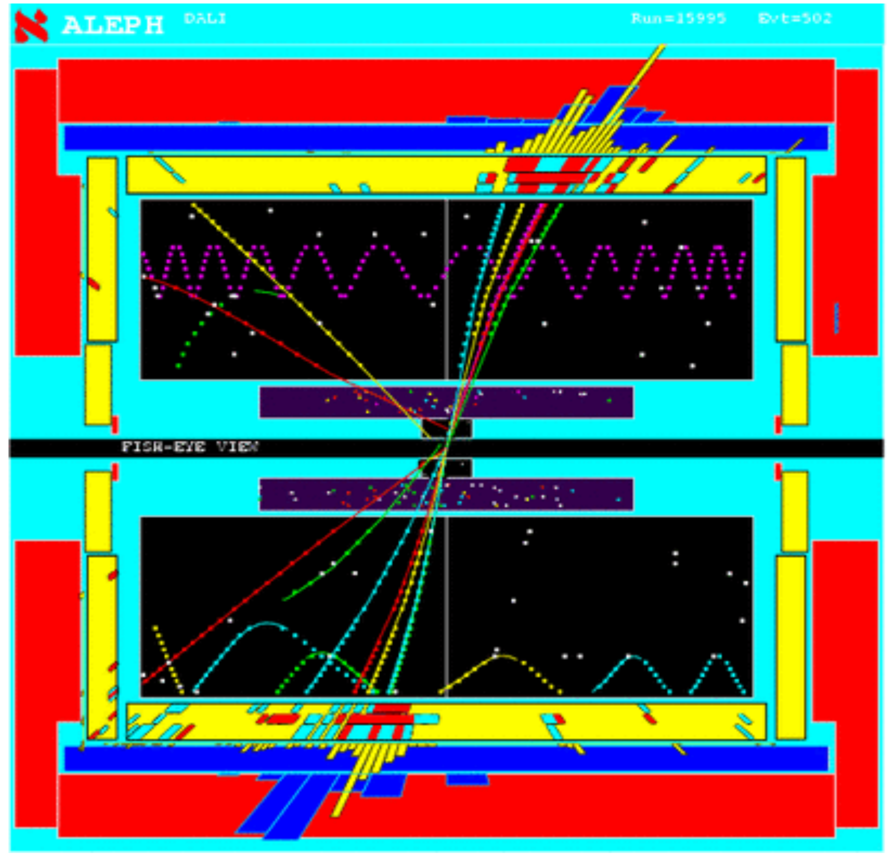
$$y = y_0 + \sqrt{R^2 - (x - x_0)^2}$$

$$\boxed{y = y_0 + R\left(1 - \frac{(x-x_0)^2}{2R^2} + \dots\right)} = \left(y_0 + R - \frac{x_0}{2R^2}\right) + \frac{x_0}{R}x - \frac{1}{2R}x^2 + \dots$$

$$\boxed{= a + bx + \frac{1}{2}cx^2} \approx a + bx \text{ (for very large R)}$$

The Helix … seen in an experiment





For small momenta y is a periodic function of z
For large momenta we have a straight line as a function of z

What we need to do …

- **Once we have measured the transverse momentum and the dip angle the total momentum is**

$$P = \frac{P_\perp}{\cos\lambda} = \frac{0.3BR}{\cos\lambda}$$

- **The error on the momentum is given by the measurement errors on the curvature radius R and the dip angle λ**

$$\frac{\partial P}{\partial R} = \frac{P_\perp}{R}$$

$$\frac{\partial P}{\partial \lambda} = -P_\perp \tan\lambda$$

$$\boxed{\left(\frac{\Delta P}{P}\right)^2 = \left(\frac{\Delta R}{R}\right)^2 + (\tan\lambda\Delta\lambda)^2}$$

relative error (%)

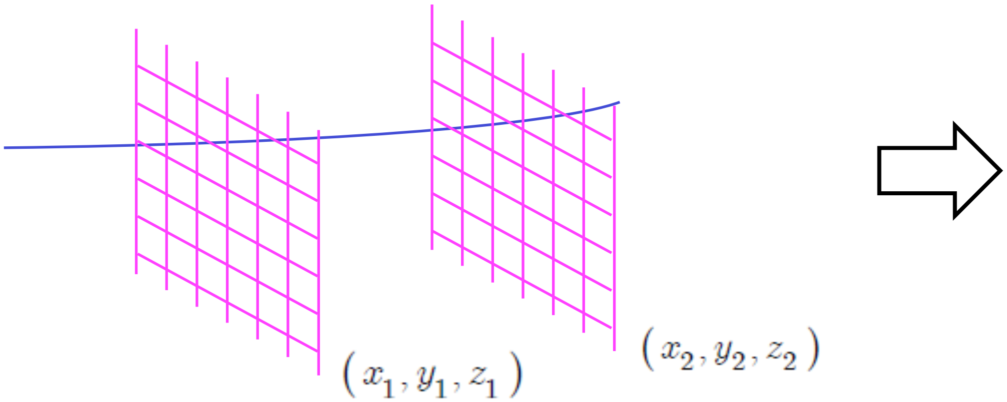
- **We need to study (for solenoid magnets)**
  - the error on the radius measured in the bending plane r - ϕ
  - the error on the dip angle in the r - *z* plane

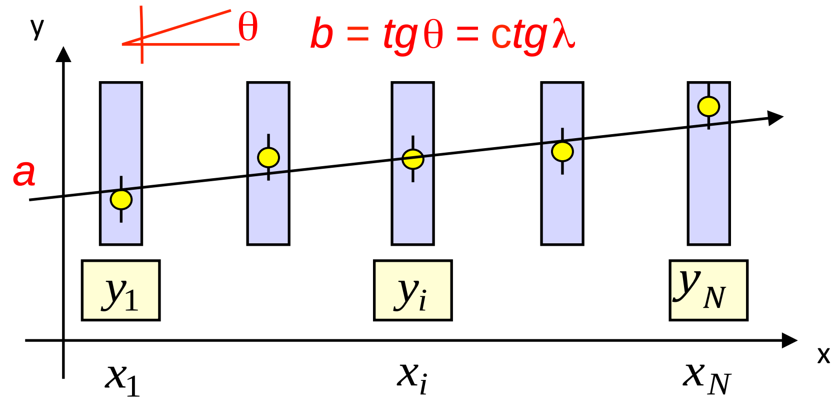
- **… and also**
  - The contribution of multiple scattering to the momentum resolution

- **Comment:**
  - in a hadron collider like LHC the main emphasis is on transverse momentum measurement
  - elementary processes take place among partons that are not at rest in the laboratory frame
  - → use momentum conservation only in the transverse plane

> 25 mins?

Using a track model …





N measurements

$$S = \sum_{i=1}^{N}\sum_{j=1}^{N}(\xi_i^{meas} - \xi_i^{fit})\ V_{y,ij}^{-1}\ (\xi_j^{meas} - \xi_j^{fit}) \quad = \sum_{i=1}^{N} \frac{(\xi_i^{\text{mess}} - \xi_i^{fit}(\theta))^2}{\sigma_i^2}$$

covariance matrix

if V is diagonal

in general: track fit in matrix formalism (x-y space)

- **f be a linear function of the parameters θ_i:** $f(x|\theta) = \theta_1\, f_1(x) + \ldots + \theta_m\, f_m(x) = \sum_{j=1}^{m} \theta_j\, f_j(x)$
  (i.e. true for both discussed cases)
- **then the expectation values for the measurement points at positions x_i are:**

$$\eta_i = \theta_1\, f_1(x_i) + \ldots + \theta_m\, f_m(x_i) = \sum_{j=1}^{m} \theta_j\, f_j(x_i) = \sum_{j=1}^{m} H_{ij}\, \theta_j$$

H_{ij} → n x m matrix

- **the minimization requirement then reads:**

$$\Rightarrow \boxed{S = (\vec{y} - H\,\theta)^T\, V_y^{-1}\, (\vec{y} - H\,\theta) \longrightarrow \text{min}}$$

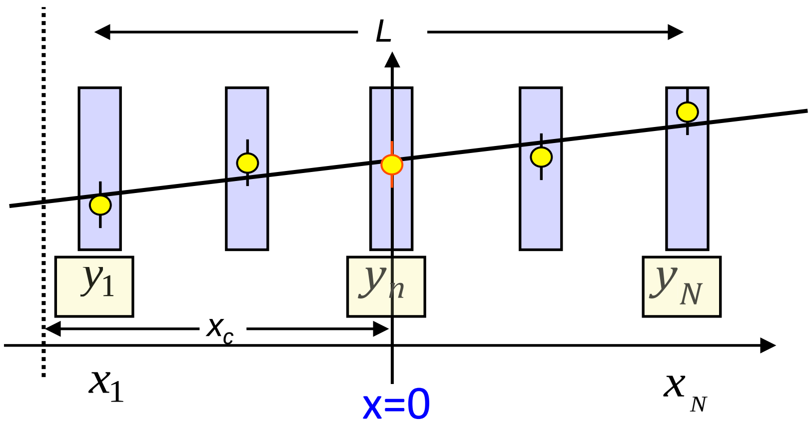
with solution $\hat{\theta} = \underbrace{\left(H^T\, V_y^{-1}\, H\right)^{-1}\, H^T\, V_y^{-1}}_{=:\, A}\, \vec{y} = A\vec{y} \quad \Rightarrow \quad \boxed{\hat{y} = \sum_{j=1}^{m} \hat{\theta}_j\, f_j(x)}$ best coord. estimate for given x

and errors $V_\theta = A\, V_y\, A^T = \left(H^T\, V_y^{-1}\, H\right)^{-1}$ (error propagation = linear trafo of V_y)

$$\Rightarrow \quad \sigma_y^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial y}{\partial \theta_i} \frac{\partial y}{\partial \theta_j}\, V_{\theta,ij} = \boxed{\sum_{i=1}^{m} \sum_{j=1}^{m} f_i(x)\, f_j(x)\, V_{\theta,ij}}$$

error of best fit coordinate

Application to a straight line



N measurements $\quad x_N - x_1 = L\,,$

$$x_i = x_1 + (i-1)\,\frac{L}{N-1} \qquad x_c = \frac{x_1 + x_N}{2} = 0$$

$$f_1(x) = 1,\;\; f_2(x) = x \qquad \theta_1 = a,\;\; \theta_2 = b$$

$$\Rightarrow \quad y = f(x|\theta) = a + b\,x$$

fit $\quad \Rightarrow \quad \hat{y} = \sum_{j=1}^{m} \hat{\theta}_j\, f_j(x)$ best estimate of y for a given x

with errors (on parameters)

$$V_\theta = A\,V_y\,A^T = \left(H^T\,V_y^{-1}\,H\right)^{-1} \quad \Rightarrow \quad \begin{cases} \sigma_a^2 = \dfrac{\sigma^2}{N} \\ \boxed{\sigma_b^2 = \dfrac{\sigma^2}{N}\,\dfrac{12(N-1)}{(N+1)\,L^2}} \\ \sigma_{ab} = 0 \end{cases}$$

cf. choice of coord. system

dipole

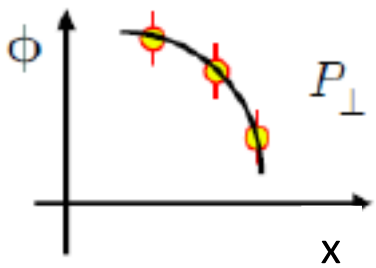
imp.par

… and errors on position estimates

$$\sigma_y^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial y}{\partial \theta_i}\,\frac{\partial y}{\partial \theta_j}\,V_{\theta,ij} \quad \Rightarrow \quad \boxed{\sigma_y^2 = \sigma_a^2 + x_0^2\,\sigma_b^2 = \frac{\sigma^2}{N} + \frac{\sigma^2}{N}\,\frac{12(N-1)}{(N+1)}\,\frac{x_0^2}{L^2}}$$

x_0 = a specificly chosen x - value

Application to a linearized circle



$$y = y_0 + \sqrt{R^2 - (x - x_0)^2}$$

$$\Rightarrow y \approx a \; + \; bx \; + \; \frac{1}{2} c x^2$$

errors

$$V_\theta = A\,V_y\,A^T = \left(H^T\,V_y^{-1}\,H\right)^{-1} \quad \Rightarrow \quad \left\{ \begin{aligned} \sigma_a^2 &= \sigma^2 \frac{3N^2 - 7}{4(N-2)N(N+2)} \\ \sigma_b^2 &= \frac{\sigma^2}{L^2} \frac{12(N-1)}{N(N+1)} \\ \sigma_c^2 &= \frac{\sigma^2}{L^4} \frac{720(N-1)^3}{(N-2)N(N+1)(N+2)} \\ \sigma_{ab} &= \sigma_{bc} = 0 \\ \sigma_{ac} &= \frac{\sigma^2}{L^2} \frac{30N}{(N-2)(N+2)} \end{aligned} \right.$$

$$\sigma_y^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial y}{\partial \theta_i} \frac{\partial y}{\partial \theta_j} V_{\theta,ij}$$

$$\Rightarrow \quad \boxed{\sigma_y^2} = \sigma_a^2 + x_0^2\,\sigma_b^2 + \frac{1}{4} x_0^4\,\sigma_c^2 + x_0^2\,\sigma_{ac}$$

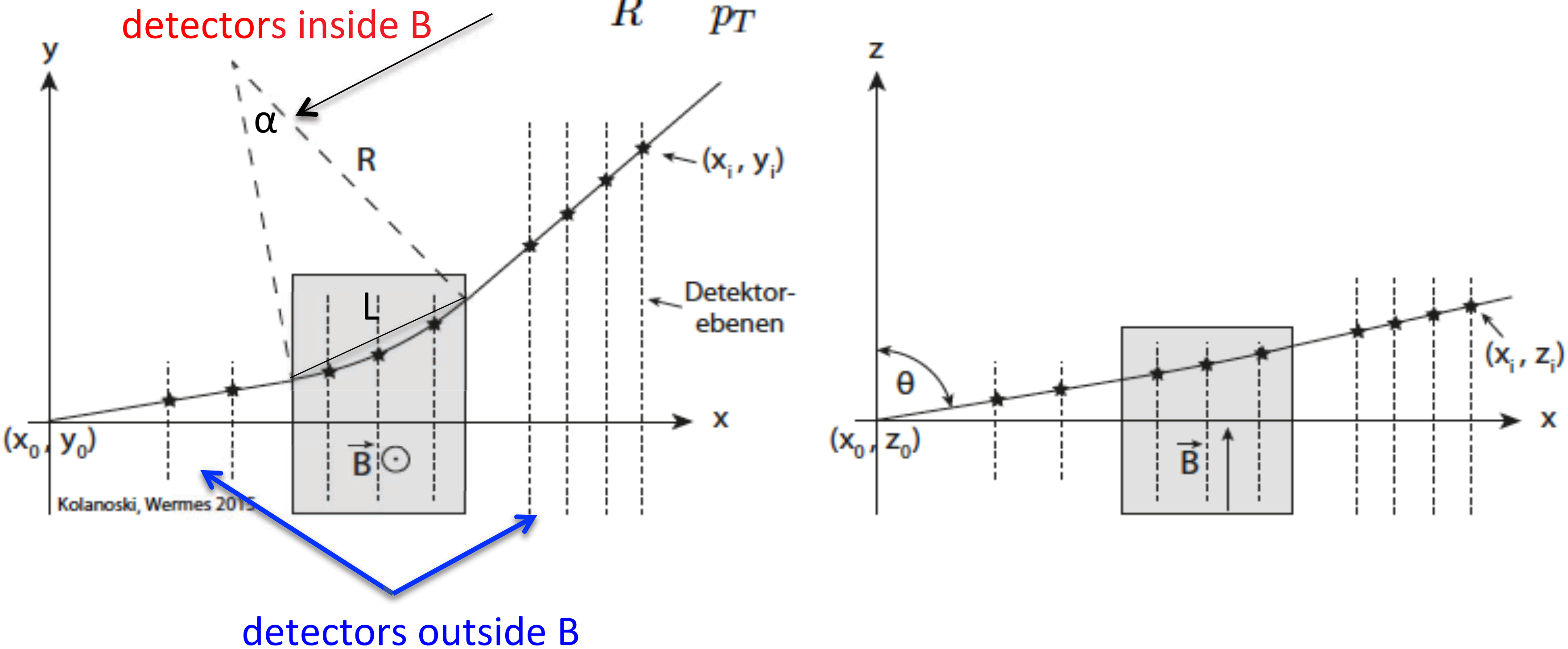
$$= \frac{\sigma^2}{N} \left(1 + \frac{x_0^2}{L^2} \frac{12(N-1)}{(N+1)} + \frac{x_0^4}{L^4} \frac{180(N-1)^3}{(N-2)(N+1)(N+2)} + \frac{x_0^2}{L^2} \frac{30N^2}{(N-2)(N+2)} \right)$$

x_0 = a specificly chosen x - value

Use cases: tracking with a dipole field

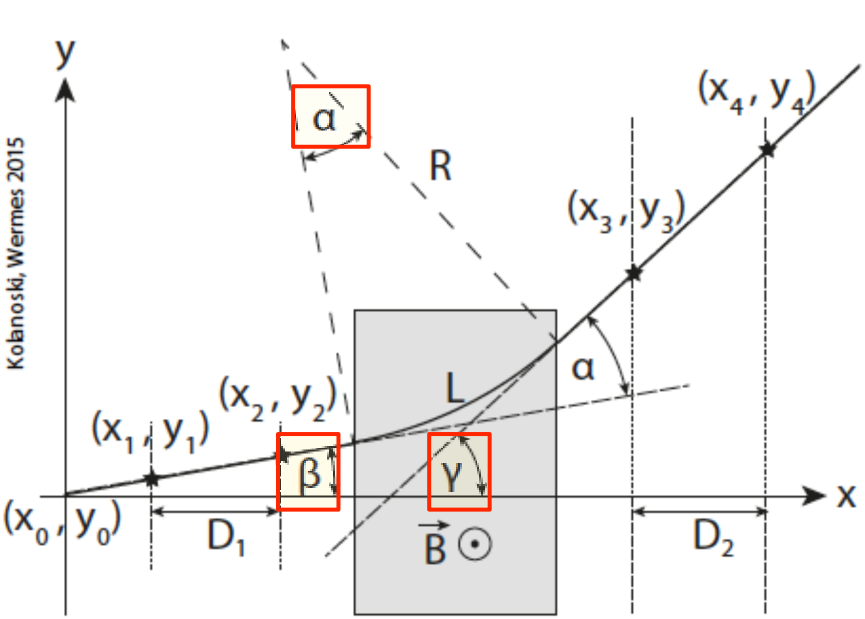
Dipole spectrometer: need a minimum of 3 points in bending plane and 2 in plane perpendicular

$$\alpha = \frac{L}{R} = \frac{q}{p_T} L\,B$$



- ❑ two straight line fits
- ❑ get bending α from difference of the slopes before and after magnet
- ❑ track model can use that straight lines should be tangential to the circle section in B
- ❑ measurements inside B are not mandatory

Use cases: tracking with a dipole field



$$\alpha = \frac{L}{R} = \frac{q}{p_T} L\, B \quad \propto \int_L B dl$$

bending power

$$\alpha = \gamma - \beta$$

$$\tan\beta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{D_1}$$

$$\tan\gamma = \frac{y_4 - y_3}{x_4 - x_3} = \frac{y_4 - y_3}{D_2}$$

$$D_1 = D_2 = D$$

(lever arm)

resolution for 2+2 measurements

$$\sigma_{\tan\beta} = \sqrt{2}\,\frac{\sigma_{\mathrm{mess}}}{D_1}$$

$$\sigma_{\tan\gamma} = \sqrt{2}\,\frac{\sigma_{\mathrm{mess}}}{D_2}$$

$$\sigma_\alpha = \sqrt{2}\,\sigma_{\mathrm{mess}}\sqrt{\frac{1}{D_1^2} + \frac{1}{D_2^2}} = \frac{2\,\sigma_{\mathrm{mess}}}{D}$$

assume equal
$\sigma = \sigma_{\mathrm{mess}}$

resolution (for 2 x N measurements)

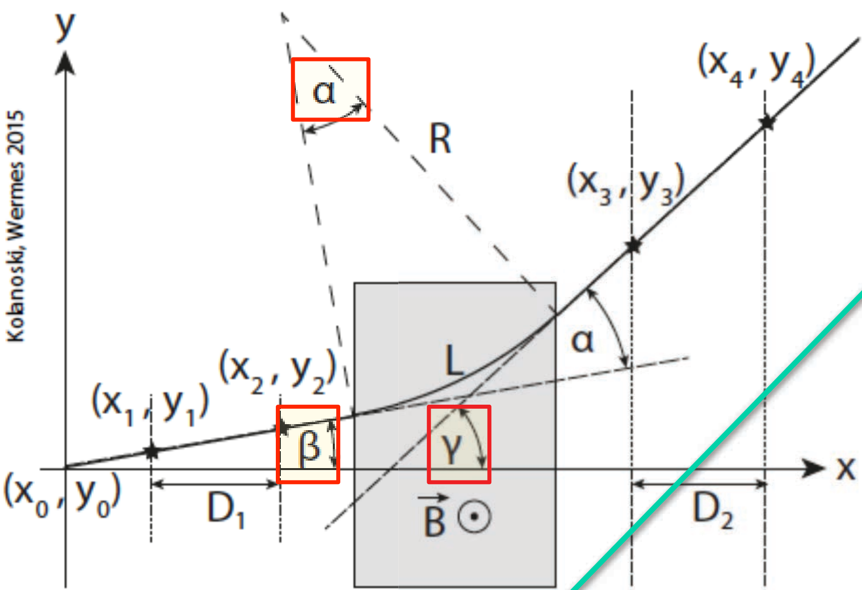
with
(from p. 25)

$$\sigma_{slope} = \frac{\sigma_{\mathrm{mess}}}{D}\sqrt{\frac{12(N-1)}{N(N+1)}}$$

$$\Rightarrow \boxed{\sigma_\alpha = \sqrt{\frac{24(N-1)}{N(N+1)}}\,\frac{\sigma_{\mathrm{mess}}}{D}}$$

$$= 2\,\frac{\sigma_{mess}}{D}\,(for\ N = 2)$$

Use cases: tracking with a dipole field



$$\alpha = \frac{L}{R} = \frac{q}{p_T} L\,B \quad \propto \int_L B\,dl$$

bending power

$$\alpha = \gamma - \beta$$

$$\tan\beta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{D_1}$$

$$\tan\gamma = \frac{y_4 - y_3}{x_4 - x_3} = \frac{y_4 - y_3}{D_2}$$

$$D_1 = D_2 = D$$

(lever arm)

$$dp_T = \frac{p_T^2}{qLB}\,d\alpha \quad \Rightarrow \quad \boxed{\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_{\text{mess}}\,p_T}{0.3\,|z|\,L\,B\,D}\sqrt{\frac{24(N-1)}{N(N+1)}}}$$

momentum resolution with dipole field

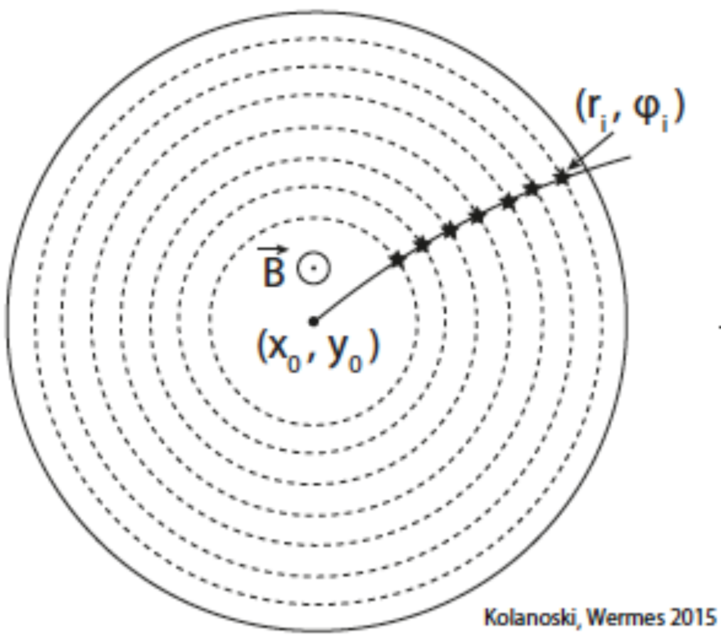
example: N = 5, σ_{mess} = 100 µm, D = 1m, B = 1.5 T, L = 2 m

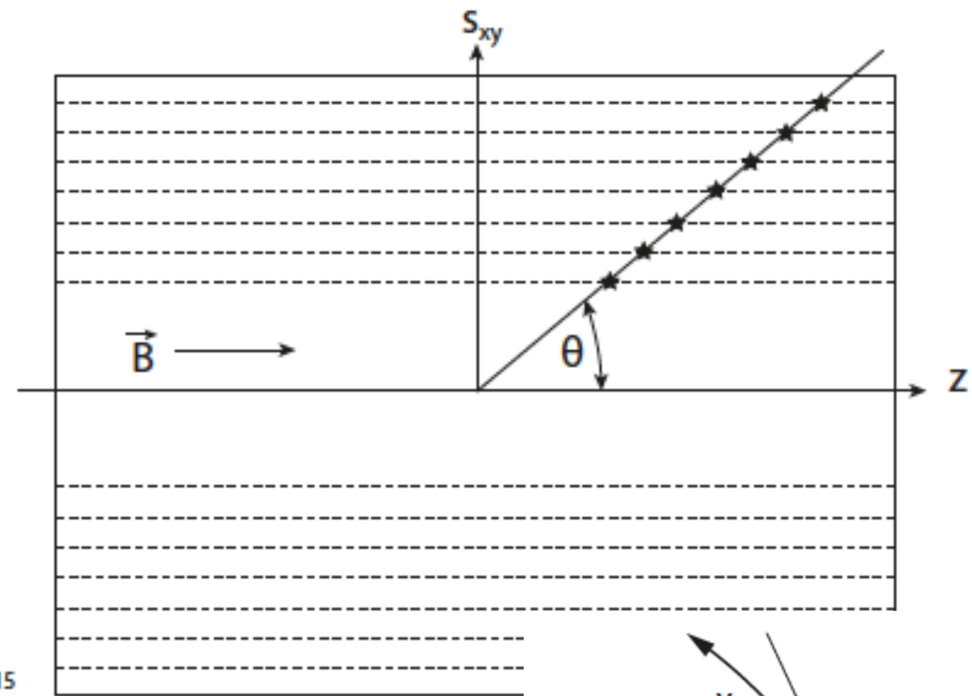
$$\Rightarrow \quad \frac{\sigma_{p_T}}{p_T} = 0.2 \cdot 10^{-3}\,p_T/(\mathrm{GeV/c})$$

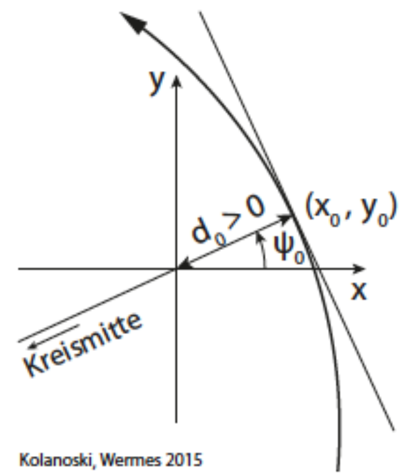
i.e. 0.2% @ 10 GeV/c

Use cases: solenoid

Solenoid spectrometer: can often use beam center as "beam constraint"







measure: (r_i, ϕ_i) or $(r_i\ \phi_i, z_i)$

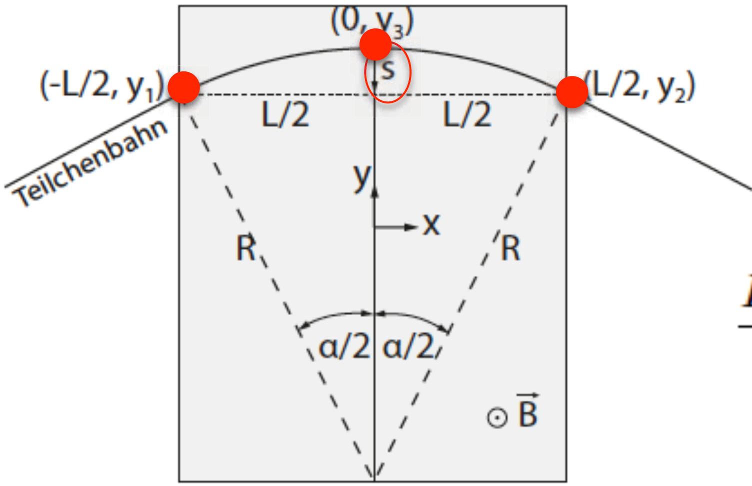
helix model with **5 parameters:**

$$\boxed{\kappa = \frac{\pm 1}{R}, \psi_0, d_0, \theta, z_0}$$

$$x_0 = d_0 \cos\psi_0 \,, \qquad y_0 = d_0 \sin\psi_0$$

Use cases: solenoid

the sagitta s



$$p_T = |q|\, B\, R = \frac{q\, B}{\kappa}$$

The precision of a measurement of the momentum in a homogeneous B-field is determined by the precision with which the sagitta s can be measured.

$$\left.\begin{aligned} \frac{R-s}{R} &= \cos\frac{\alpha}{2} \approx 1 - \frac{\alpha^2}{8} \\ \frac{L}{2\,R} &= \sin\frac{\alpha}{2} \approx \frac{\alpha}{2} \end{aligned}\right\} s = \frac{R\,\alpha^2}{8} = \frac{1}{8}\frac{L^2}{R} = \frac{1}{8} L^2\, |\kappa|$$

hence $\boxed{\sigma_\kappa = \frac{8}{L^2}\,\sigma_s}$

if we measure y at the three distinct points ● we have (N=3):

$$s = y_3 - \frac{y_1 + y_2}{2} \quad \Rightarrow \quad \sigma_s = \sqrt{\sigma^2_{\mathrm{mess}} + \frac{1}{4}\, 2\, \sigma^2_{\mathrm{mess}}} = \sqrt{\frac{3}{2}}\,\sigma_{\mathrm{mess}} \quad \Rightarrow \quad \sigma_\kappa = \frac{\sqrt{96}}{L^2}\sigma_{mess}$$

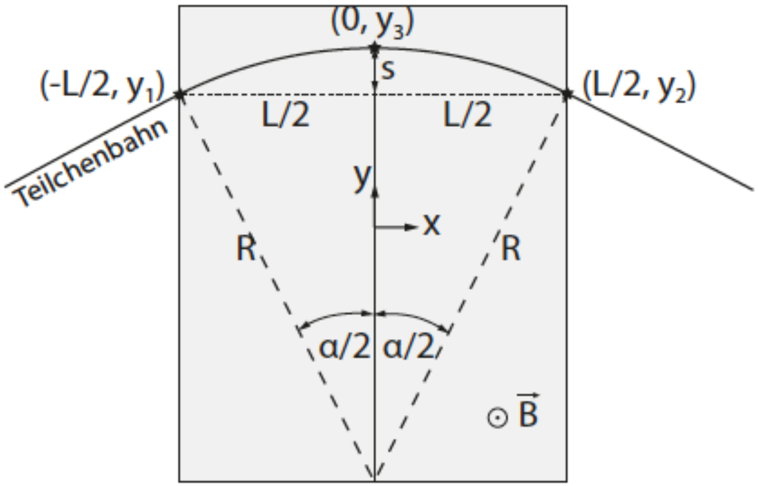
for N measurements use linearized circle approximation and matrix formalism (see p. 26)

for N > 10

$$\boxed{\sigma_\kappa = \frac{\sigma_{\mathrm{mess}}}{L^2}\sqrt{\frac{720(N-1)^3}{(N-2)N(N+1)(N+2)}}} \quad \boxed{\approx \frac{\sigma_{\mathrm{mess}}}{L^2}\sqrt{\frac{720}{N+4}}}$$

Use cases: solenoid

the sagitta s



$$p_T = |q|\,B\,R = \frac{q\,B}{\kappa}$$

$$\Rightarrow \quad \sigma_{p_T} = \frac{p_T^2}{|q|\,B}\sigma_\kappa \overset{(!)}{=} \frac{p_T^2}{0.3\,|z|\,B}\sigma_\kappa$$

Hence we get for N equidistant points:

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\text{mess}} = \frac{p_T}{0.3|z|}\frac{\sigma_{\text{mess}}}{L^2 B}\sqrt{\frac{720}{N+4}}$$

Gluckstern formula
NIM 24 (1963) 381

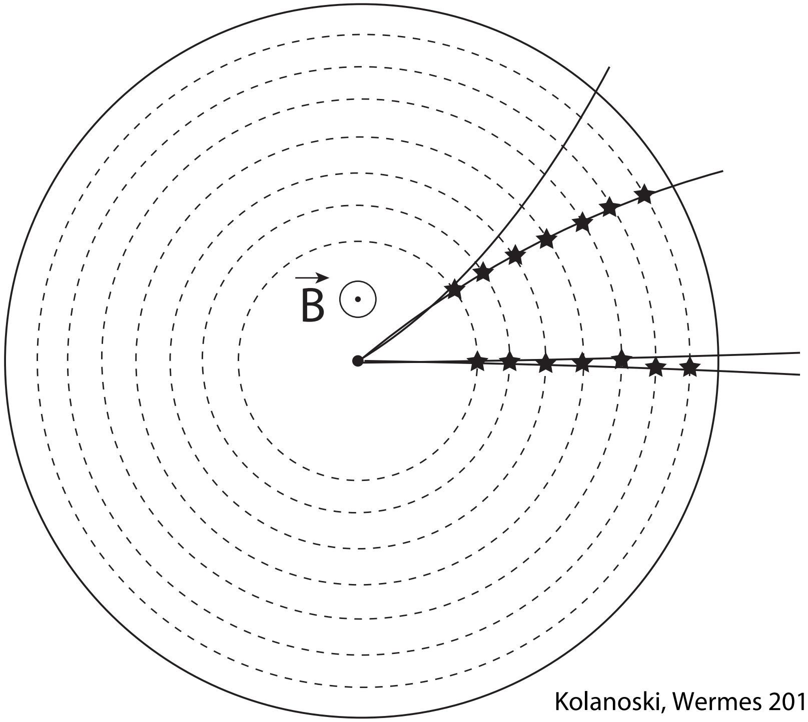
$$[p_T] = \text{GeV}/\text{c}, \;\; [L] = \text{m}, \;\; [B] = \text{T}$$

dependencies for optimization

- optimize σ_{mess} until other effects dominate (e.g. multiple scattering, next)
- $1/L^2$: the longer the better
- better with N, but only $1/\sqrt{N}$
- linear better with B-field strength (!) … but more confusion if many tracks
- rel. error ~p_T (=> p_T of very stiff tracks cannot be measured) .

Note: resolution not symmetric and not gaussian in p_T, only ~in κ -> generate random no's. in κ

The sign of charge of a particle



Kolanoski, Wermes 2015

- The largest momenta that can be resolved (with nσ) wrt the sign of charge can be calculated with the condition

$$\frac{1}{R} = |\kappa| = \frac{0.3B}{p_T} > n\sigma_\kappa \Rightarrow |p_t| < \frac{0.3B}{n\sigma_\kappa}$$

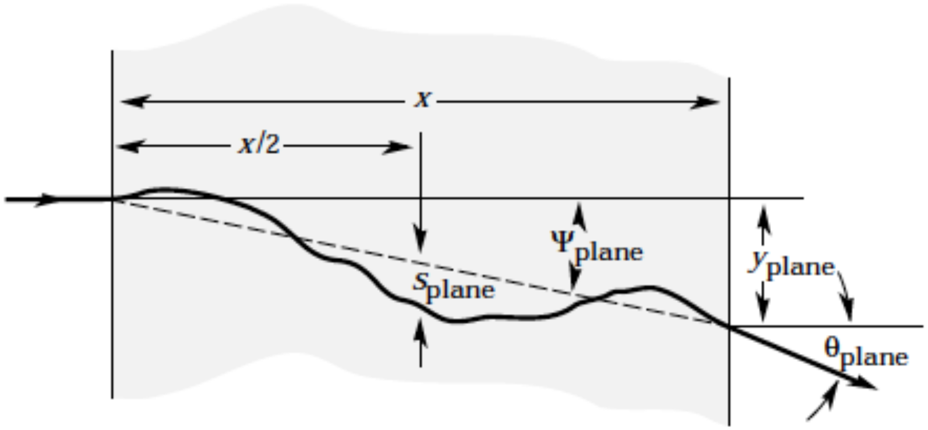
- with $\sigma_\kappa \approx \frac{\sigma_{\text{mess}}}{L^2}\sqrt{\frac{720}{N+4}}$

we get

$$\boxed{p_T < \frac{0.3\, z\, B\, L^2}{n\sigma_{\text{mess}}}\sqrt{\frac{N+4}{720}}}$$

example: N = 12, L=1m, B = 1T, σ_{mess} = 100 µm, z = 1 => p_T < 220 GeV/c for n = 2

The influence of Multiple Coulomb Scattering

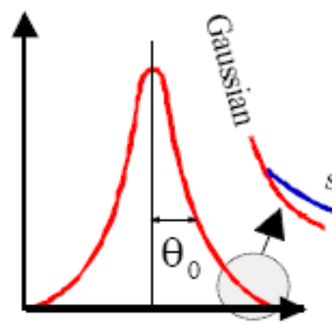


charged particles undergo multiple Coulomb scattering processes when passing through matter

average deviation in a thickness x constitutes additional contribution to sagitta:

$$\langle s_{plan} \rangle = \frac{1}{4\sqrt{3}}\, x\, \theta_0 \quad \approx \sigma_{sagitta}$$

- The scattering angle has a distribution that is almost gaussian
- At large angles deviations from gaussian distributions appear that manifest as a long tail going as $\sin^{-4}\theta/2$ (Moliere theory).
- In "thick" detectors the distribution of the lateral displacement y_{plane} should also be considered.



$$\frac{1}{\sin^4 \frac{\theta_p}{2}}$$

radiation length

$$\theta_0 = \frac{13.6\,\mathrm{MeV/c}}{p\beta} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \frac{x}{X_0}\right)$$

$$\boxed{\sigma_\kappa = \frac{8}{L^2}\sigma_s} = \frac{8}{L^2}\frac{1}{4\sqrt{3}}\, x\, \theta_0 \overset{x \approx L}{=} \sqrt{\frac{4}{3}}\frac{\theta_0}{L} = \boxed{\frac{0.0136\,\mathrm{GeV/c}}{p\,\beta\, L} z \sqrt{\frac{L/\sin\theta}{X_0}} \left(\underset{N=3}{\sqrt{1.33}} - \underset{N>10}{\sqrt{1.43}}\right)}$$

Total momentum resolution

$$= \frac{0.0136 \cdot \sqrt{1.43}}{0.3}$$

$$\sigma_{p_T} = \frac{p_T^2}{|q|\, B}\sigma_\kappa = \frac{p_T^2}{0.3\,|z|\, B}\sigma_\kappa \quad \Rightarrow$$

in GeV/c, Tesla units

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\text{MS}} = \frac{0.054}{L\, B\, \beta}\sqrt{\frac{L/\sin\theta}{X_0}}$$

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\text{mess}} = \frac{p_T}{0.3|z|}\frac{\sigma_{\text{mess}}}{L^2 B}\sqrt{\frac{720}{N+4}}$$ for N>10

$$[p_T] = \text{GeV/c},\ [L] = \text{m},\ [B] = \text{T}$$

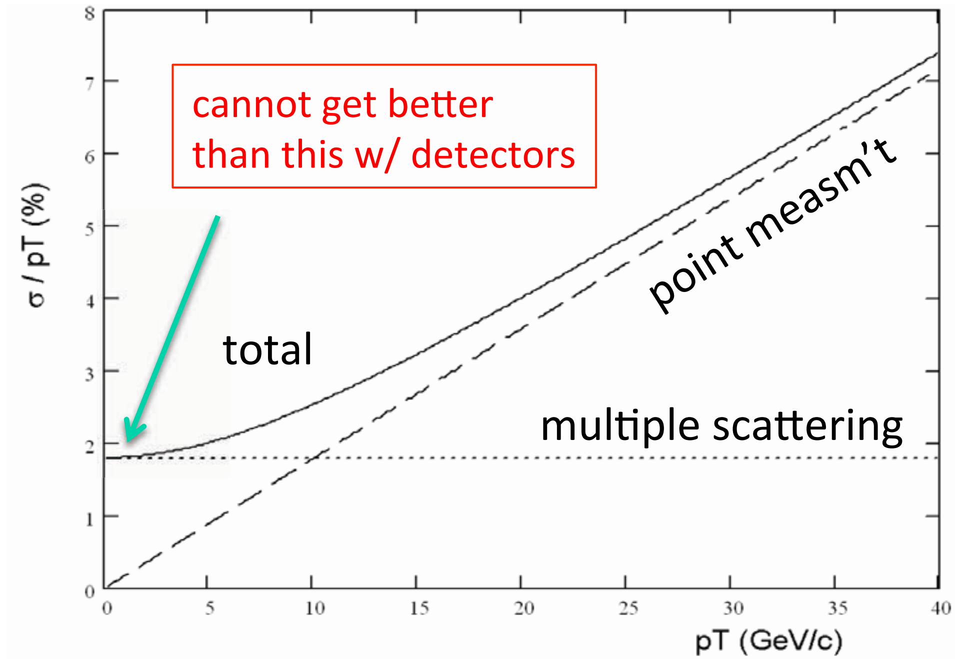
$$\boxed{\frac{\sigma_{p_T}}{p_T} = \sqrt{\left(\frac{\sigma_{p_T}}{p_T}\right)^2_{\text{mess}} + \left(\frac{\sigma_{p_T}}{p_T}\right)^2_{\text{MS}}}}$$

example: OPAL

L = 1.6 m, B = 0.435 T, N = 159, σ_{mess}= 135 µm

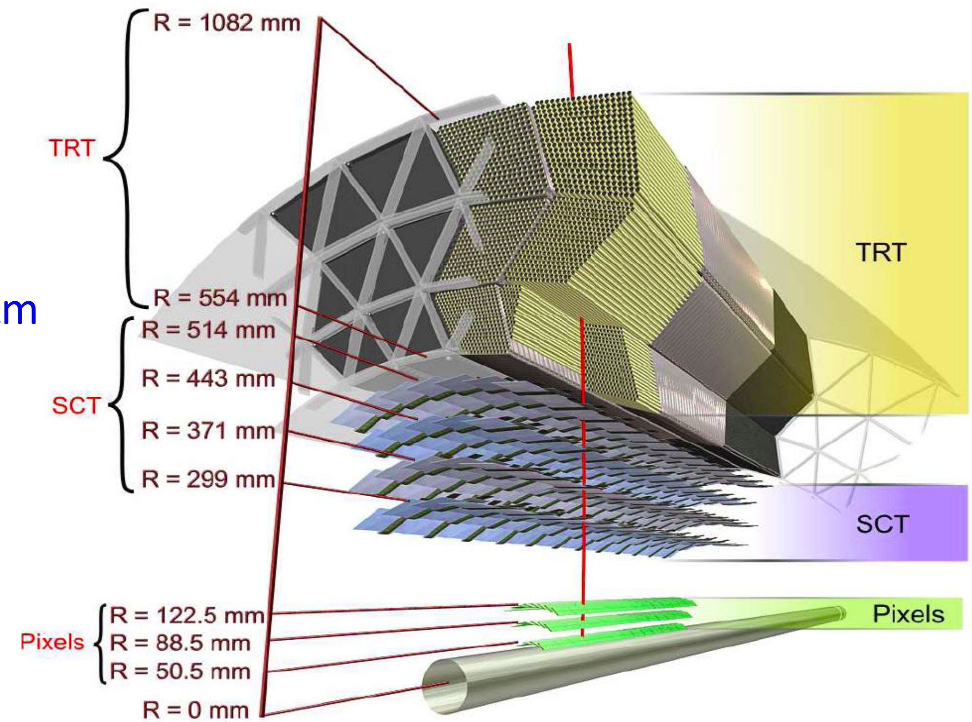
$$\frac{\sigma_{p_T}}{p_T} = \sqrt{(0.0015\, p_T)^2 + (0.02)^2}$$

p_T in GeV



Example: Momentum Resolution in ATLAS

- We can now give a rough estimate of the momentum resolution of the ATLAS tracking systems (pixels + SCT + TRT)



Simplifications:

- assume high momenta (no MS)
- R_{min} = 5.05 cm, R_{max} = 1082cm
- pixels (5cm to 12cm) N=3 (up to 2012), σ = 12µm
- SCT (30cm to 55cm) N=4 layers, σ = 16 µm
- TRT (55cm to 105 cm) N = 36, σ = 170 µm
  → use as a single point with σ = 28 µm
  at R = 80 cm (= $R_{max} \Rightarrow$ L = 75 cm)
- N = 3 + 4 + 1 = 8
- σ = 12, 16, 28 µm → < σ > ~16 µm

$$\Rightarrow \quad \sigma_\kappa \approx \frac{\sigma_{\text{mess}}}{L^2} \times 7.56 \quad \leftarrow \sqrt{\frac{720(N-1)^3}{(N-2)N(N+1)(N+2)}}$$

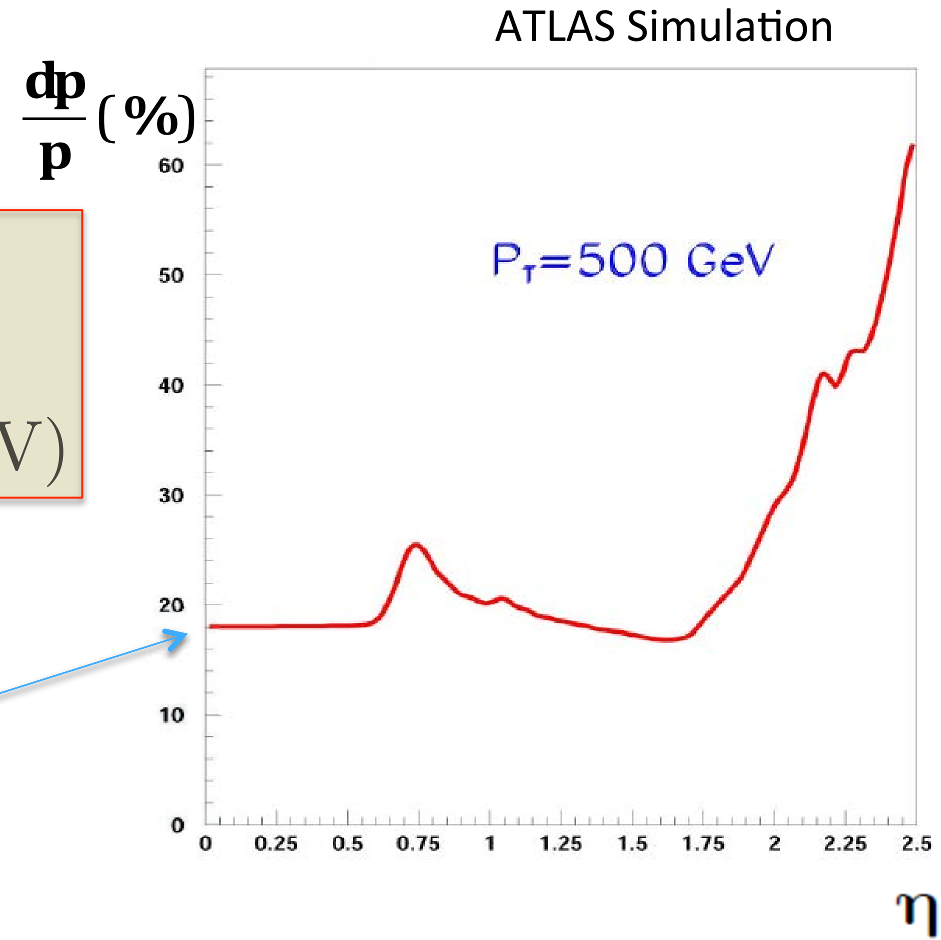
ATLAS Momentum Resolution (cont'd)

B = 2 T

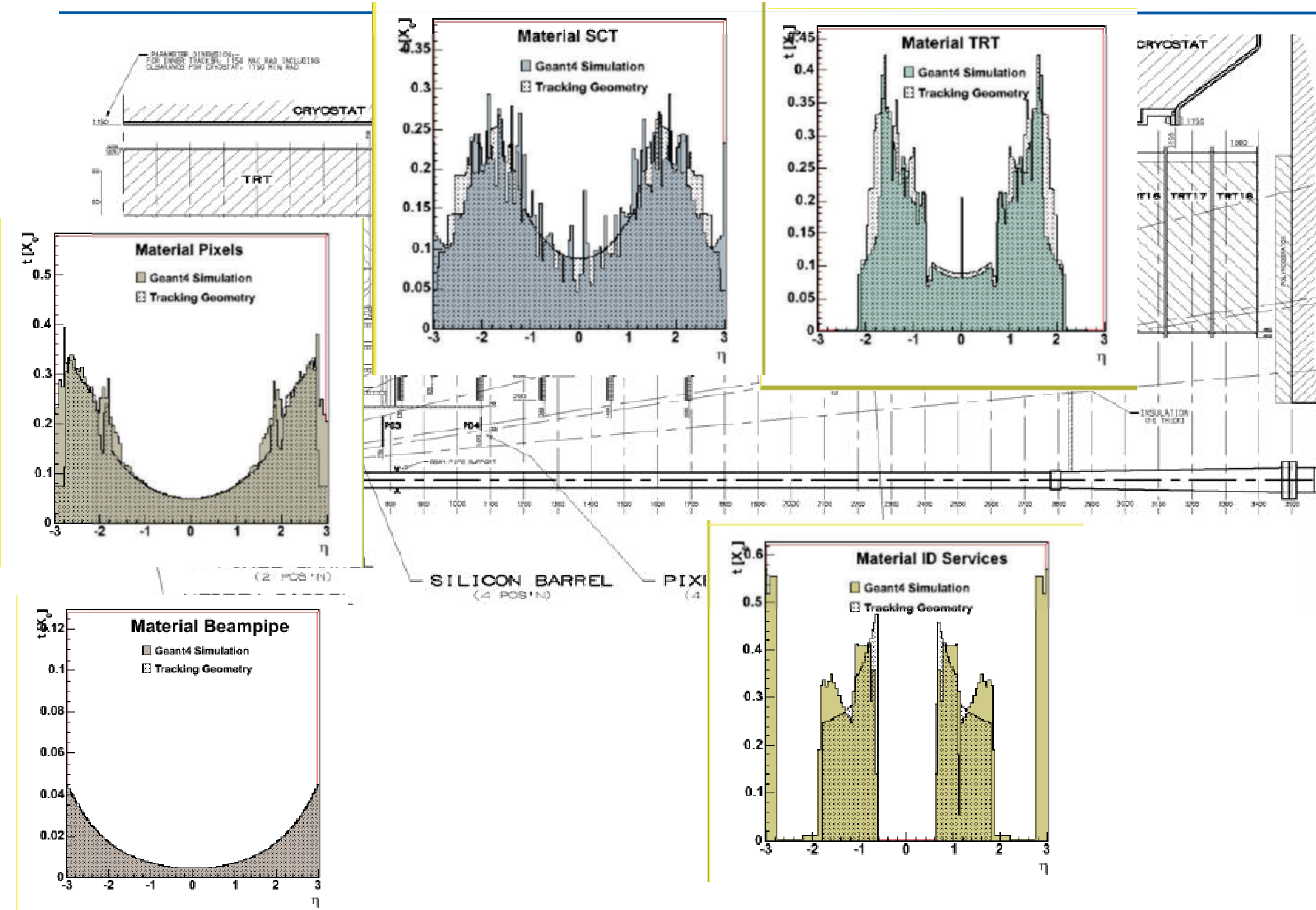
$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_{mess}}{0.3BL^2} p_T \times 7.56$$
$$= 3.6 \times 10^{-4} \; p_T/(\text{in GeV})$$

at p_T = 500 GeV

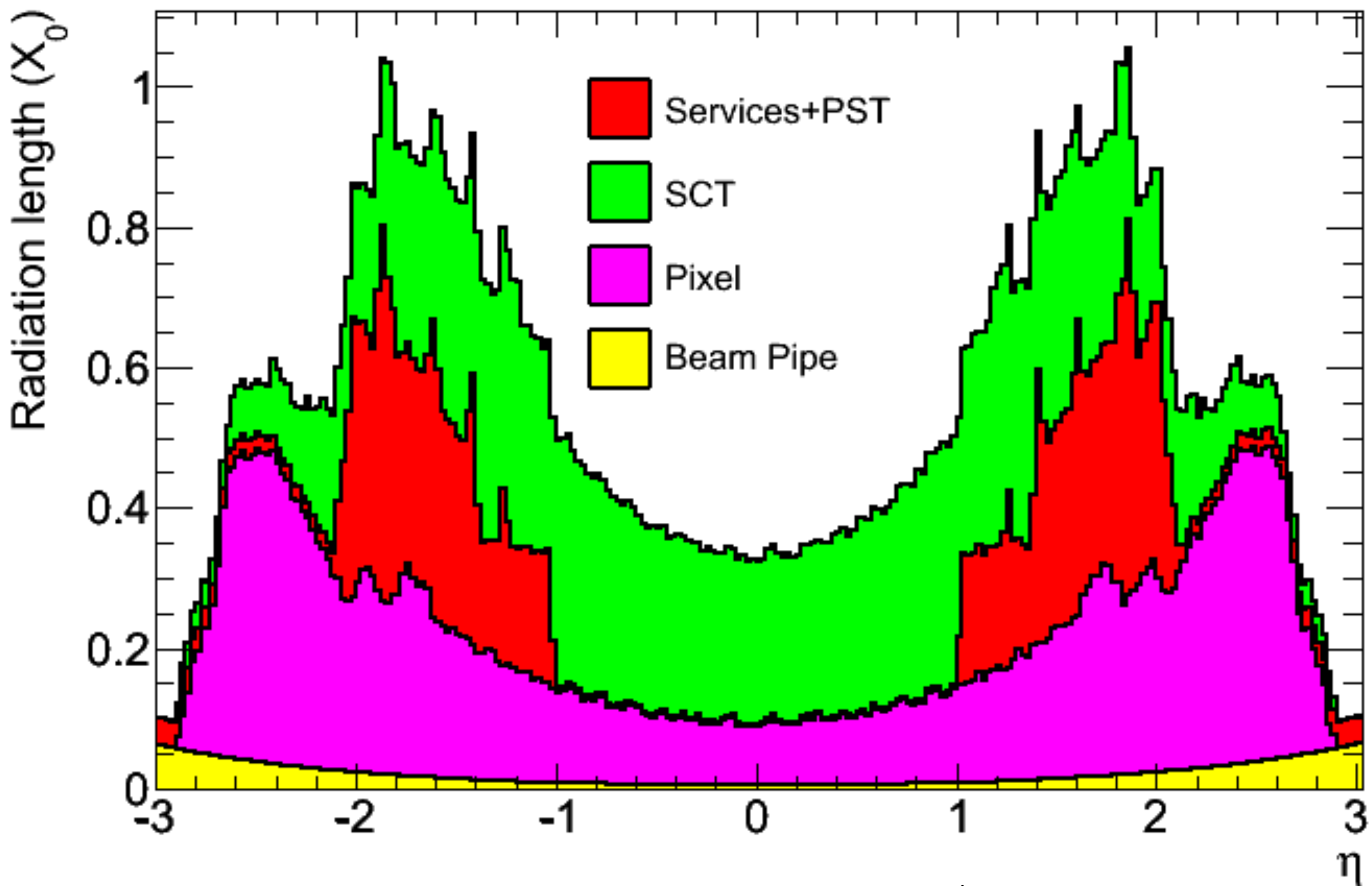
$$\frac{\mathrm{dp}}{\mathrm{p}} - 18\%$$



… why is the resolution not constant ?



Material in ATLAS



$$\eta = -\ln \tan\theta/2$$

Momentum Resolution with M.S. in ATLAS

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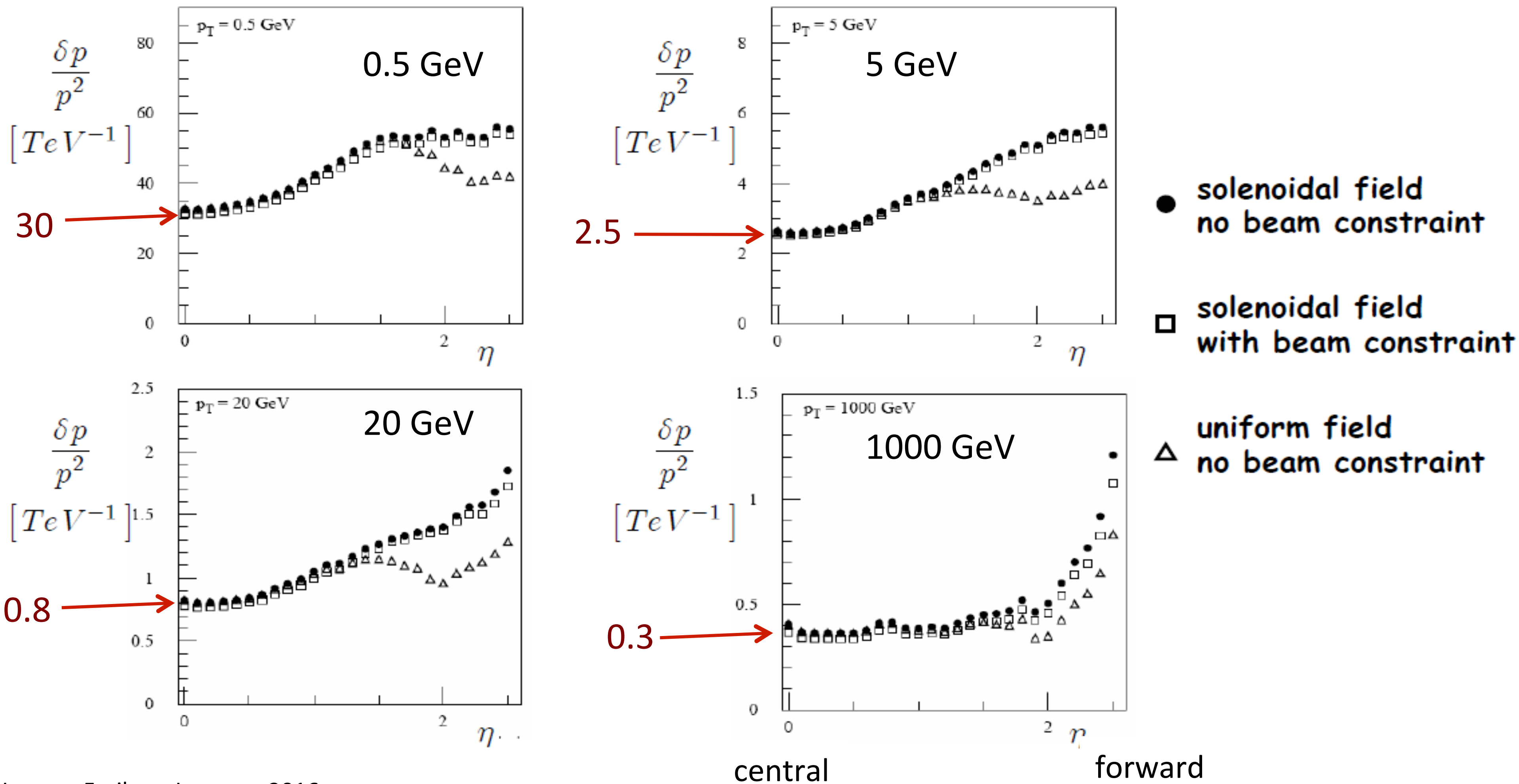
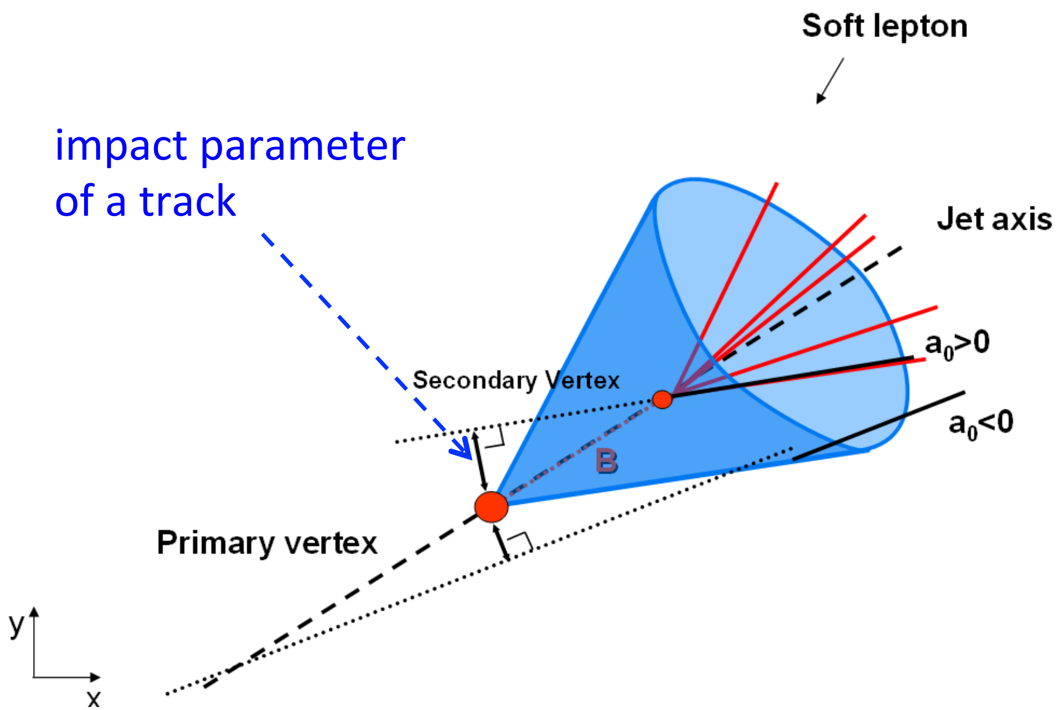
$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\text{mess}} = \frac{p_T}{0.3|z|}\frac{\sigma_{\text{mess}}}{L^2 B}\sqrt{\frac{720}{N+4}} \qquad \frac{\sigma_{p_T}}{p_T} = \sqrt{\left(\frac{\sigma_{p_T}}{p_T}\right)^2_{\text{mess}} + \left(\frac{\sigma_{p_T}}{p_T}\right)^2_{\text{streu}}}$$

as a function of η



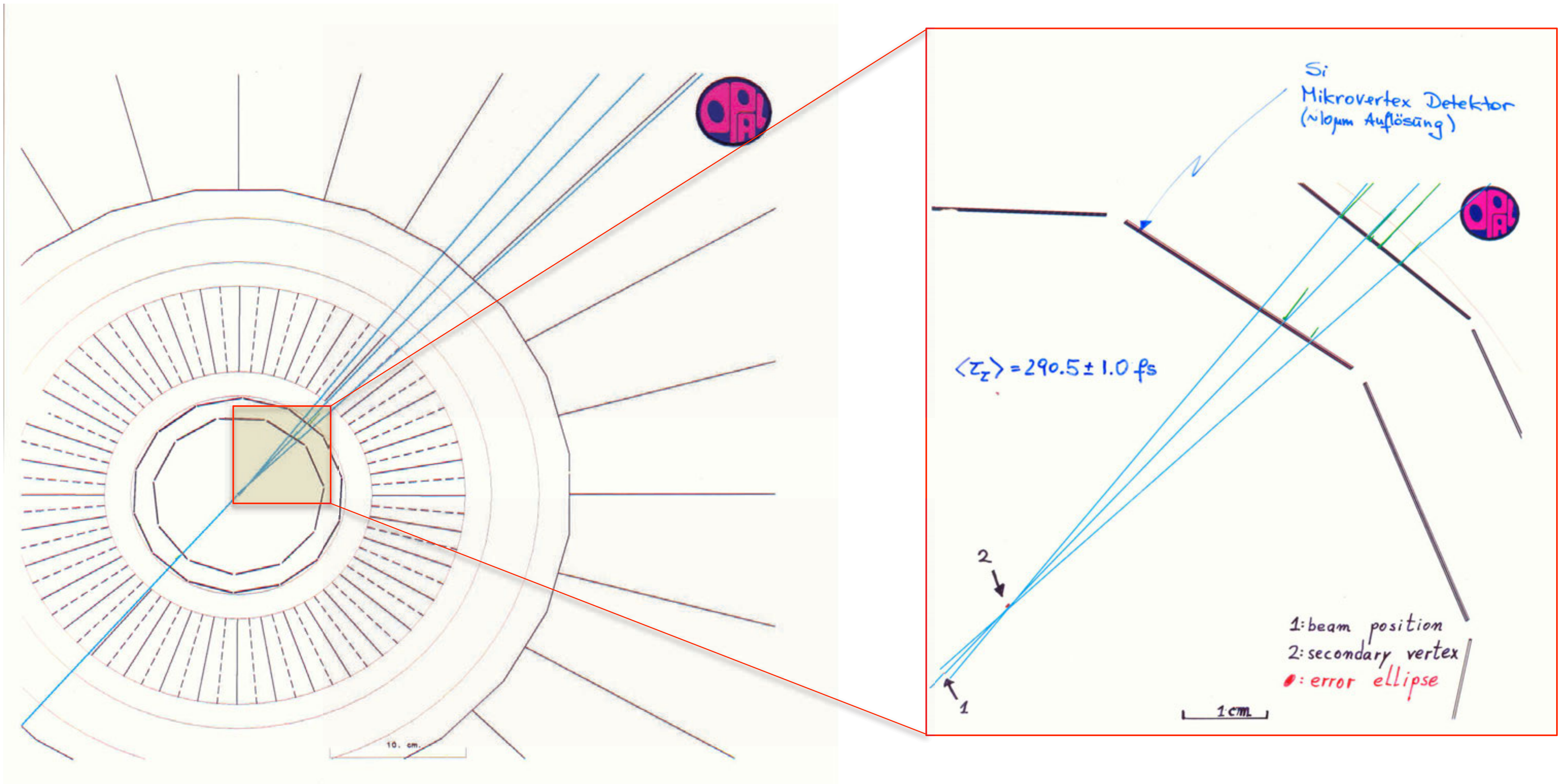
Vertex detection and measurement

vertex detectors measure the "impact parameter"

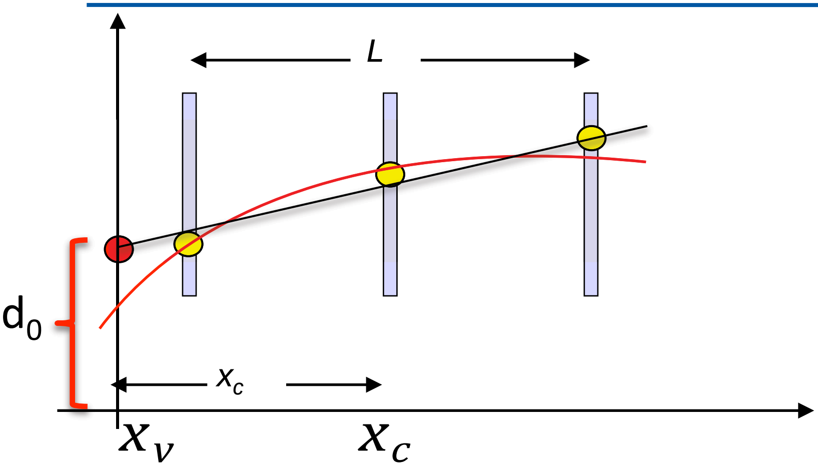


skip to end?

Extrapolation to the (primary) vertex



Impact parameter resolution



must extrapolate to vertex
either using

$$y = a + bx$$

or $$y = a + bx + \frac{1}{2}cx^2$$ i.e. with curvature

$$d_0 \approx y - y_0$$

extrapolation dist. / meas. distance

$$r := \frac{x_0}{L}$$

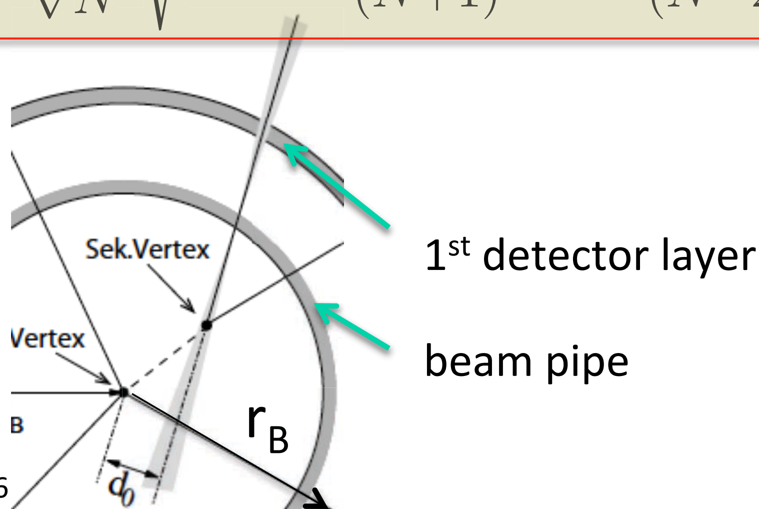
linear

$$\sigma_{d_0} = \sigma_y = \sqrt{\sigma_a^2 + x_0^2\,\sigma_b^2} = \frac{\sigma_{\text{mess}}}{\sqrt{N}}\sqrt{1 + \frac{12(N-1)}{(N+1)}r^2}$$

parabolic

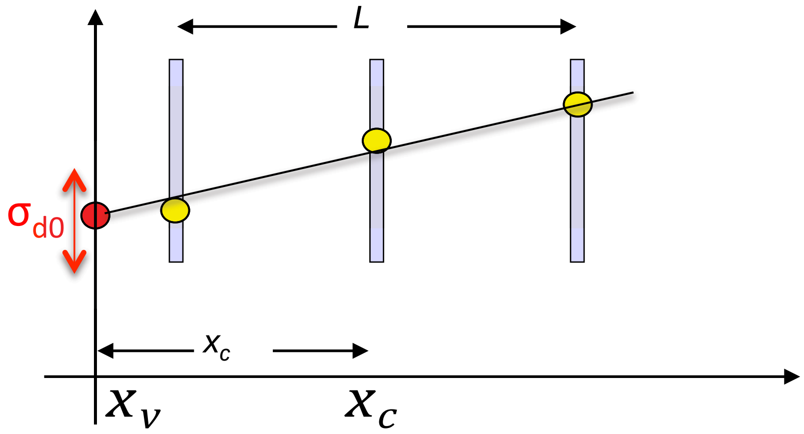
$$\sigma_{d_0} = \frac{\sigma_{\text{mess}}}{\sqrt{N}}\sqrt{1 + r^2\frac{12(N-1)}{(N+1)} + r^4\frac{180(N-1)^3}{(N-2)(N+1)(N+2)} + r^2\frac{30N^2}{(N-2)(N+2)}}$$

multiple scattering:



$$\sigma_{d_0}^{MS} = \theta_0\, r_B$$

What we conclude …



here only linear case

$$\sigma_{d_0} = \frac{\sigma_{\text{mess}}}{\sqrt{N}} \sqrt{1 + \frac{12(N-1)}{(N+1)} \frac{x^2}{L^2}} \oplus \theta_0 r_B$$

We should have

- small measurement errors σ_{mess}
- large lever arm L
- place first plane as near as possible to the production point: small x

Increasing the number of points also improves the resolution on d_0 but only as $1/\sqrt{N}$

The technology most often used is
Si - detectors

PRO – high resolution σ_{mess} ~ 10 µm

CON - expensive
- small N
- small L
- small X_0 => large θ_0

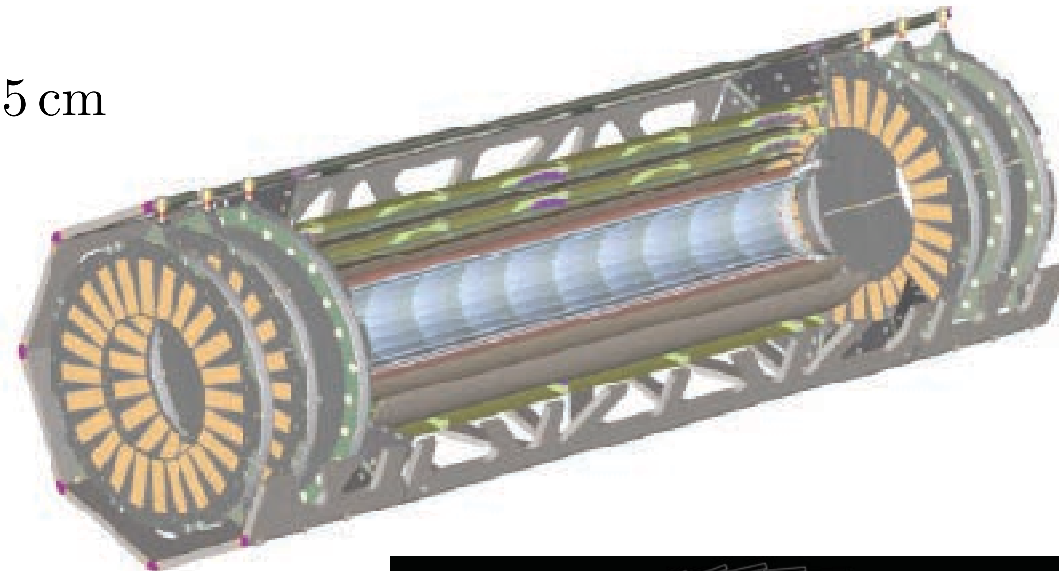
Example: ATLAS Pixel Detector

$$N = 3, \ \sigma = 10\mu m$$

$$x_1 = 5.05\mathrm{cm}, \ \mathrm{x}_2 = 8.85\,\mathrm{cm}, \ \mathrm{x}_3 = 12.25\,\mathrm{cm}$$

$$L = 7.3\mathrm{cm}, \ r = x_2/L = 1.22$$

$$\sqrt{1 + \frac{12(N-1)}{(N+1)} r^2} = 3.15$$



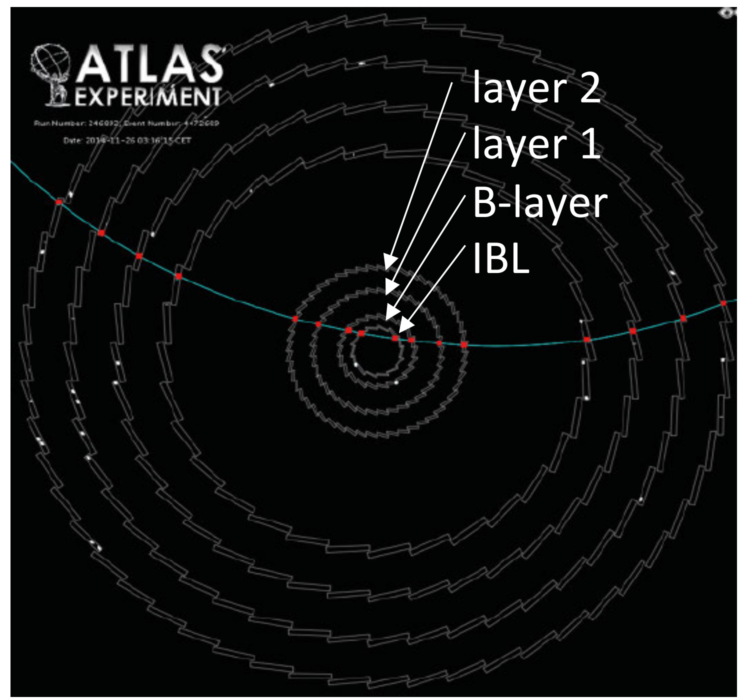
Impact parameter resolution

$$\boxed{\sigma_{d_0} = 18.2\,\mu\mathrm{m}}$$ (linear, i.e. no field)

$$\sigma_{d_0} = 45.5\,\mu\mathrm{m}$$ (extrapolation with B-field)

Note

- if curvature is used for extrapolation with N=3 the error on d_0 gets larger by a factor ~2.5.
- however, curvature is measured by the entire inner detector => error on d_0 similar to linear case



With new IBL (N=4, x_0 = 3.55 cm)

$$\sigma_{d_0} = 12.5\,\mu\mathrm{m}$$

Tracker resolutions with Multiple Scattering

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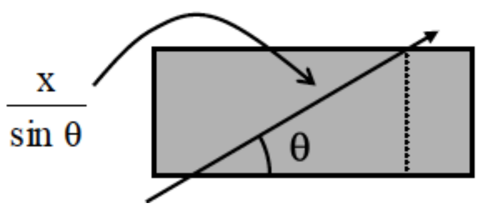
- For low momentum tracks the momentum resolution and the impact parameter resolution are dominated by multiple scattering
- The amount of material actually traversed by the particles depends on the polar angle



- the momentum resolution tends to

$$\frac{\sigma_p}{p^2} \rightarrow k_p \frac{\sqrt{x/X_0}}{p\sqrt{\sin\theta}}$$

- the impact parameter resolution tends to

$$\sigma_{d_0} \rightarrow k_{d_0} \frac{\sqrt{x/X_0}}{p\sqrt{\sin\theta}}$$

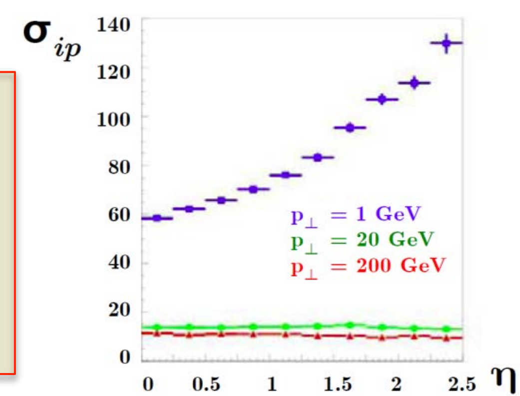
- Since the MS error and the point measurement error are independent of each other, the total error is the sum in quadrature of the 2 terms with and w/o MS
- For the ATLAS detector Monte Carlo studies have shown that the resolutions on momentum and impact parameter can be parametrized as

$$\frac{\sigma_{p_T}}{p_T^2} = 0.00036 \oplus \frac{0.013}{p_T\sqrt{\sin\theta}} (\mathrm{GeV})^{-1}$$

or

$$\frac{\sigma_{p_T}}{p_T} = 0.04\%\, p_T \oplus \frac{1.3\%}{\sqrt{\sin\theta}} (\mathrm{GeV})^{-1}$$

$$\sigma_{d_0} = 11\mu m \oplus \frac{73\mu m}{p_T\sqrt{\sin\theta}}$$



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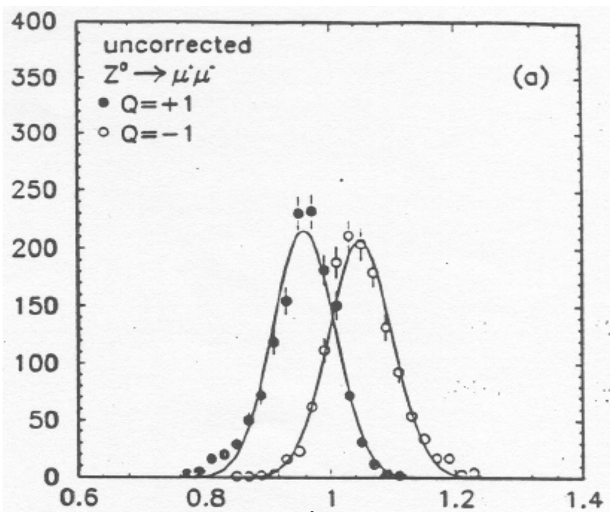
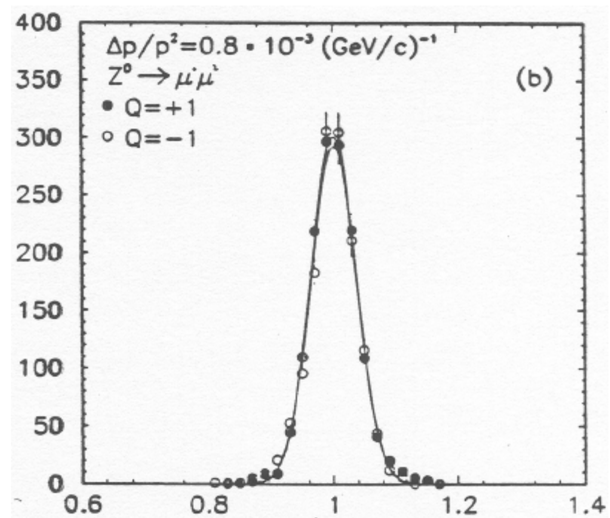
Backup

Systematic Effects: Misalignment

- As an example consider a systematic effect as seen in the ALEPH TPC
- The resolution can be studied using muon pairs produced in $e^+ e^-$ annihilation at the Z^0 peak
- The muons are produced back to back and have exactly half the c.m. energy
- The plot shows the momentum reconstructed separately for positive and negative muons
- As the plot clearly shows, the error is quite large
- To account for this error $\delta \sim 1\ mm$ needed !
- => Magnetic field distortion !
- A correction procedure is essential !
- The following plot shows the same distribution after proper magnetic distortion corrections are applied



E_{beam}/p_{track}



E_{beam}/p_{track}

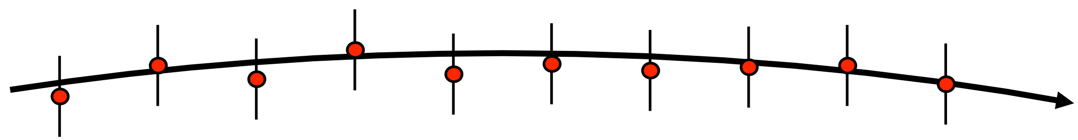
Kalman Filter

- Applications of Kalman Filter:
  - navigation
  - radar tracking
  - sonar ranging
  - satellite orbit computation
  - stock prize prediction
- It is used in all sort of fields
  - Eagle landed on the moon using KF
  - Gyroscopes in airplanes use KF
- Usually the problem is to estimate a "state" of some sort and its uncertainty
  - location and velocity of airplane
  - track parameters of charged particles in HEP experiments
- However we do not observe the "state" directly.
- We only observe some measurements from sensors which are noisy:
  - radar tracking
  - charged particle tracking detectors
- As an additional complication the state evolves (in time or in propagation) with its own uncertainties: process stochastic noise
  - deviation from trajectory due to random wind (radar)
  - multiple scattering (HEP)
- In case of tracking in HEP instead of time we consider the evolution of the track parameter at the discrete subsequent layers, where the detectors perform the measurement

Kalman Filter

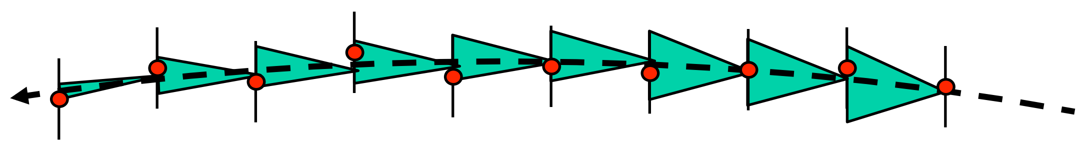
- ❑ It is best to start the filter from the end of the track
- ❑ After all the measurements have been used (filtered) it is possible to build a procedure that
  - uses the (stored) intermediate results of the filter
  - gives the best parameter estimation at any point
- ❑ This is the "smoother" algorithm

production vertex



direction of flight →

production vertex



← direction of filter